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Discharge Response Analysis and Hydrologic Similarity: The Interrelation Between the Geomorphologic IUH and the Storm Characteristics

IGNACIO RODRÍGUEZ-ITURBE, GUSTAVO DEVOTO, AND JUAN B. VALDÉS

Graduate Program in Hydrology and Water Resources, Universidad Simón Bolívar, Caracas, Venezuela

It is shown that the dynamic parameter v of the geomorphologic instantaneous unit hydrograph (IUH) can be taken as the velocity at the peak discharge time for a given rainfall-runoff event in a basin. This transforms the time variant IUH throughout the event into a time invariant IUH in each storm occurrence. The errors which the a priori estimation of the velocity in the IUH may cause in the calculation of the peak and time to peak of the runoff discharge are estimated for different types of basins and storms; the relative weights of the storm characteristics and the drainage network parameters in the prediction procedure are also studied in detail. Drainage basins are defined as hydrologically similar if they have the same IUH when kinematic conditions are kept the same; through the geomorphologic IUH theory it is shown that the controlling parameter in hydrologic similarity is $R_L^{0.43}/L_\Omega$.

INTRODUCTION

General expressions for the instantaneous unit hydrograph (IUH) have been derived as functions of the geomorphologic parameters of a basin. The two most important parameters for a characterization of the IUH are its peak q_p and time to peak t_p, which vary from storm to storm and also during a given storm as functions of the velocity v of the flow at each time interval [*Rodríguez-Iturbe and Valdés,* 1979].

In this paper an attempt is made to investigate the problem of estimating the peak storm discharge Q_p and its time of occurrence T_p from the geomorphologic IUH. Since the velocity of flow is unknown to the designer, and, moreover, since this velocity changes during the life of the discharge response, two problem areas of interest are the following:

1. What kind of errors in Q_p and T_p can be expected from the uncertainty in the variable v?

2. What kind of errors in Q_p and T_p can be expected when assuming a constant value for the velocity of flow during a storm event? What flow velocity should be used at different time intervals during the storm?

To explore the above areas, a fundamental tool is available in the rainfall-runoff models developed in hydrology in the last 15 years. Three of the basins described in the companion paper by *Valdés et al.* [1979], with the same very detailed rainfall-runoff representation, have been used in this investigation to study the problem areas mentioned before.

The geomorphologic IUH derivation clearly shows the two types of features of the response function: those depending only on the catchment characteristics and those which also depend on the rainfall intensity and duration and whose influence is reflected in the parameter v. The interrelation between these two types of features in combination with the rainfall intensity i and duration t_r produces the storm peak discharge parameters Q_p and T_p. The relative importance of the above type of features should shed some light on the nature of flood discharges occurring in natural watersheds and may also further serve the purpose of orienting the study of the much elusive and crucial problem of hydrologic similarity tackled later in this paper.

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FLOW VELOCITY CHARACTERIZATION

A storm of certain intensity constant throughout the duration t_r is assumed to occur with a uniform spatial pattern over a watershed. *Rodríguez-Iturbe and Valdés* [1979] discuss in detail the assumption, justified by several other investigators, that the flow velocity at any given moment during the storm can be taken as more or less constant throughout the basin. The problem lies now in the fact that the above characteristic velocity for the basin as a whole changes throughout time with the result that the IUH not only changes from storm to storm but also during the storm. Since using a time-varying IUH which depends on a time-varying velocity would greatly complicate any inferences about Q_p and T_p, both for practical purposes and for theoretical generalizations, an attempt was made regarding the possibility of describing the time variant velocity with a single characteristic value.

Three of the basins described by *Valdés et al.* [1979], with different geomorphologic and physiographic characteristics, were tested for a range of storms of varying intensities and durations. By means of the detailed rainfall-runoff model implemented for each basin, histograms of the velocity distribution over the whole period of outflow were obtained at the outlet of the basin. Figure 1 shows typical results of the analysis. In all cases the velocity distribution exhibits a relatively small variance in the sense that most of the flow is concentrated in a not too wide interval near the peak velocity. The variance of the distribution diminishes when increasing either the intensity of rainfall, its duration, or both. This will tend to indicate that an attempt to use the geomorphologic IUH with a constant velocity equal to that at the peak discharge is a justifiable approach for the estimation of Q_p and T_p.

The kinematic characteristics of the response process are then assumed to be synthetized in the maximum velocity expected during the outflow. We should now study the goodness of the above assumption. For this purpose the parameters of the IUH, q_p and t_p, for the three basins in question were estimated for a range of flow velocities going from 0.5 to 7 m/s. To simplify the analysis, a triangular form was assumed for the IUH, since it is known that for prediction purposes the form is not important as long as the peak and time to peak of the IUH are correctly estimated [*Henderson,* 1963]. Thus a set

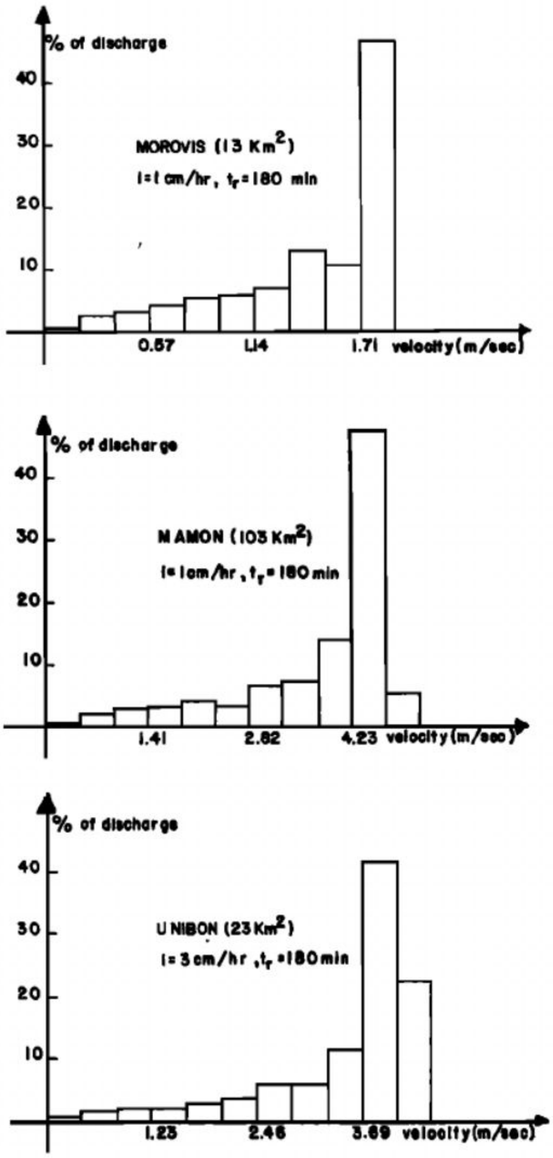

Fig. 1. Examples of velocity histograms obtained by rainfall-runoff modeling at the outlet of different basins.

of triangular IUH's was obtained for each basin, each IUH corresponding to a different flow velocity assumption.

