A Rainfall-Runoff Analysis of the Geomorphologic IUH

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The instantaneous unit hydrograph (IUH) derived from the geomorphologic characteristics of a basin and a timing component, the velocity of the discharge, was presented by Rodríguez-Iturbe and Valdés (1979). To analyze this geomorphologic IUH in real world basins, a study was carried out on several basins in Venezuela and Puerto Rico. The geomorphologic IUH for each basin was compared with the IUH's derived from the discharge hydrograph produced by a physically based rainfall-runoff model of the same basins. The effects that the nonlinearities of the rainfall-runoff model have on the derivation of the IUH are analyzed, and further, controlled experiments are carried out in which the IUH is derived under constant velocity conditions. The geomorphologic IUH's and the ones obtained in the experiments are remarkably similar in all the basins analyzed.

INTRODUCTION

The geomorphologic instantaneous unit hydrograph (IUH) developed in the companion paper by *Rodríguez-Iturbe and Valdés* [1979] is used here to represent the surface runoff response function of real world basins. In particular, a physical basis for representing the nonlinearities of the response function of a basin in terms of a time-varying geomorphological IUH is presented. This involves a time-varying velocity parameter which varies during a storm and from storm to storm. Results of numerical experiments with a nonlinear rainfall-runoff model are used to verify and illustrate the theory of the time-varying geomorphologic IUH.

DESCRIPTION OF THE BASINS

Two very different regions are considered in the experiments. The first region is in the northern part of Puerto Rico with very high levels of annual precipitation, whereas the second area is in the central part of Venezuela near the city of Barquisimeto, where the annual levels of precipitations are only one third of those of the Puerto Rico area.

Indio Basin Area (Puerto Rico)

The two basins selected within the Indio basin area are the Morovis basin with an area of 13 km^2 and the Unibon basin with an area of 23 km^2. Both of those rivers are tributaries to the Indio River, which itself drains into the Cibuco River, which goes into the Atlantic Ocean near the city of Vega Baja.

The Morovis and Unibon basins are located between 66°30′ and 66°15′W longitude and 18°15′ and 18°30′N latitude. The general layout of the two basins and their location on the island are shown in Figure 1.

Mean annual precipitation over the basin is between 60 and 90 in. and is relatively uniform throughout the year, as is shown in Figure 1.

Mamon Basin Area (Venezuela)

The Mamon River basin is located in the central part of Venezuela with an area of 103 km^2. The general layout of the basin is shown in Figure 2. The climate is almost of the desert type with a mean annual precipitation of 500 mm (20 in.), which is less than one third of the precipitation on the Puerto Rico area, but it is also distributed uniformly throughout the year. The potential annual evapotranspiration is very high (2000 mm). The entire basin is characterized by low round hills extensively eroded with some alluvial plains among them. The complete lack of a forest cover has made the slopes very unstable and subject to erosion.

Owing to the climate characteristics the streams, although almost dry most of the time, have large floods with very high velocities which cause large erosions in the slopes. Figure 3 shows a stream in the basin having almost vertical banks.

Geomorphologic Analysis of the Basins

Horton's numbers R_A, R_B, and R_L, which quantitatively represent Horton's laws of basin drainage composition, were estimated for all basins using Strahler's ordering procedure. A visual fit was performed in each one of the diagrams. In the case of R_B the lines were drawn through the point $N_\Omega = 1$.

Horton diagrams for all basins are displayed in Figure 4, and Horton numbers for the four basins are given in Table 1. As can be seen from Table 1, all numbers fall within limits usually found in nature. The entire Indio basin, which was not used in this study, had an area of extremely previous limestone, and thus the R_A number did not fall within the usual limits. This is a case of geologic control where Horton's laws do not apply.

The bifurcation ratios R_B of the four basins are very similar. This is not the case, however, with the other two ratios. The Venezuelan basins have a very high drainage density, almost 10 times that of the basins in Puerto Rico. This is somewhat reflected in the values of R_A and R_L, which are larger for the Puerto Rican basins than for the Venezuelan basins. As mentioned earlier, the Mamon basin has large floods with very high velocities, although the streams are dry most of the time. In this basin, high velocities, complete lack of forest cover, and soil type tend to produce a very high value of drainage density.

Basin orders ranged from 6 for the whole Mamon basin to 3 for the basins in Puerto Rico.

DESCRIPTION OF THE EXPERIMENTS

To test the geomorphologic IUH of *Rodríguez-Iturbe and Valdés* [1979], controlled numerical experiments were carried out to obtain the IUH's of the four basins described earlier under different dynamic conditions. A very detailed modeling of each basin was made in which every stream segment was modeled as an individual segment in the rainfall-runoff model developed originally by *Schaake* [1971]. This model is based on the continuity equation and on the kinematic wave approximations to the equations of motion; the reasons for

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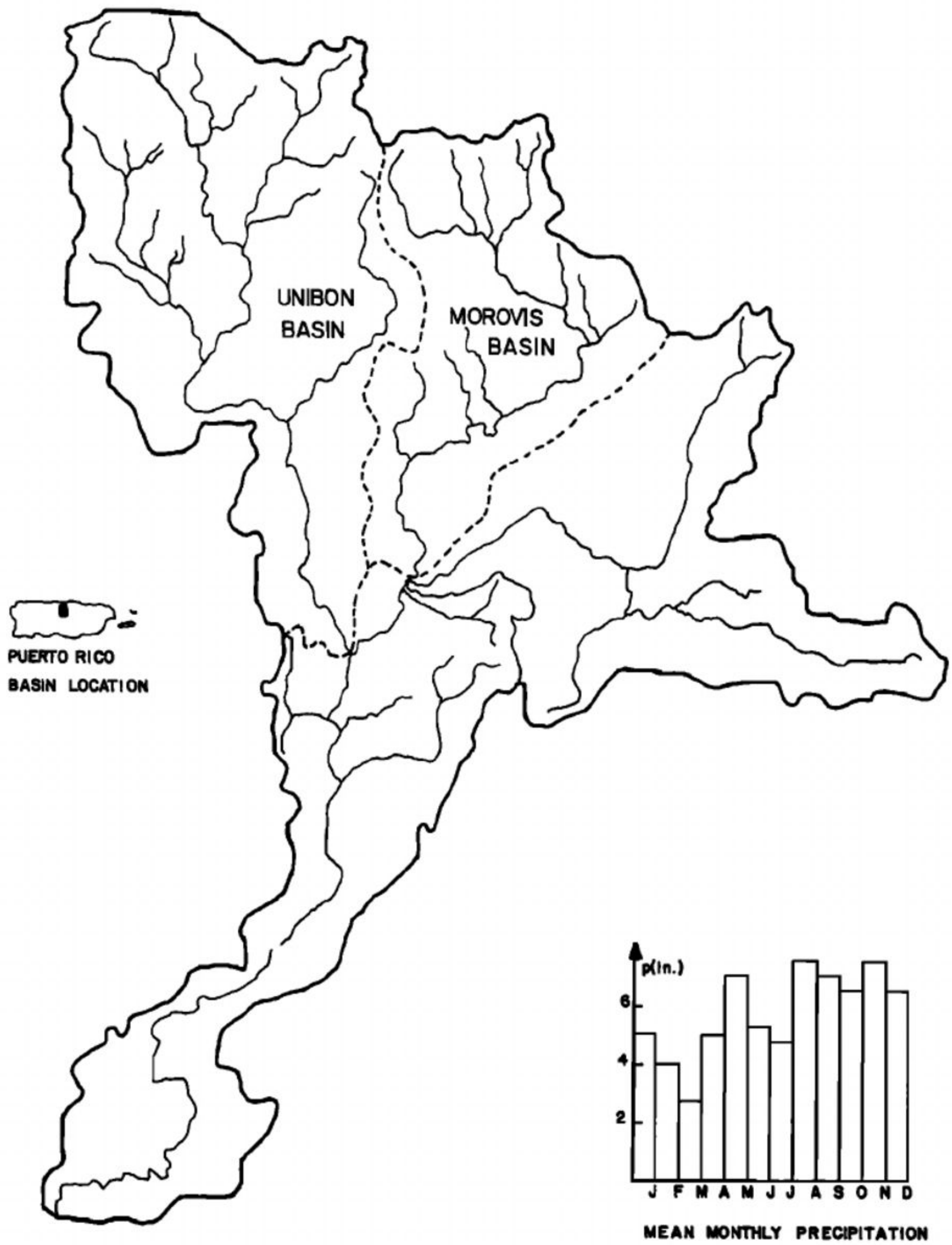

Fig. 1. Indio basin (Puerto Rico) with the Morovis and the Unibon subbasins.

choosing this particular model are its great simplicity and the extensive familiarity the authors had with the model during the last few years. Furthermore, models based on the kinematic wave have been applied to several basins in Puerto Rico and Venezuela with very good results. See, for example, *Resource Analysis, Inc.* [1976] and *Rodríguez-Iturbe* [1974], among others. An expanded form of this model is readily available [*Dawdy et al.*, 1978]. The rainfall-runoff model allows for spatial and temporal variability of the precipitation and gives options to represent infiltration either by Horton's equation or the method of the *U.S. Soil Conservation Service* [1971]. Instead of directly using historical rainfall-runoff data a nonlinear rainfall-runoff model was used to verify the geomorphological IUH for two main reasons. First, the historical data contain substantial uncertainties in the spatial and temporal distribution of the storm, in the occurrence of the infiltration losses, and in the true discharge. Second, nonlinearities in the surface runoff process affect the apparent IUH derived from historical data.