Storms of different intensity and duration were then convoluted with each set of IUH's to obtain the Q_p and T_p predicted by the geomorphologic hydrograph. These Q_p and T_p were afterward compared with the equivalent values obtained when the same storms were simulated with an impervious rainfall-runoff representation of each watershed.

Tables 1–3 give examples of the results obtained when comparing Q_p^* and T_p^* from the rainfall-runoff model with the Q_p and T_p obtained from the use of the geomorphologic IUH with a velocity equal to the velocity given in the rainfall-runoff model at the moment of the peak discharge v_p^*. R_B, R_A, and R_L stand for the Horton parameters: bifurcation, area, and length ratios as defined by *Rodríguez-Iturbe and Valdés* [1979]. L_Ω represents the length of the highest-order stream.

The comparisons are very encouraging, suggesting that the geomorphologic IUH can indeed become a useful tool in many regions where very little or no data are available to the hydrologist. They also suggest that the approach can be an avenue to compare responses arising from different basins when different storms are involved.

Since v_p^* is unknown to the hydrologist, the sensitivity of Q_p and T_p to different assumptions on v_p^* was studied, estimating the percentage errors $|Q_p - Q_p^*|/Q_p^*$ and $|T_p - T_p^*|/T_p^*$ for the same kind of experiment described before. Figures 2–4 show examples of the results obtained in this analysis. It is observed that when v_p^* is above 2 m/s, one may err in the estimation of the velocity to be used in the geomorphologic IUH without producing serious errors in the estimation of the peak discharge and time to peak. Nevertheless, in storms with smaller flow velocities, relatively small errors in the kinematic parameter of the IUH will lead to large errors in the estimation of Q_p and T_p. For design considerations the above observations are somewhat tranquilizing, since one is normally interested in the response of the basin to storms of critical nature which produce large flow velocities. The engineer could estimate Q_p and T_p for different velocity values in the IUH and make a decision type of analysis, accounting for the uncertainty in the velocity.

The above analysis also agrees with the well-known danger of using unit hydrographs derived for storms of different character than the hydrologist wishes to study. Furthermore, it explains in an objective manner the different unit hydrographs that can be obtained in the same basin when performing the estimation in a routine type of numerical scheme applied to different rainfalls and their corresponding hydrographs.

STORM AND DRAINAGE NETWORK CHARACTERISTICS: THEIR RELATIVE WEIGHTS

The assumption of a certain form for the IUH will lead to explicit relationships connecting the storm and drainage network characteristics in regard to their joint action to produce the peak outflow discharge. *Henderson* [1963] has shown that for an IUH of a triangular form the following relationship holds:

$$\frac{Q_p}{Q_e} = \frac{2t_r}{t_b}\left(1 - \frac{t_r}{2t_b}\right) \tag{1}$$

TABLE 1. Examples of the Comparisons for the Mamon Basin of the Peak Discharges and Time to Peak Obtained From the Geomorphologic IUH With the Equivalent Values Q_p^*, T_p^* Obtained by Rainfall-Runoff Experiments

	v, m/s	T_p, min	Q_p, m³/s
	i = 1 cm/h, t_r = 3 hours		
v_p^* = 4.6	4.0	186	281
Q_p^* = 271	4.5	180	284
T_p^* = 180	5.0	164	285
	i = 1 cm/h, t_r = 2 hours		
v_p^* = 4.4	4.0	141	236
Q_p^* = 238	4.5	135	252
T_p^* = 130	5.0	130	265
	i = 1 cm/h, t_r = 1 hour		
v_p^* = 3.3	3.0	113	112
Q_p^* = 113	3.5	103	128
T_p^* = 110	4.0	96	143
	i = 1 cm/h, t_r = 0.5 hour		
v_p^* = 2.2	2.0	125	40
Q_p^* = 40	2.5	104	50
T_p^* = 130	3.0	91	59

Where R_A = 4.5, R_B = 3.5, R_L = 2.1, sixth order, L_Ω = 12.25 km, and A = 103 km².

TABLE 2. Examples of the Comparisons for the Unibon Basin of the Peak Discharges and Time to Peak Obtained From the Geomorphologic IUH with the Equivalent Values Q_p^*, T_p^* Obtained by Rainfall-Runoff Experiments

	v, m/s	T_p, min	Q_p, m³/s
	i = 3 cm/h, t_r = 3 hours		
$v_p^* = 4.1$	2.5	185	192
$Q_p^* = 194$	3.0	170	194
$T_p^* = 180$	4.0	127	194
	i = 3 cm/h, t_r = 2 hours		
$v_p^* = 4.0$	3.5	126	188
$Q_p^* = 188$	4.0	122	193
$T_p^* = 120$	4.5	113	194
	i = 3 cm/h, t_r = 1 hour		
$v_p^* = 3.2$	2.5	95	98
$Q_p^* = 109$	3.0	86	113
$T_p^* = 100$	3.5	81	127
	i = 3 cm/h, t_r = 0.5 hour		
$v_p^* = 2.2$	1.5	104	33
$Q_p^* = 44$	2.0	84	43
$T_p^* = 100$	2.5	72	53

Where $R_A = 5.6$, $R_B = 4$, $R_L = 2.8$, third order, $L_\Omega = 8.6$ km, and $A = 23$ km².

where Q_p is the peak discharge produced by a rainfall of constant intensity i and duration t_r, Q_e is the equilibrium discharge iA (A, area of the basin), and t_b is the base time of the IUH.

Since $q_p \times t_b = 2$, one may rewrite (1) as

$$\frac{Q_p}{Q_e} = t_r \cdot q_p \left(1 - \frac{t_r \cdot q_p}{4}\right) \qquad (2)$$

Using

$$q_p = \frac{1.31}{L_\Omega} R_L^{0.43} v \qquad (3)$$

[*Rodríguez-Iturbe and Valdés*, 1979], one may estimate the ratio Q_p/Q_e for different combinations of t_r, v, L_Ω, and R_L.

TABLE 3. Examples of the Comparisons for the Morovis Basin of the Peak Discharges and Time to Peak Obtained From the Geomorphologic IUH with the Equivalent Values Q_p^*, T_p^* Obtained by Rainfall-Runoff Experiments

	v, m/s	T_p, min	Q_p, m³/s
	i = 3 cm/h, t_r = 3 hours		
$v_p^* = 3.0$	2.5	182	112
$Q_p^* = 112$	3.0	159	112
$T_p^* = 180$	3.5	136	112
	i = 3 cm/h, t_r = 2 hours		
$v_p^* = 2.9$	2.5	136	97
$Q_p^* = 103$	3.0	129	105
$T_p^* = 130$	3.5	124	110
	i = 3 cm/h, t_r = 1 hours		
$v_p^* = 2.3$	2.0	100	49
$Q_p^* = 55$	2.5	89	59
$T_p^* = 90$	3.0	82	68
	i = 3 cm/h, t_r = 0.5 hour		
$v_p^* = 1.5$	1.0	130	14
$Q_p^* = 21$	1.5	95	20
$T_p^* = 100$	2.0	77	26

Where $R_A = 5$, $R_B = 3.2$, $R_L = 2.7$, third order, $L_\Omega = 8$ km, and $A = 13$ km².

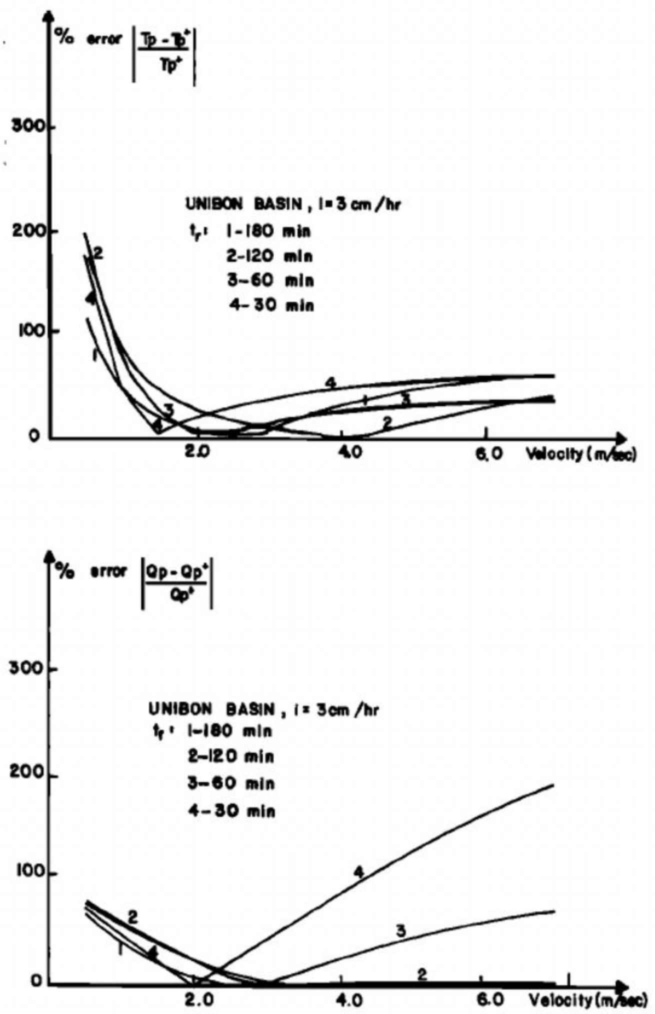

Fig. 2. Examples of the errors made in the estimation of Q_p and T_p for several storms in the Unibon basin when different velocities are used in the geomorphologic IUH.

Figures 5 and 6 show examples of the above dependence. It is interesting to notice the following:

1. When v increases, holding t_r constant, there exists a considerable increase of the ratio Q_p/Q_e. This means that storms of the same duration and intensity produce different peak responses depending on the flow rates in the drainage network previous to the storm.

2. Any correlation about peak responses of storms of different duration should take into account the flow velocity parameter, or the analysis will produce either misleading results or lack of correlation.

3. The importance of the Horton length ratio R_L depends heavily on the characteristics of the storm and the internal scale parameter L_Ω. Thus one observes that in all graphs it is below a certain value of L_Ω that R_L starts playing an important role. For large values of L_Ω the relation Q_p/Q_e changes very little through the whole range of R_L; this region is followed by a boundary after which R_L seriously influences the ratio Q_p/Q_e. For the same duration of rainfall t_r an increase in velocity will shift lines of the same L_Ω from the region where R_L does not have an influence to the region where R_L plays an important role. The same observation is valid when the velocity is held constant but the rainfall duration increases.

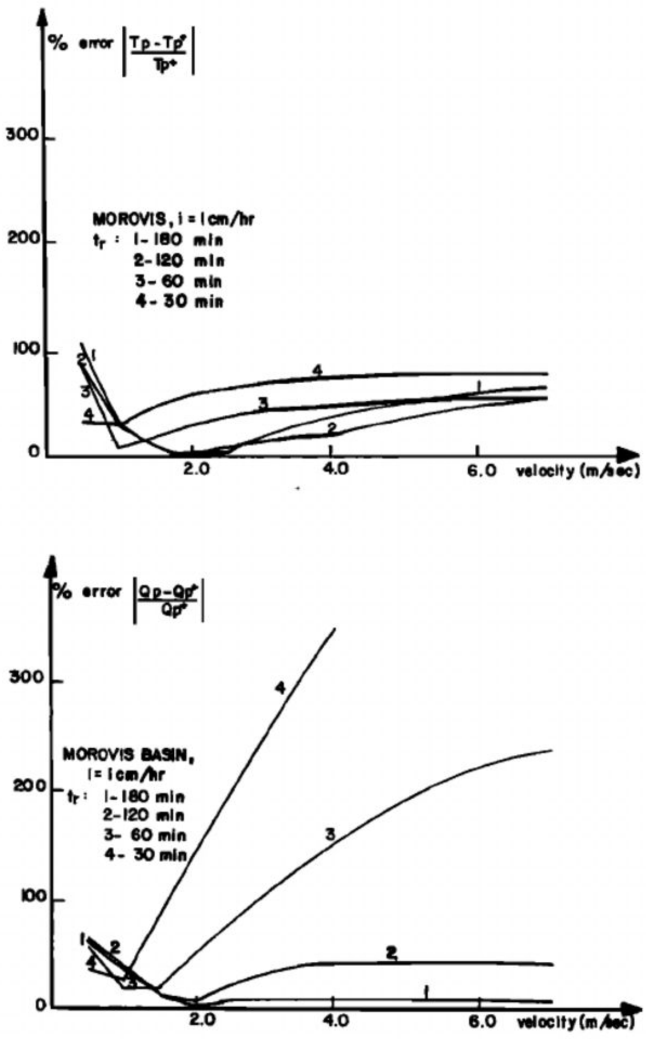

Fig. 3. Examples of the errors made in the estimation of Q_p and T_p for several storms in the Morovis basin when different velocities are used in the geomorphologic IUH.

4. The internal scale parameter L_Ω is a crucial factor in the variability of the ratio Q_p/Q_e.

Hydrologic Similarity

Two basins will be defined as similar from a hydrologic point of view if when excited by a unit impulse of rainfall, their response is the same, given that the flow velocities are kept the same in both cases. Two comments should be made regarding the above definition. The first one concerns the fact that we have defined hydrologic similarity through the IUH and not through comparisons of forms, areas, slopes, etc. We feel the most representative feature of the behavior of a system is its response function, and any attempt addressed to compare the discharge behavior of different basins through a general framework should be based on the structure of an IUH.

The second comment concerns the fact that kinematic conditions are kept the same in both basins. The theory of the geomorphologic IUH [*Rodríguez-Iturbe and Valdés*, 1979] and the experiments presented by *Valdés et al.* [1979] show the flow velocity v as the kinematic parameter which controls both the peak q_p and time to peak t_p of the IUH. Thus the IUH depends, besides on the input itself, on the antecedent conditions of the basin, namely, on the flow conditions in the drainage network at the moment we impose the unit impulse of rainfall. Since these conditions are independent of the event under study, the response to a unit input, we are forced to establish that they are identical in order to try a general study of the problem.