The numerical experiments to verify the geomorphologic IUH were made using storms of constant intensity and uniformly distributed over the catchment. The duration of the experimental storms was longer than the time to equilibrium. Further, to simplify the problem of how to deal with infiltration losses and because the unit hydrograph theory deals with effective rainfall, the basins are assumed to be impervious throughout the controlled experiments.

IUH Derivation Through a Rainfall-Runoff Model

The outflow discharge $Q(t)$ for a storm of duration t_r and constant intensity i_0 is given by the IUH theory as

$$Q(t) = \int_0^t h(\tau) i(t - \tau)\, d\tau \tag{1a}$$

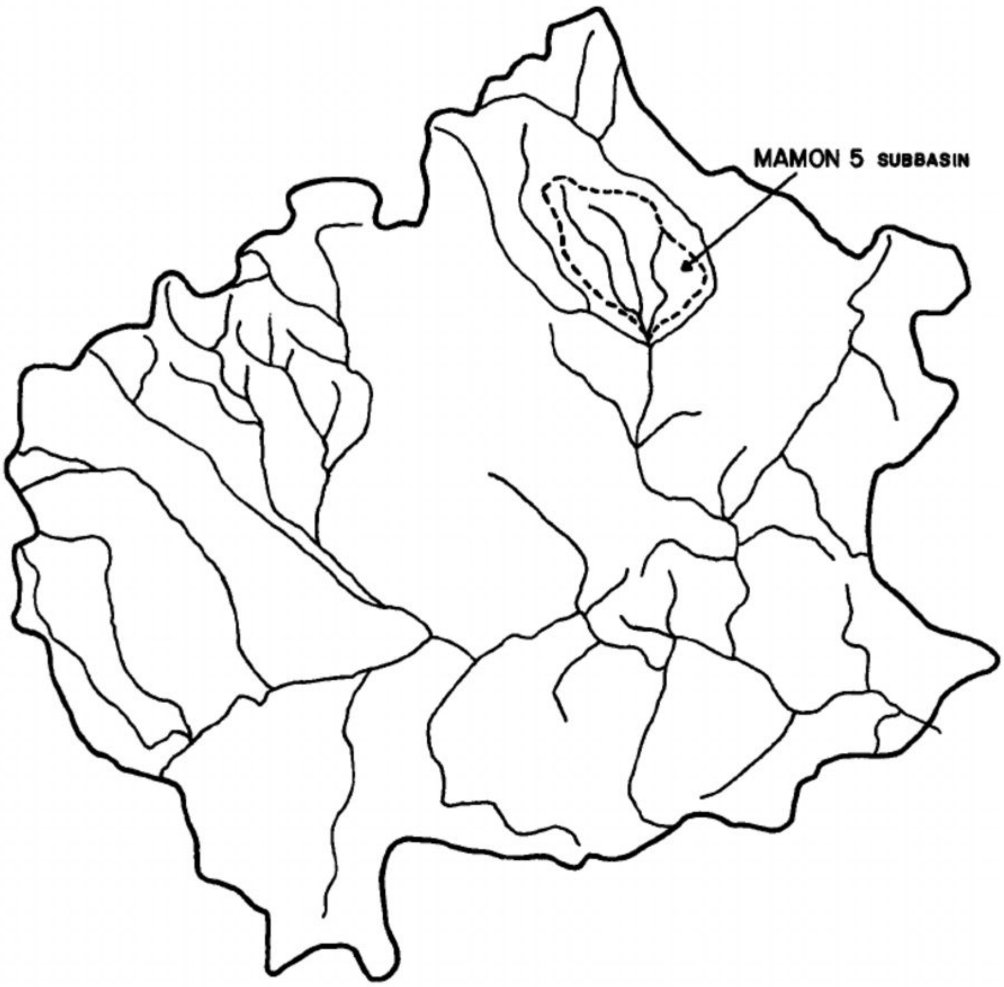

Fig. 2. Mamon basin (Venezuela).

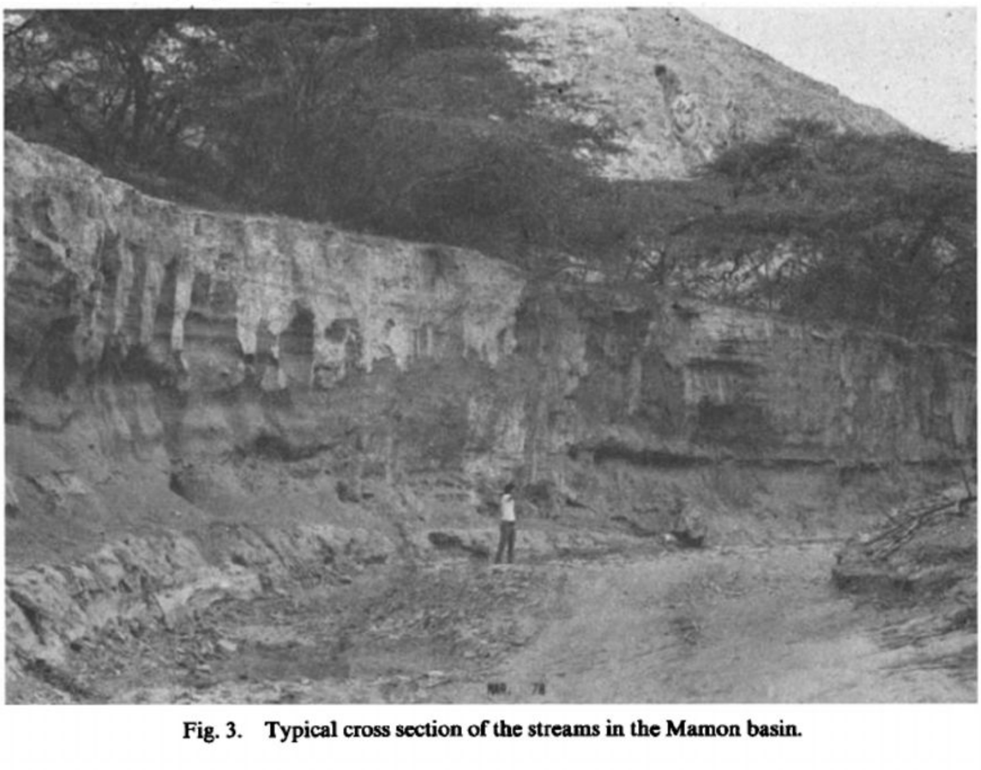

Fig. 3. Typical cross section of the streams in the Mamon basin.

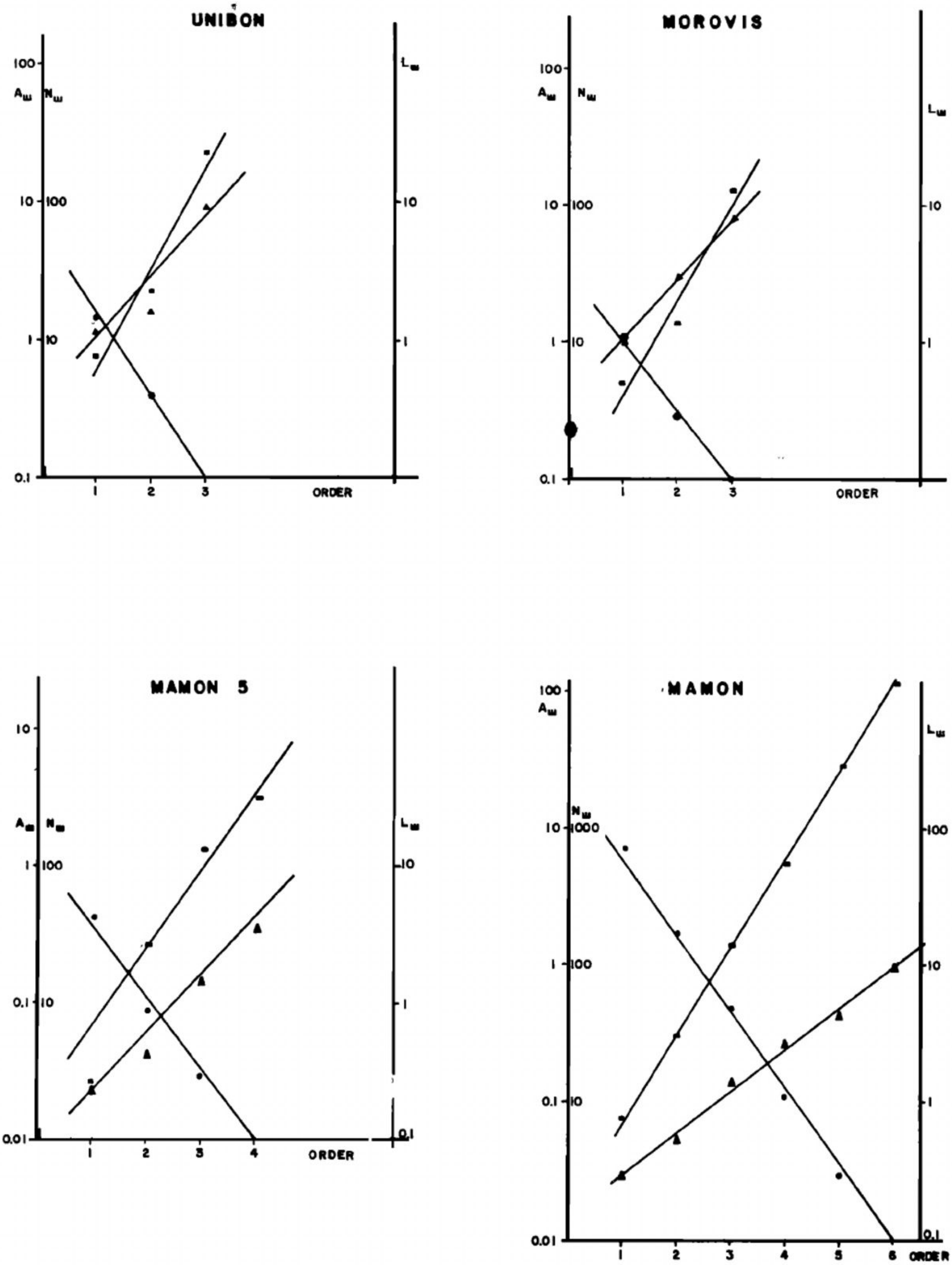

Fig. 4. Geomorphologic parameters of the four basins under study.

TABLE 1. Geomorphologic Parameters of the Four Basins Analyzed

Basin	Area, km^2	L_Ω, km	R_B	R_A	R_L	Order
Morovis (Puerto Rico)	13.0	8.0	3.2	5.0	2.7	3
Unibon (Puerto Rice)	23.0	8.6	4.0	5.6	2.8	3
Mamon (Venezuela)	103.0	12.25	3.5	4.5	2.1	6
Mamon 5 (Venezuela)	3.15	3.59	3.3	3.8	2.5	4

or

$$Q(t) = \int_0^t h(t-\tau)i(t)\,d\tau \tag{1b}$$

where

$$i(t) = i_0 \quad \text{if} \quad t < t_r$$

$$i(t) = 0 \quad \text{if} \quad t > t_r$$

and $h(\tau)$ is the ordinate of the IUH. The derivative of $Q(t)$ gives the ordinates of the IUH. For a storm of infinite duration, $t_r = \infty$, the derivative of (1) is

$$\frac{dQ(t)}{dt} = \frac{d}{dt}\int_0^t h(\tau)i_0\,d\tau$$

and, using Liebniz's rule,

$$\frac{dQ(t)}{dt} = i_0 \cdot h(t) \tag{2}$$

To derive the IUH for a storm lasting longer than the time to equilibrium but less than infinite, the precipitation input is defined as follows [*Schaake,* 1965]:

$$i(t) = i_0[u(t) - u(t-t_r)] \tag{3}$$

where

$$u(t) = 0 \qquad t < 0$$

$$u(t) = 1 \qquad t \geq 0$$

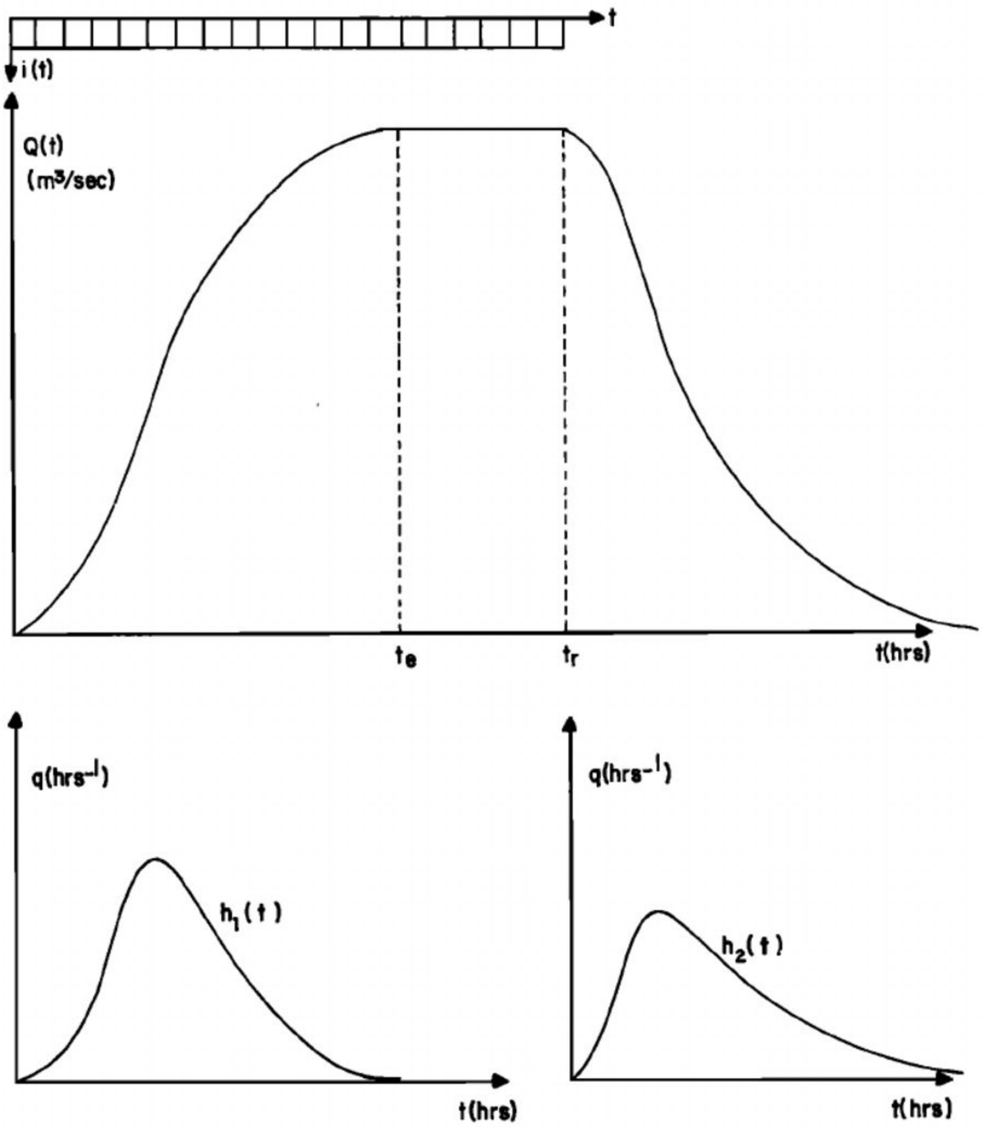

Fig. 5. Description of the rainfall-runoff experiments to derive the response function of a basin.