Our first goal in this section is the design of two basins which, although different in many parameters, especially in the appearance of their drainage network, will be hydrologically similar according to the theory of the geomorphologic IUH. Since we wish identical q_p and t_p, we will need the same $IR = q_p \cdot t_p$, which *Rodriguez-Iturbe and Valdés* [1979] have shown to be independent of velocity and size and equal to

$$IR = 0.58\,(R_B/R_A)^{0.55} \tag{4}$$

The same IR was chosen for the two basins, and then two different values of R_A were picked. Through (4) the corre-

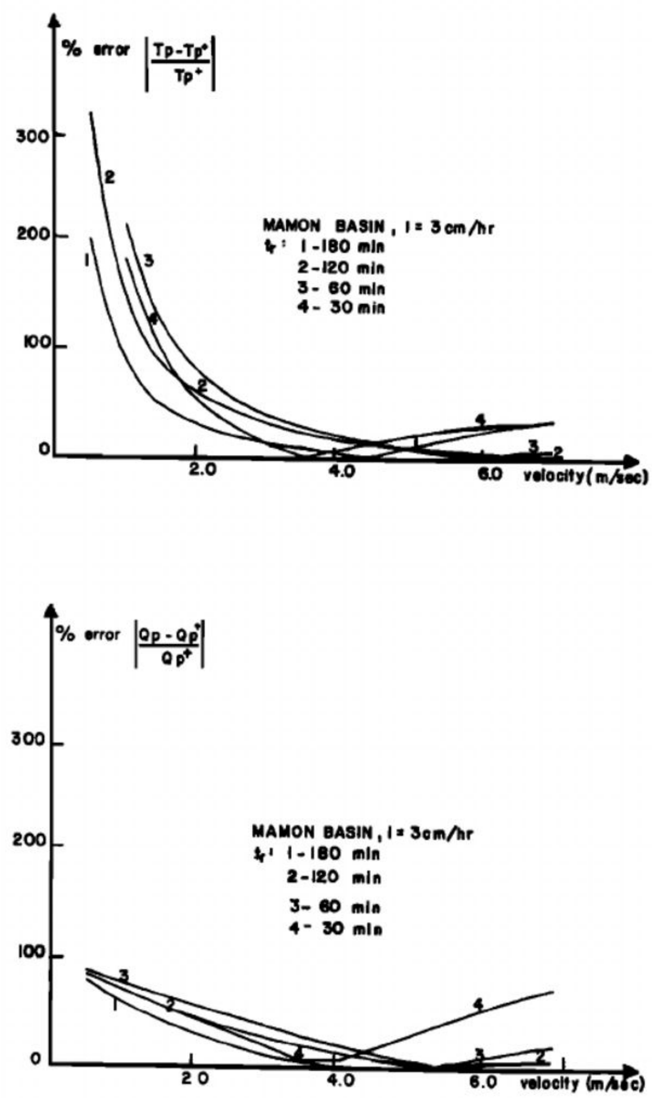

Fig. 4. Examples of the errors made in the estimation of Q_p and T_p for several storms in the Mamon Basin when different velocities are used in the geomorphologic IUH.

sponding values of R_B were established. Assuming $\Omega = 3$ for both basins, N_1 and N_2 are obtained. In this experiment the same area is given to both basins (100 km²), and through R_A we calculate $\bar{A}_1$ and $\bar{A}_2$. Regarding the choice of the $\bar{L}_\omega$ we proceeded in the following manner: $\bar{L}_1$ was chosen through the functional form

$$\bar{L}_1 = \eta \bar{A}_1^{1/\beta} \tag{5}$$

For this, nine different basins were analyzed to obtain values of η and β with some resemblance to reality. The only purpose of (5) is to avoid a choice of $\bar{L}_1$ totally independent of $\bar{A}_1$, which has already been chosen. Different R_L were used but in such a manner that the ratio $R_L^{0.43}/L_\Omega$ in the expression of q_p remains the same in both basins.

The channels of order 1 draining to orders 2 and 3 were distributed according, approximately, to the expressions of p_{12} and p_{13} given by *Rodríguez-Iturbe and Valdés* [1979]. For the areas draining directly to each order ω stream, the vector $\theta_i(0)$ of initial state probabilities was used. Table 4 shows a summary of the characteristics of the two basins, and Figures 7 and 8 give the schematic of their drainage network.

The individual lengths of the streams of orders 1 and 2 were obtained by sampling from a log normal distribution with mean $\bar{L}_\omega$ and variance $\bar{L}_\omega/2$; the same procedure was adopted

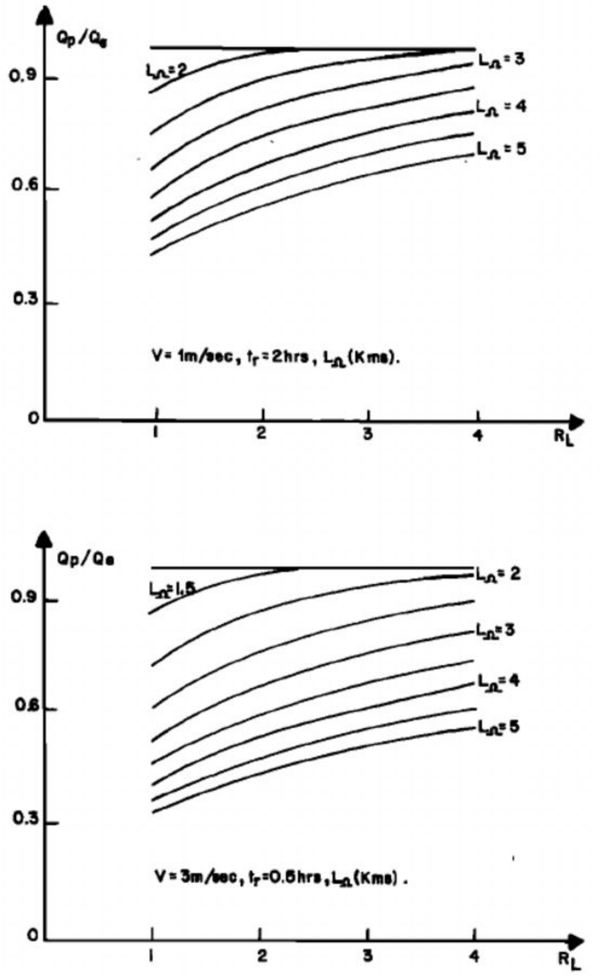

Fig. 5. Relative weights of the storm characteristics and the drainage basin properties int he peak discharge from a rainfall event.