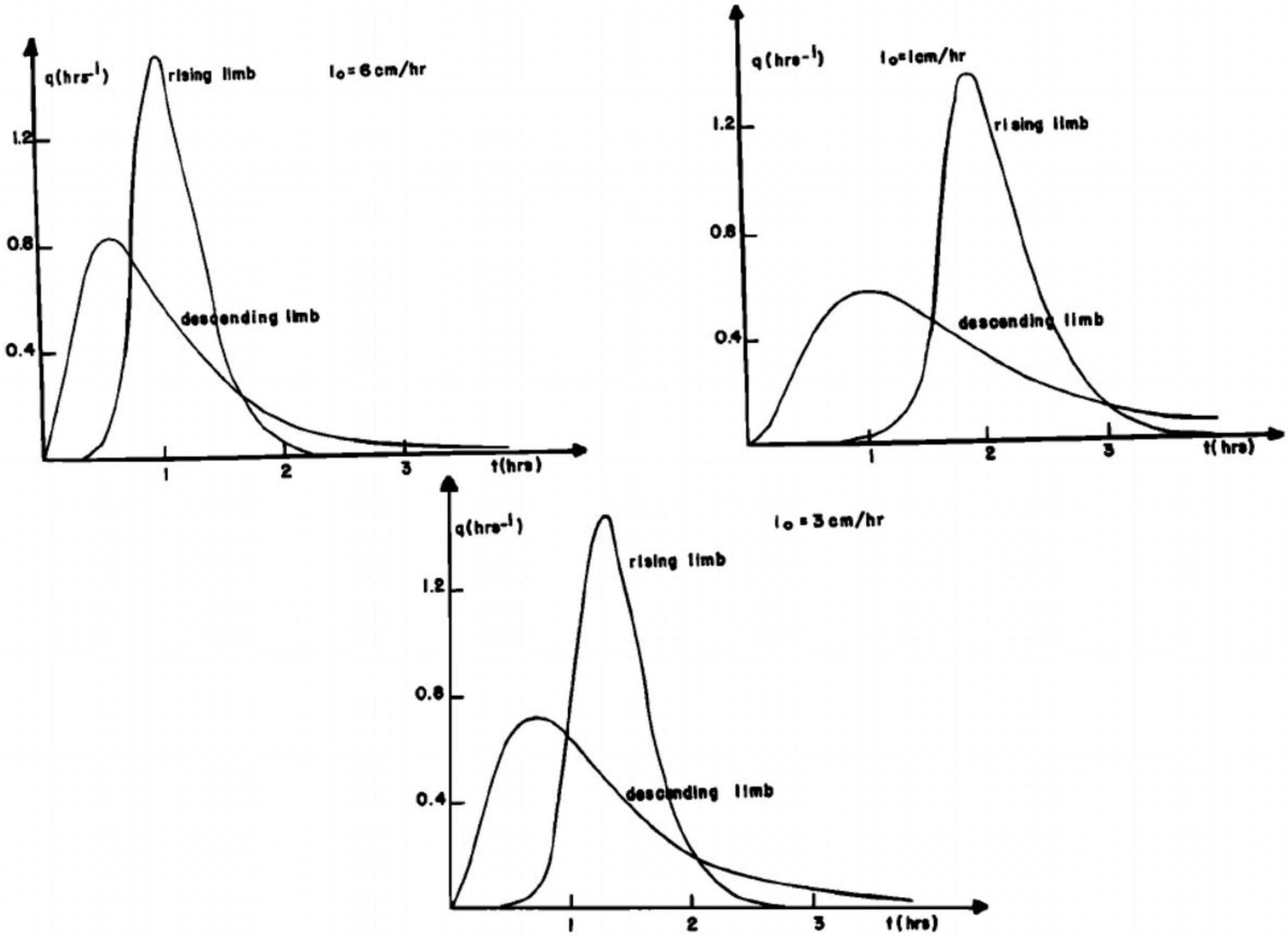

Fig. 6. Rainfall-runoff-derived IUH's for the Unibon basin for different storm intensities.

and IUH's may be obtained from both the rising limb and the descending limb of the hydrograph. Thus

$$\frac{dQ(t)}{dt} = i_0 h(t) \qquad \text{for} \quad t < t_r \tag{4a}$$

$$\frac{dQ(t)}{dt} = i_0[h(t) - h(t - t_r)] \qquad \text{for} \quad t > t_r \tag{4b}$$

The derivatives in (4a) are equal to zero when steady state is reached, and two expressions are obtained from (4), one to compute the IUH ordinates from the rising limb of the hydrograph, i.e.,

$$\frac{dQ(t)/dt}{i_0} = h_1(t) \tag{5a}$$

and other one for the IUH ordinates computed from the descending limb of the discharge hydrograph, i.e.,

$$-\frac{dQ(t)/dt}{i_0} = h_2(t) \tag{5b}$$

[*Schaake*, 1965].

This procedure is illustrated by Figure 5 and was applied to the four basins mentioned earlier for several storm intensities ranging from 1 to 6 cm/h. The derived IUH's are shown in Figures 6–9. As can be seen from these figures, the IUH's are quite different both for the same storm but computed from the rising or descending limbs of the outflow hydrograph and also for different storm intensities. This is because the rainfall-runoff model which is based on the kinematic wave is a nonlinear representation of catchment runoff. Thus for the same storm the IUH computed from the rising limbs takes longer to reach the peak discharge, and it reaches a higher value than the one obtained from the descending limbs. Further, although the IUH ordinates were normalized by the storm intensity, the IUH's computed from the rising limb for different storm intensities are not the same. This is also due to the nonlinearities of the rainfall-runoff model. The IUH's compared from the rising limbs for different storm intensities for the four basins are shown in Figure 10.

A comparison of the IUH's obtained from the rainfall-runoff model with the geomorphologic IUH is not possible at this stage because it is necessary to define a velocity to compute the geomorphologic IUH. This velocity, of course, changes during the experimental storm from zero at the beginning until it reaches a maximum at equilibrium time, remains constant until the storms ends, and then starts to decrease. Because the rates of change of velocity are larger in the part of the S curve where the rising limb IUH is computed than for the case of the descending limb IUH, the comparison of the geomorphologic IUH for a given velocity, say, that at the time of equilibrium, will tend to give a better fit with the descending limb IUH than with the rising limb IUH. Although this timing component v allows the geomorphologic IUH to vary from storm to storm and also through a given storm, representing as a linear time variant response function the response of a nonlinear system, the purpose of the controlled experiments was to derive an IUH from the rainfall-runoff model in which the velocity was kept constant.

In a conversation with the authors, John Schaake, of the

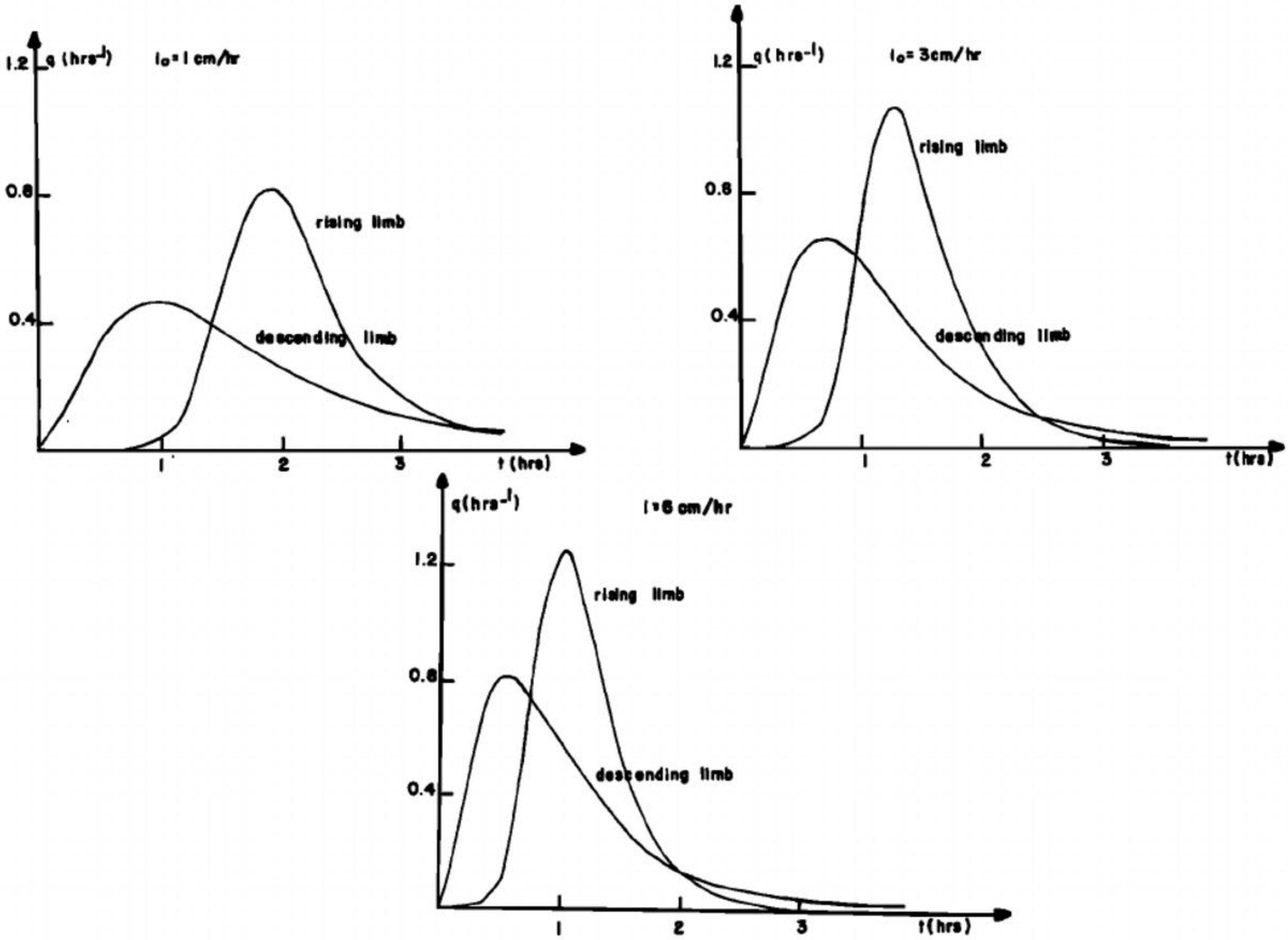

Fig. 7. Rainfall-runoff-derived IUH's for the Morovis basin for different storm intensities.

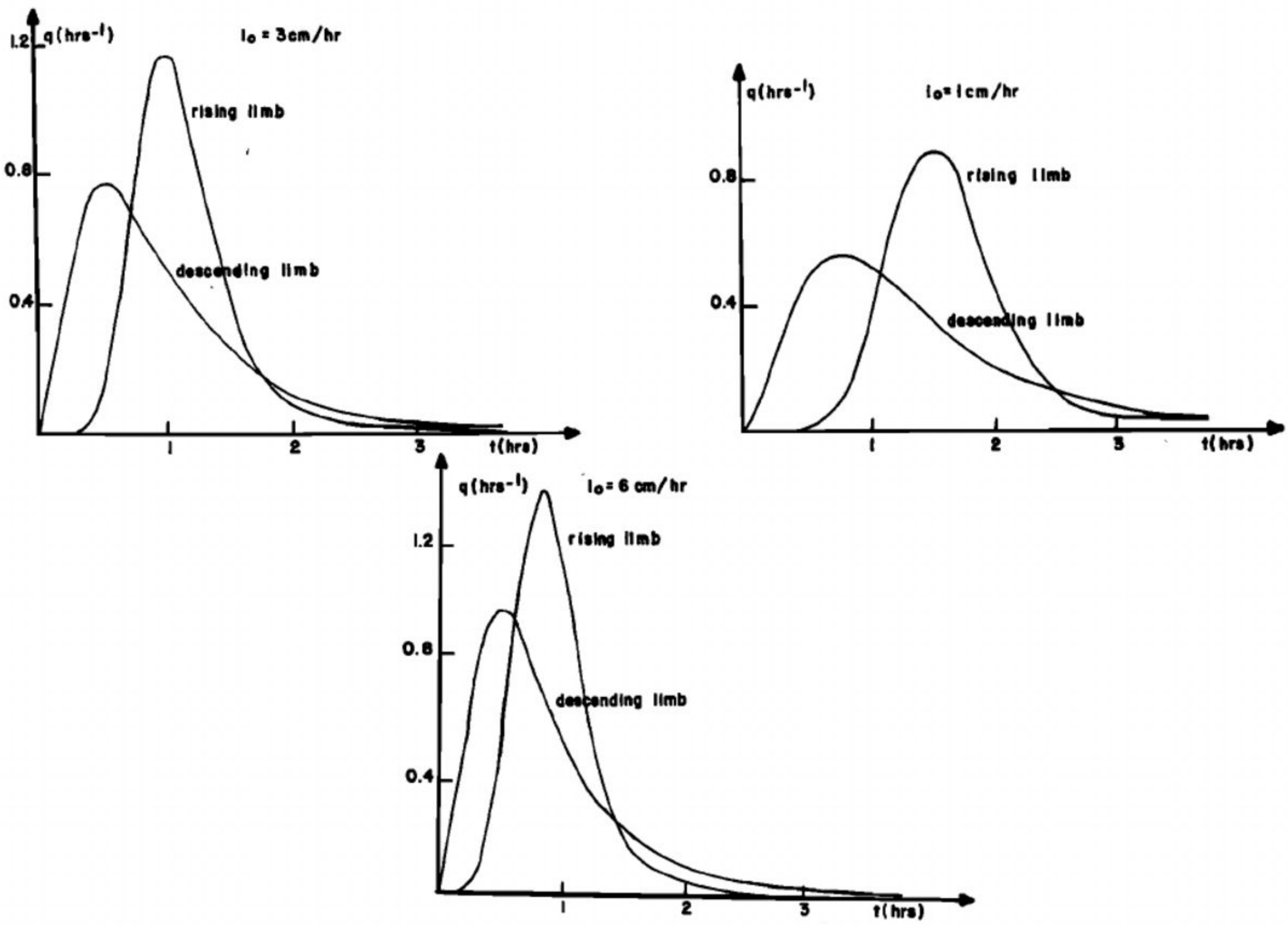

Fig. 8. Rainfall-runoff-derived IUH's for the Mamon basin for different storm intensities.