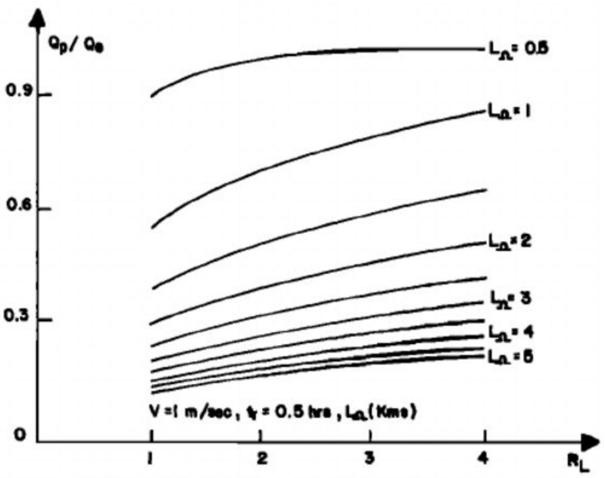

Fig. 6. Relative weights of the storm characteristics and the drainage basin properties in the peak discharge from a rainfall event.

for the individual areas of orders 1 and 2. The slopes were chosen in such a manner that the average fall in streams of different order remains approximately the same as established by the so-called law of average falls found by *Yang* [1971]. Finally, the average overland slopes were chosen equal to 40%, 30%, and 20% for the first, second, and third orders, respectively.

The two basins described above were then represented in a very detailed manner by means of the rainfall-runoff model first described by *Schaake* [1971]. In the representation each link of the network was considered a separate segment of the model, and zero infiltration was assumed.

The first experiment consists of the estimation of the IUH for both basins using three different storms. The hyetograph for the three storms is shown in Figure 9, where $i_0 = 1$, 3, and 6 cm/h. The procedure is as the one described by *Valdés et al.* [1979], the IUH being estimated from the derivative of the hydrograph obtained either in the recession or the rising limb (they are equal) corresponding to the incremental pulse of the rainfall intensity. The kinematic conditions in both basins are kept the same because, having the same total area and the same average width of channel, at equilibrium time they have the same flow velocity.

TABLE 4. Geomorphological Characteristics of Basins 1 and 2 Used in the Experiments of Hydrologic Similarity

	Basin 1	Basin 2
Ω	3	3
R_A	4	6
R_B	3	4.5
N_1	9	21
N_2	3	5
$A = A_3$, km²	100	100
$\bar{A}_1$, km²	6.25	2.77
$\bar{A}_2$, km²	25	16.66
$\bar{L}_1$, km	4.58	2.87
R_L	1.5	2
$\bar{L}_2$, km	6.88	5.73
$\bar{L}_3$, km	10.32	11.46
θ_1 (0)	0.5625	0.5652
θ_2 (0)	0.2625	0.3242
θ_3 (0)	0.1750	0.1143
p_{12}	0.8667	0.7569
p_{13}	0.1333	0.2431

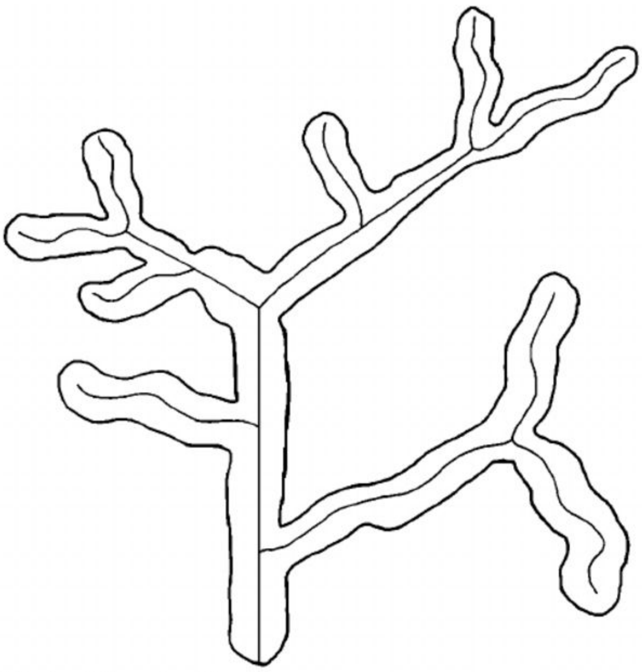

Fig. 7. Schematic of basin 1 used in the experiments of hydrologic similarity.

Figure 10 shows typical examples of the IUH obtained in this experiment; it is observed that the agreement is very good in all cases. This we feel is not something that could be expected beforehand; one basin has 51 segments, and the other 123 segments in the rainfall-runoff representation. The number of different basins of order 3 and of the same external scale area, which could be constructed fulfilling Horton's laws, is practically infinite, and nevertheless, two particular networks were built up which, although looking quite different to the naked eye, were predicted to have the same IUH.

The second experiment consisted of the comparison of two pairs of basins where the scale of lengths is 1:2. The two watersheds designed in the first experiment have areas of 100 km²; in each of them all stream lengths were multiplied first by a factor of 3 in order to have two basins of 300 km². This, nevertheless, does not change the parameters R_B, R_L, R_A, p_{ij}, and $\theta_i(0)$, which, as well as the connectivity of the network, remain the same as before. The factor of 3 was applied in order

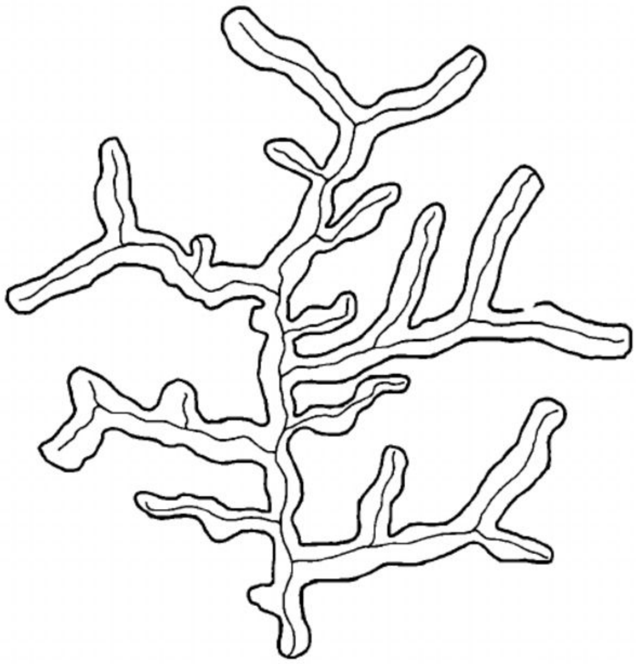

Fig. 8. Schematic of basin 2 used in the experiments of hydrologic similarity.

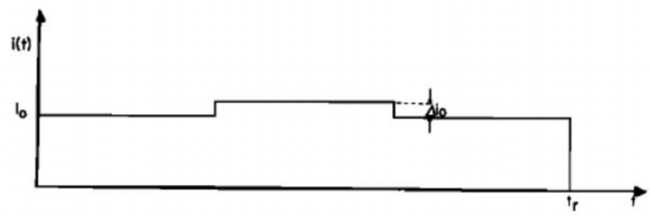

Fig. 9. General hyetograph imposed upon the rainfall-runoff model to obtain the IUH to be compared with the geomorphologic theory ($\Delta i_0 = 0.10_{i0}$).