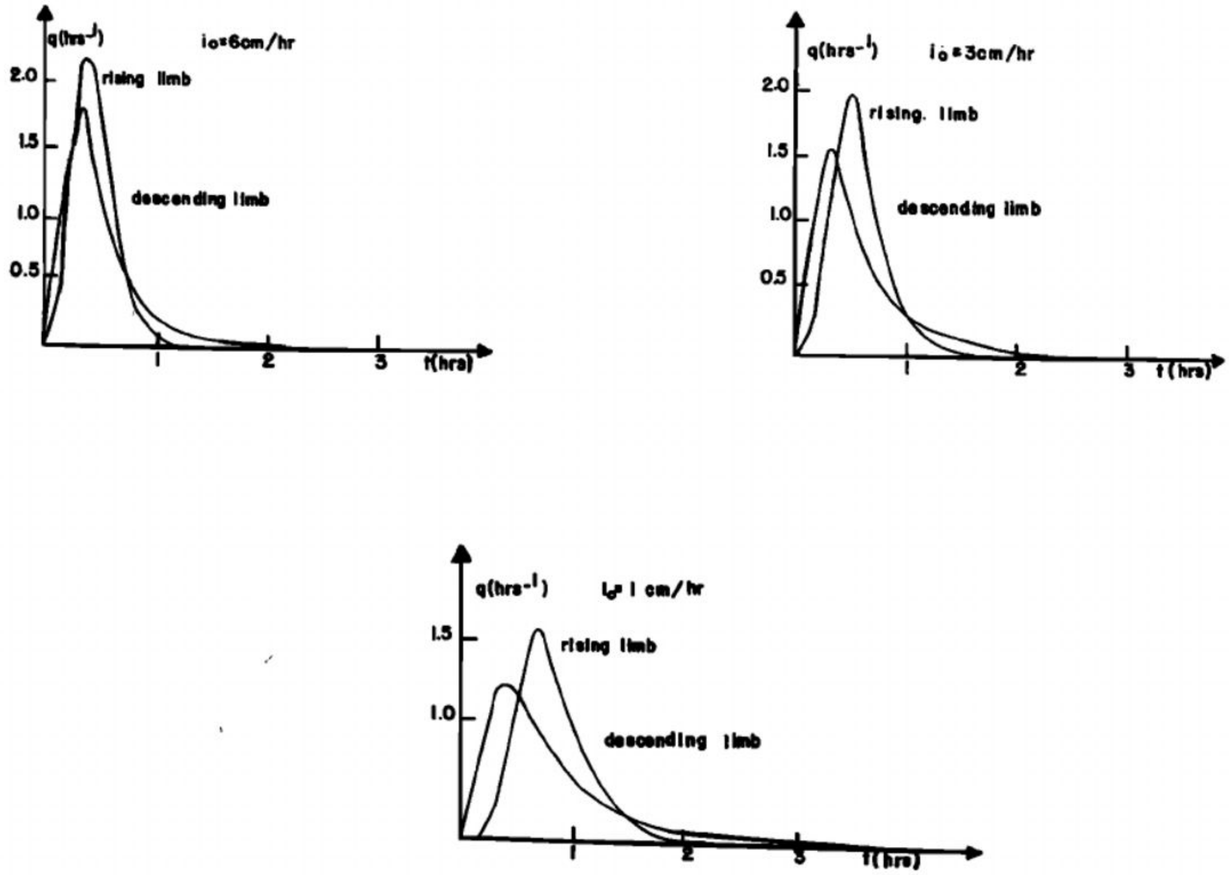

Fig. 9. Rainfall-runoff-derived IUH's for the Mamon 5 basin for different storm intensities.

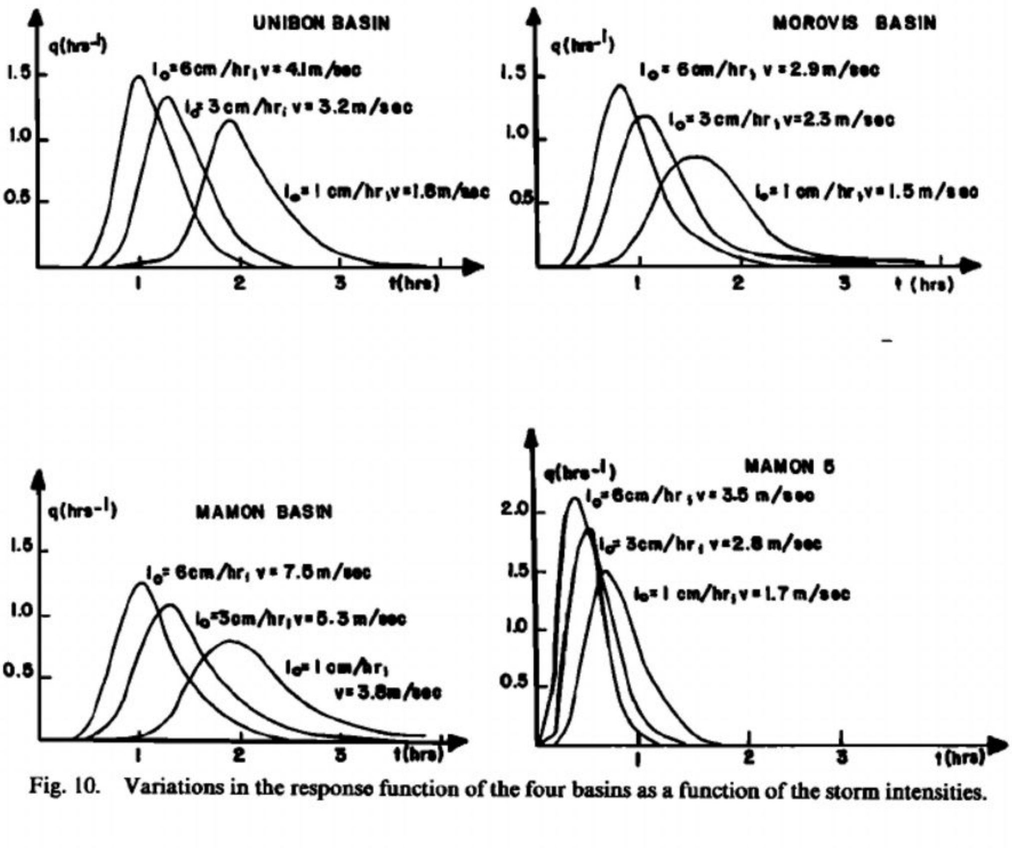

Fig. 10. Variations in the response function of the four basins as a function of the storm intensities.

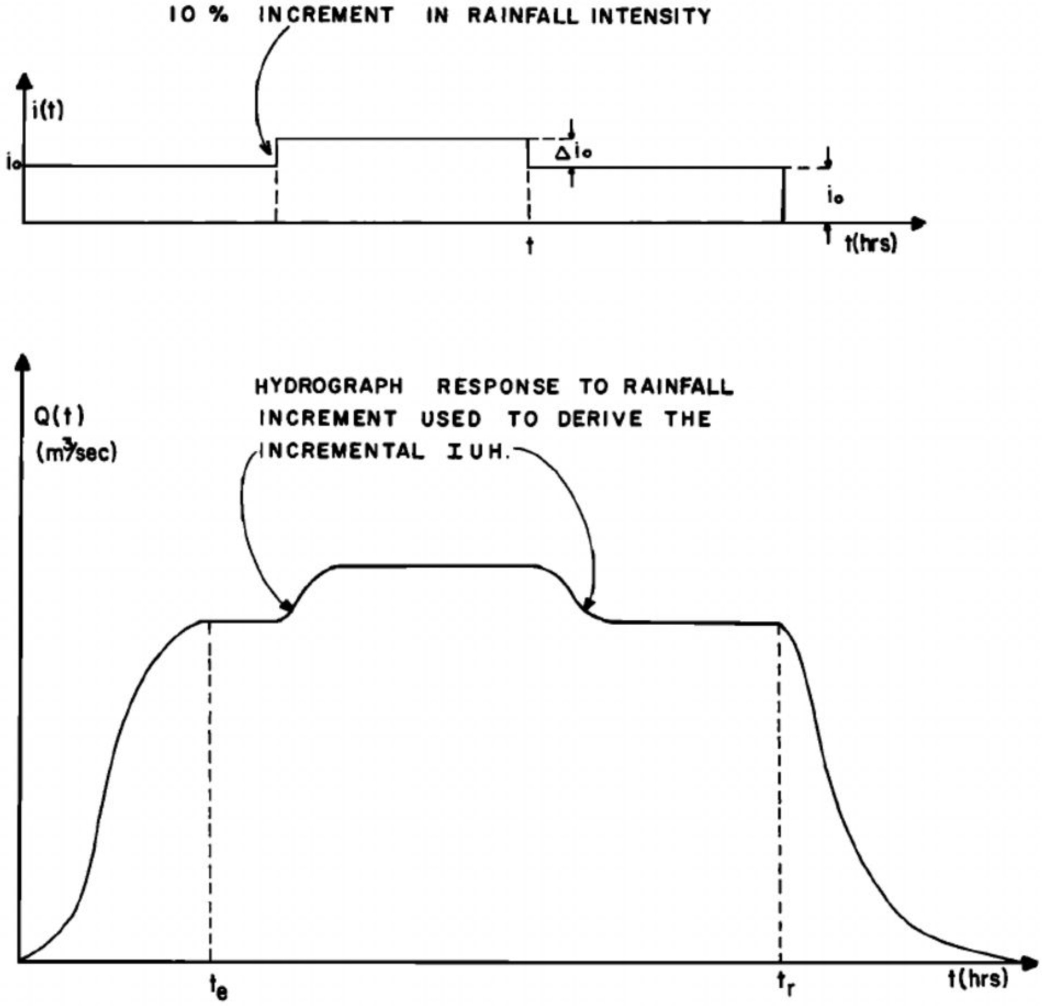

Fig. 11. Description of the incremental IUH experiment.