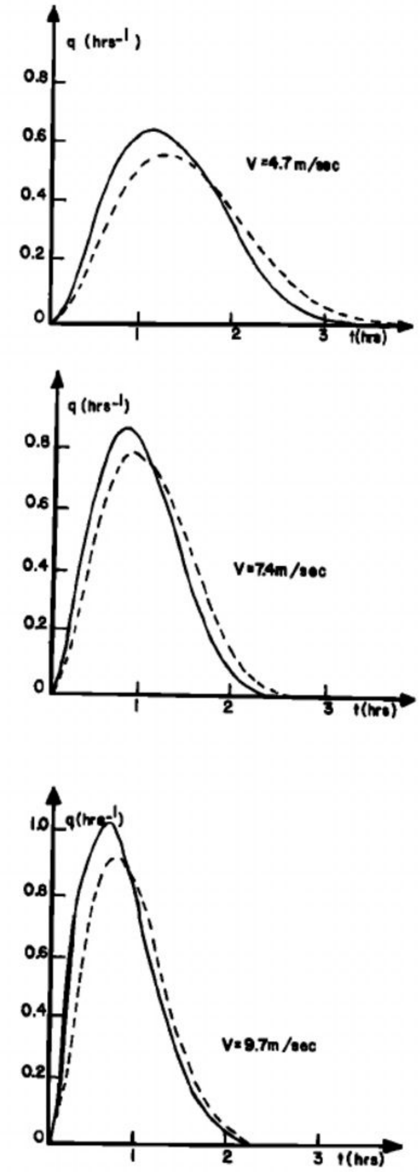

Fig. 10. IUH's of basin 1 (dashed lines) and 2 (solid lines) for different kinematic conditions.

not to have very small basins when the scale of lengths is reduced by 2, leading to a reduction by 4 in the area. Thus we now have two pairs of basins, one pair of 300 km² and the other pair of 75 km². We will compare each one of the larger ones with its corresponding reduction in scale 1:2. In order to have the same kinematic conditions the rainfall-runoff representation of the larger basins was subjected to an intensity of rainfall 2 times larger than that of the smaller basins in each of the runs carried out. To avoid very large and unrealistic velocities, the experiments are now carried out with rainfall intensities ranging from 0.2 to 1.0 cm/h, and, as before, after reaching equilibrium an incremental pulse in the rainfall intensity is introduced equal to 10% of the base intensity.

Figure 11 is typical of the results of the experiment. The theory of the geomorphologic IUH indicates that in the corresponding pairs of basins the peak of the smaller one should be double that of the larger one and the time to peak should be one half owing to the fact that they have the same R_L and the same R_A/R_B and the length L_Ω has been reduced by a factor of 2. The results from the rainfall-runoff model confirm extremely well this prediction.

One important point to bring up at this moment is the observation that for the same kinematic conditions the effect of size or scale in the IUH does not come through the area of the basin but rather through the length of the streams reflected in the parameter L_Ω. Neither the expression of q_p nor that of t_p contains the area; they involve R_L, R_A/R_B, and L_Ω in addition to the flow velocity v. Moreover, they are general expressions for watersheds of any order Ω. This will suggest that basins of very different areas, number of sources, orders, etc. may indeed have the same IUH as long as $R_L^{0.43}/L_\Omega$, which controls q_p, and $L_\Omega \cdot (R_B/R_A)^{0.55} R_L^{-0.38}$, which controls t_p, remain the same.

A third experiment was designed to verify the above observation. We started with basin 1 used in the first experiment (shown in Figure 7) and made this basin grow to a fourth-or-

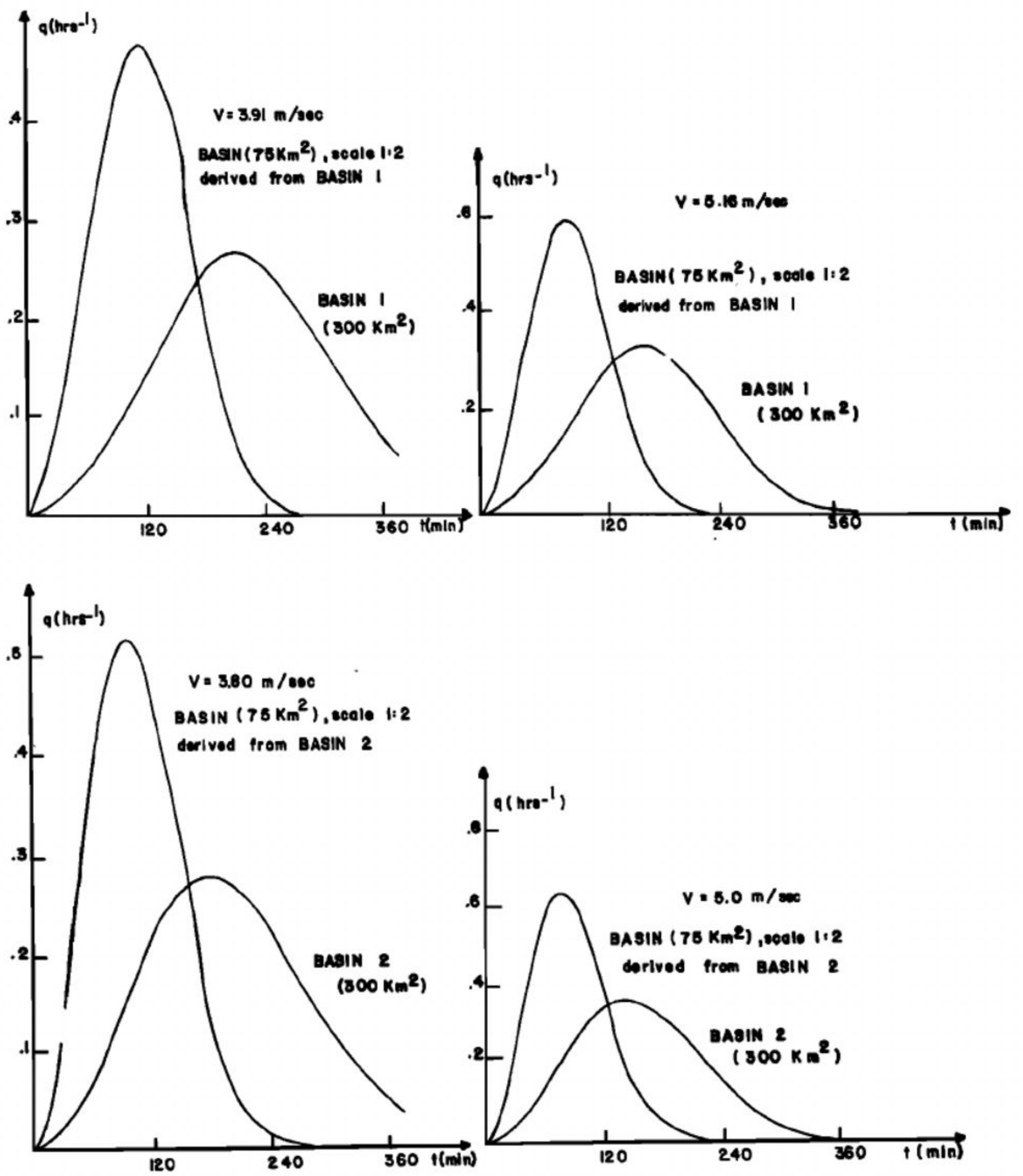

Fig. 11. Examples of the IUH's obtained in the second experiment of hydrologic similarity. The theory suggests that the peak and time to peak of the corresponding pair of hydrographs should be in a 1:2 relationship.