U.S. National Weather Service, suggested the realization of a so-called 'incremental IUH' experiment in which, after the discharge reaches a plateau at the time of equilibrium, the intensity of rainfall is incremented by 10% of the original intensity i_0. This experiment is illustrated in Figure 11. Owing to the small increase in rainfall intensity, the velocity following the increment in the rainfall remains practically constant and essentially equal to the velocity prior to the increment. The interesting point is that the IUH's computed from rising limbs and descending limbs of the hydrograph response to the incremented pulse of rainfall are practically the same for all intensities and for all basins. Figures 12–15 show examples of these comparisons. Nevertheless, as can be seen from Figure 16, the incremental IUH's are not the same for different storm intensities for the same basin. This is because although the IUH's are derived under a constant velocity condition, this velocity is not the same for the different storms but is higher with the increase of the storm intensity.

The velocity at the time of equilibrium v_e, given by the rainfall-runoff model, is now used to derive the geomorphologic IUH for each basin and for each storm intensity. The geomorphologic IUH's are then compared with the IUH's derived from the rainfall-runoff models using the incremental IUH experiments. The comparison is very satisfactory, as shown in Figures 17–20. Thus the variation from storm to storm of the response function of the basin, which is a nonlinear function of storm intensities, is satisfactorily handled by the dynamic component of the geomorphologic IUH. The variation within the storm of the response function, which is also due to the nonlinearities of the system, could also be handled by a time variant geomorphologic IUH in which v is changing with time.

The effect that the above variations have on the discharge peak Q_p and time to peak discharge T_p has been fully analyzed by *Rodríguez-Iturbe et al.* [1979].

SUMMARY AND CONCLUSIONS

The instantaneous unit hydrograph derived as a function of the geomorphologic parameters of a basin by *Rodríguez-Iturbe and Valdés* [1979] seems to be a workable approach to obtain the response function of a basin.

This has been verified through controlled numerical experiments which were carried out on four basins in Venezuela and Puerto Rico with areas ranging from 3 to 103 km² in which a geomorphologic analysis and a detailed rainfall-runoff modeling were made. The instantaneous unit hydrographs derived from the discharge hydrograph of the basins for different storm intensities were compared among themselves and with the geomorphologic IUH. No spatial or temporal variation of the precipitation was considered, and the basins were assumed to be completely impervious, since IUH theory deals with rainfall excess. As the experiments showed, there is danger of

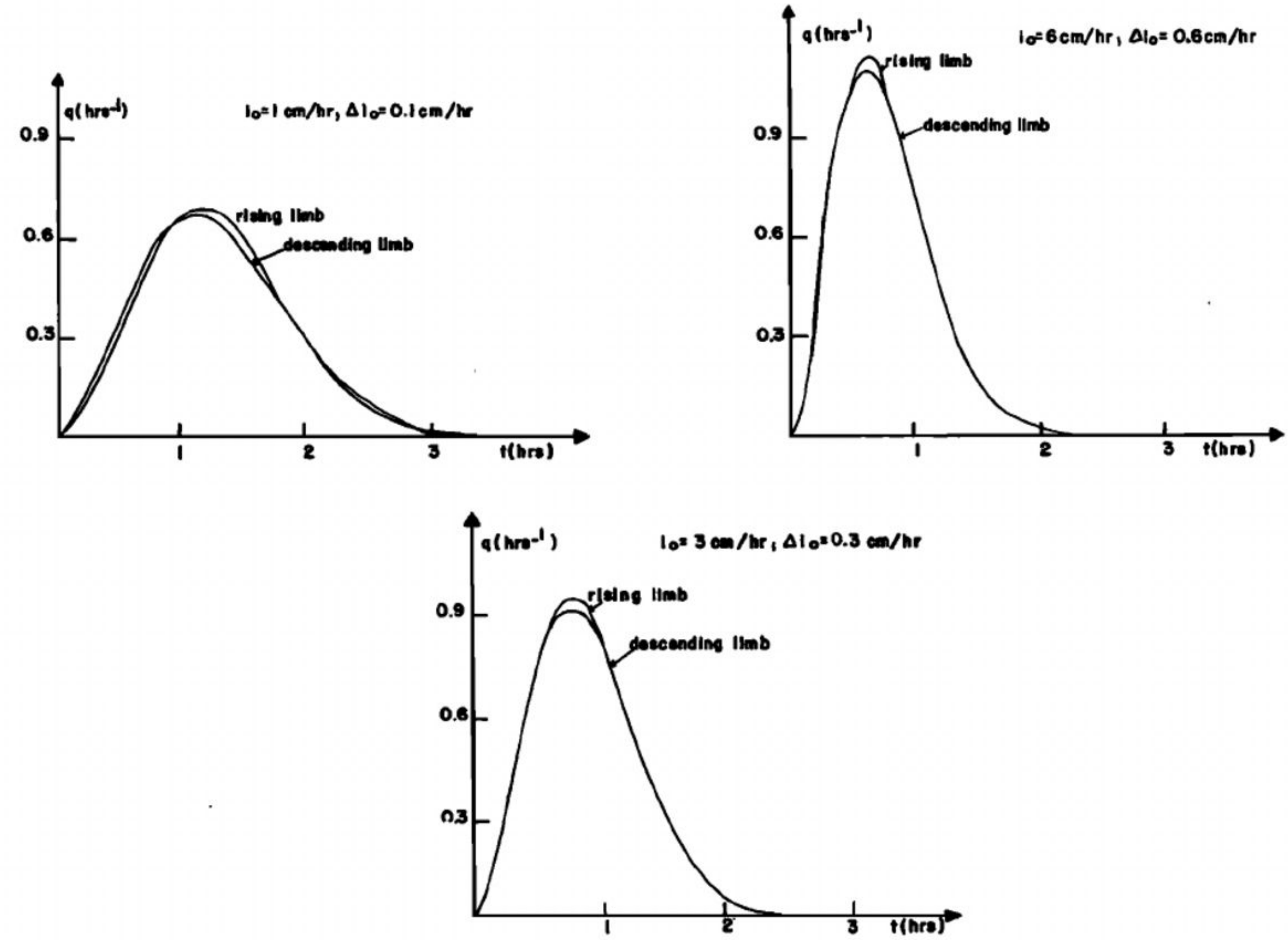

Fig. 12. Incremental IUH's for the Unibon basin for several storm intensities.