der basin in the following manner: R_B, R_L, R_A, and L_Ω remain the same as in basin 1. Since the new basin is of order 4, we have 27 first-order streams with average length of 3.06 km and $\bar{A}_1 = 2.7$ km^2. The transition probabilities for a fourth-order basin are given by

$$p_{12} = \frac{2}{R_B} + \frac{(2R_B - 1)(R_B^2 - 2R_B)}{R_B^2(2R_B - 1) + R_B(R_B^2 - 1) + (R_B^2 - 1)(R_B - 1)} \quad (6)$$

$$p_{13} = \frac{(R_B^2 - 1)(R_B - 1)}{R_B^2(2R_B - 1) + R_B(R_B^2 - 1) + (R_B^2 - 1)(R_B - 1)} \quad (7)$$

$$p_{14} = \frac{(R_B^2 - 1)(R_B - 1)(R_B - 2)}{R_B^3(2R_B - 1) + R_B^2(R_B^2 - 1) + R_B(R_B^2 - 1)(R_B - 1)} \quad (8)$$

$$p_{23} = \frac{R_B - 2}{2R_B - 1} + \frac{2R_B}{R_B^2} \quad (9)$$

$$p_{24} = \frac{(R_B - 1)}{R_B(2R_B - 1)} \cdot (R_B - 2) \quad (10)$$

[*Rodríguez-Iturbe et al.*, 1978].

The streams of orders 1 and 2 were distributed in basin 3 according, approximately, to the above expressions, and the lengths and areas of the individual streams were generated in the same manner as in construction of básins 1 and 2. Figure 12 shows basin 3, which has a total area of 173 km^2, almost twice the area of basin 1. A simple inspection of both basins 1 and 3 shows how different they are in characteristics and appearance; nevertheless, our prediction is that they should have the same IUH. Figure 13 shows an example of the results obtained from the rainfall-runoff model; in this case the same kinematic conditions were maintained through the use of different rainfall intensities in both basins. The agreement is again quite satisfactory.

The above experiments suggest that for the same kinematic conditions, two basins may be considered hydrologically similar when they have identical $R_L^{0.43}/L_\Omega$ and $L_\Omega \cdot (R_B/R_A)^{0.55} R_L^{-0.38}$. Since for the values of R_L encountered in nature we may assume that $R_L^{0.43} \simeq R_L^{0.38}$, two basins will be similar when they have equal values of $(R_L^{0.43}/L_\Omega)$ and (R_B/R_A), where L_Ω should be expressed in kilometers when comparing different values of $R_L^{0.43}/L_\Omega$ [*Rodríguez-Iturbe and Valdés*, 1979]. The relative weights of R_L and L_Ω in the discharge response of a watershed were studied before in this paper.

The role of R_B/R_A can be focused on from a different perspective. The parameter IR (equation (4)) is a constant independent of the internal scale L_Ω and the kinematic condition v. For a triangular IUH, IR represents twice the area of the triangle defined by the origin, the abscissa t_p, and the ordinate q_p. We may think for a moment that our IUH is a two-state system where a drop is found at the outlet either before t_p or after t_p. Thus the most probable shape of the IUH would be as isosceles triangle where $t_p = 0.5t_b$ and $IR = 1$. With $IR = 1$ the entropy of the probability function represented in the IUH will be a maximum, and thus the most probable value of R_B/R_A would be 2.68. Nevertheless, that would be a maximization of entropy without constraints, and it is wrong, since there are physical restrictions in the relative values that R_A and R_B can take in a basin which follows Horton's geomorphological laws.

To start with, for a basin of any order Ω the probability that a drop lands in an area of order 1 is given by

$$\theta_1(0) = N_1\bar{A}_1/A_\Omega = (R_B/R_A)^{\Omega-1} \quad (11)$$

and thus R_A has to be larger than or equal to R_B. Moreover, if $R_A = R_B$, $\theta_1(0) = 1$, and then $\theta_i(0)$ for $i > 1$ should be equal to

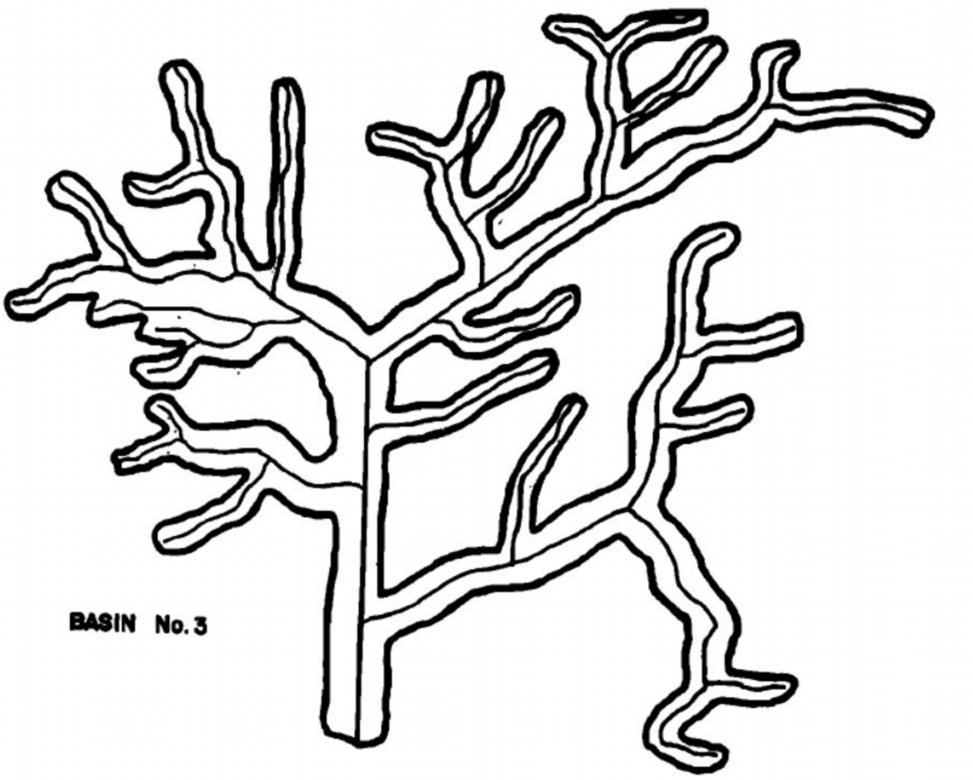

Fig. 12. Schematic of basin 3 used in the third experiment of hydrologic similarity.

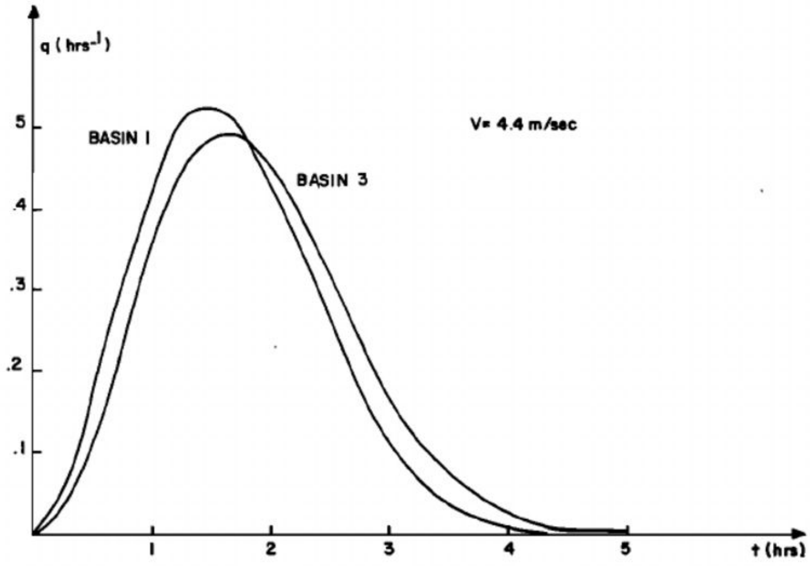

Fig. 13. IUH's of basins 1 and 3 which the theory suggests should have the same response function.

zero. Thus R_A should be larger than R_B; we have checked in the literature, and this was always the case except in some cases where the estimation of R_B was improperly done (for example, the point $N_\Omega = 1$ was not taken into account, and the line log N_ω versus log ω is improperly drawn). The expressions for $\theta_2(0)$, $\theta_3(0)$, etc. impose further restrictions in the values of R_B/R_A for those probabilities to be smaller than 1. A calculation was run by *Rodríguez-Iturbe and Valdés* [1979] for basins of orders 3, 4, and 5, computing the values of the $\theta_i(0)$ for many values of R_B and R_A. They found that the $\theta_i(0)$ were positive for $R_B/R_A \leq 0.80$.

Since the entropy of the IUH is a maximum for the largest R_B/R_A, we will suggest that in nature the most probable value of this ratio is around 0.80 and its range of variation should be quite narrow, since the number of particles (drops) involved in the IUH is very large, making the most probable state almost a certain one for practical purposes.

Figure 14 shows a plot of the values of R_B/R_A found by *Morisawa* [1962], *Woodyer and Brookfield* [1966], and *Valdés et al.* [1979]. The ordinate is the entropy H,

$$H = -\sum p \cdot \log p = -IR/2 \log IR/2 - (1 - IR/2) \log (1 - IR/2) \quad (12)$$

It is observed that most of the values are between 0.7 and 0.94. Although of course more data should be analyzed and, moreover, the estimation of R_A and R_B should be made under very objective and uniform criteria, the results suggest that indeed R_B/R_A is pretty constant in nature.

The work of *Shreve* [1966] and more recently of *Smart* [1972] gives a theoretical foundation to Horton's findings. Shreve shows that the most probable network configurations have R_B in the range encountered in nature; nevertheless, to our knowledge no explanation has been offered for the values of R_A displayed by natural basins. We suggest that our analysis explains the value of the ratio R_B/R_A and thus the values of R_A.

The above analysis will also suggest that, given the small range of variation which is shown by R_B/R_A, the controlling parameter in hydrologic similarity is

$$I = R_L^{0.43}/L_\Omega \quad (13)$$

for basins under the same kinematic conditions.

The ratio $R_b/R_A = 0.8$ leads to a value of IR in (4) of 0.513; this value has an interesting implication which is now explored. The convolution of an uniform rainfall with a triangular IUH yields an expression of the time to peak T_p for the outflow hydrograph of the type

$$T_p = t_p + t_R \cdot \left(1 - \frac{q_p \cdot t_p}{2}\right) \quad (14)$$

where t_R stands for the rainfall duration [*Rodríguez-Iturbe et al.*, 1978]. Equation (14) is then written as

$$T_p = t_p + 0.75 t_R \quad (15)$$

Snyder [1938] suggested an empirical equation in his synthetic unit hydrograph analysis of the type

$$t_{lR} = t_l + 0.25(t_R - t_r) \quad (16)$$

where t_{lR} is the adjusted lag time for a rainfall excess of t_R (in hours) and t_l is the lag time from the centroid of a unit rainfall

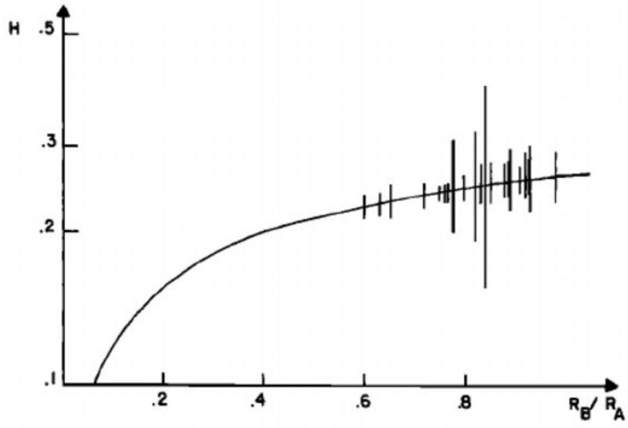

Fig. 14. Entropy of the IUH as a two-state function dependent on R_B/R_A.

excess of duration $t_r = t_l/5.5$ hours to the peak of the unit hydrograph. It is a function of the length of the main stream channel from the outlet to the divide; it also depends on an empirical coefficient and on the length along the main channel to a point opposite the watershed centroid.

Snyders's lag times are measured from the centroid of the rainfall hyetograph to the peak of the hydrograph. If we take an instantaneous rainfall, (16) becomes

$$t_{IR} = t_p + 0.25t_R \tag{17}$$

and this is equivalent to (15) when we measure T_p from the origin, since $T_p = 0.50t_R + t_{IR}$.

Final Comments

A link has been established between the geomorphologic structure and the hydrologic response of a basin. This link brings order to the infinite variety of hydrologic responses encountered in nature, but obviously this research is just a first step in that direction. Since we are dealing with a feedback system in which hydrology is not only a consequence but also a cause in the geomorphology of a basin, the above link is only the first branch of a loop in tying together the geomorphologic and hydrologic structure of natural watersheds. The most probable IUH can be viewed under the constraint of conservation of energy. The IUH itself as a function of velocity can be transformed to a representation of the kinetic energy produced by a unit input of rainfall imposed upon the basin. Since this kinetic energy, now related to geomorphologic parameters, is the result of a potential energy which can also be expressed as function of geomorphology, we want to suggest that the second branch of the loop between hydrology and geomorphology may also be explored under this framework. An example of this mutual influence is the explanation for the values of R_A suggested in this paper using the geomorphologic IUH. Research in progress at Universidad Simón Bolívar is addressed to the explicit establishment of the second branch of the loop. We wish to close by emphasizing that the value of the controlled experiments in the analysis of hydrologic similarity is only of a relative character; until the second branch of the loop is established, or at least explored in a quantitative manner, the artificial buildup of basins for purposes of comparing their hydrologic response may conduce to forms and proportions unacceptable to nature and which may still fulfill Horton's laws. This again comes to the point that this research and its companion papers are only a first step in the linking of geomorphologic structures and hydrologic response and that the satisfactory understanding of one branch of the loop is at least partially dependent on the understanding of the other branch.

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