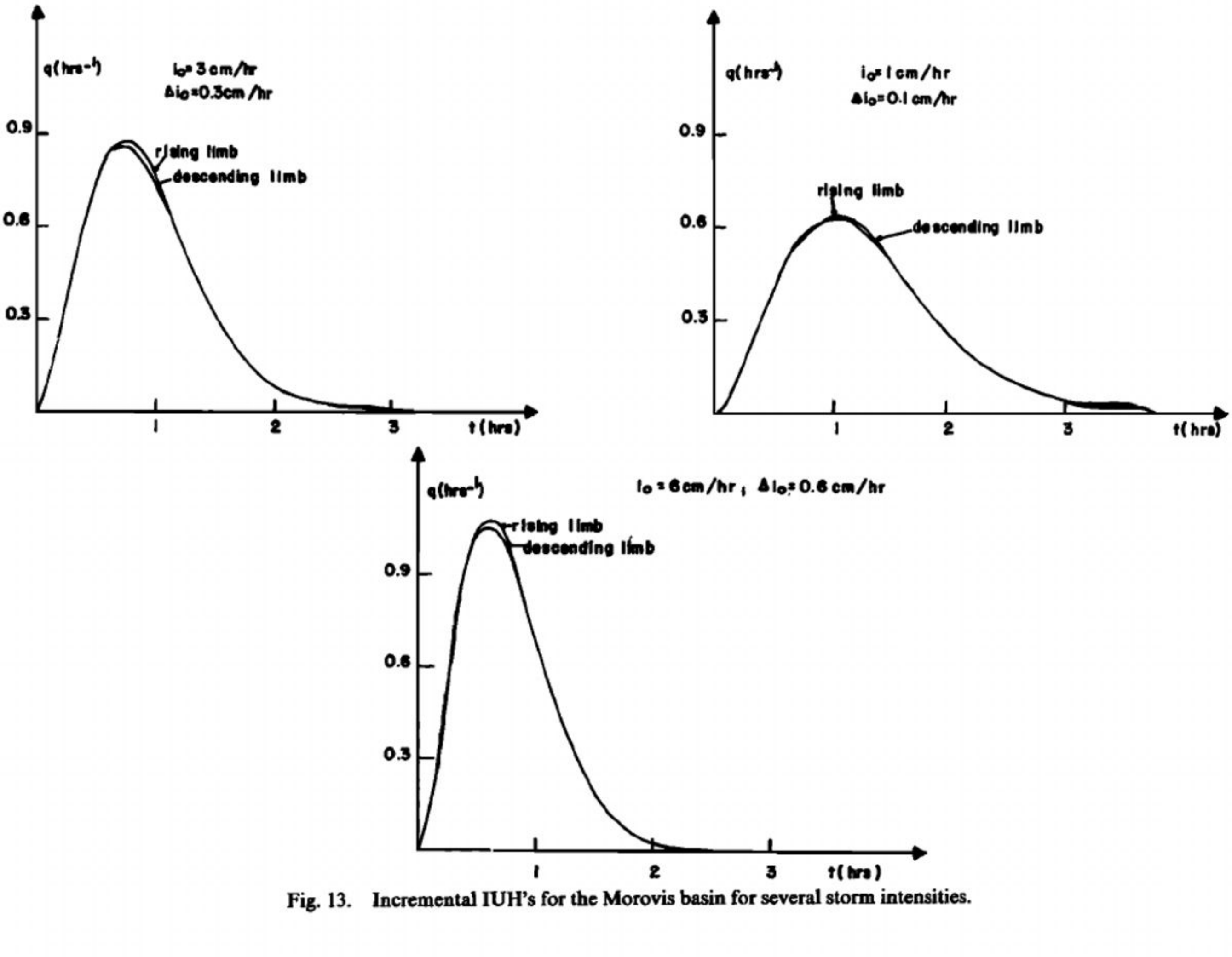

Fig. 13. Incremental IUH's for the Morovis basin for several storm intensities.

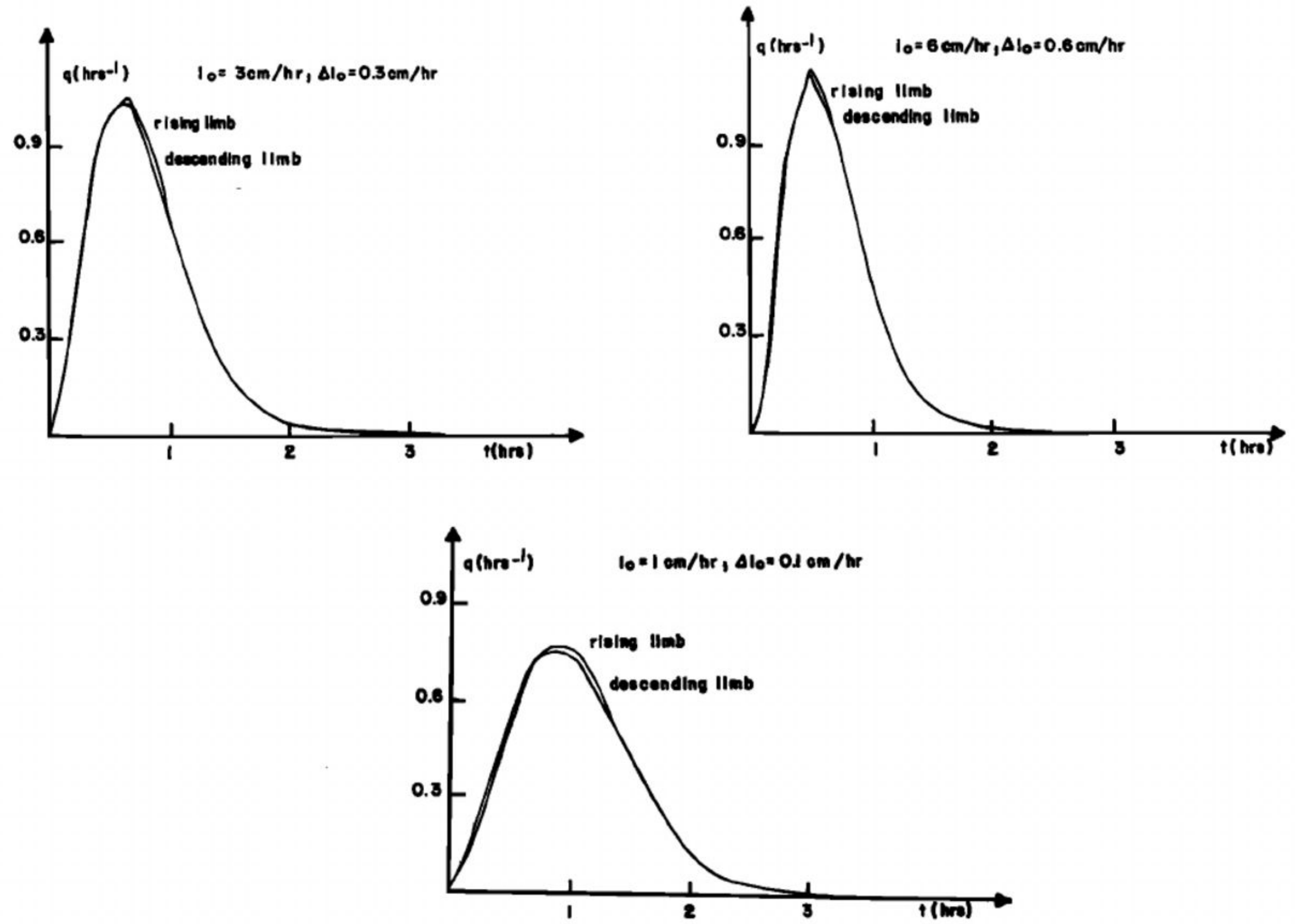

Fig. 14. Incremental IUH's for the Mamon basin for several storm intensities.

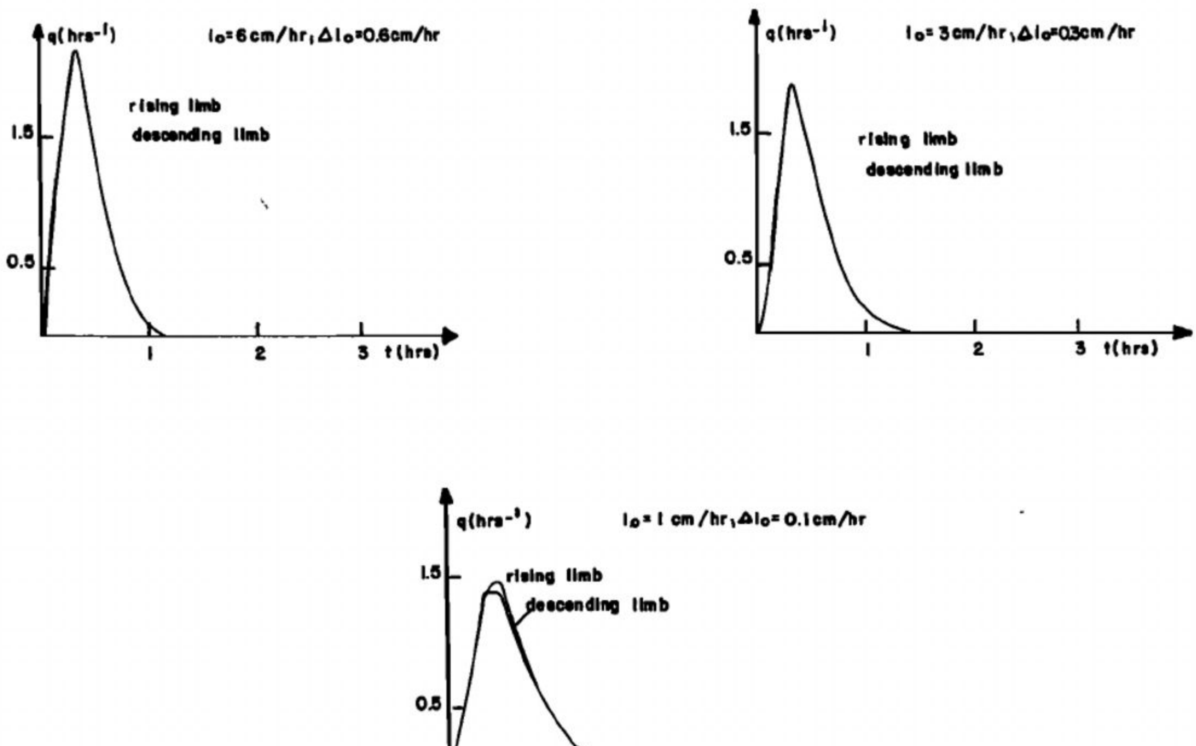

Fig. 15. Incremental IUH's for the Mamon 5 basin for several storm intensities.

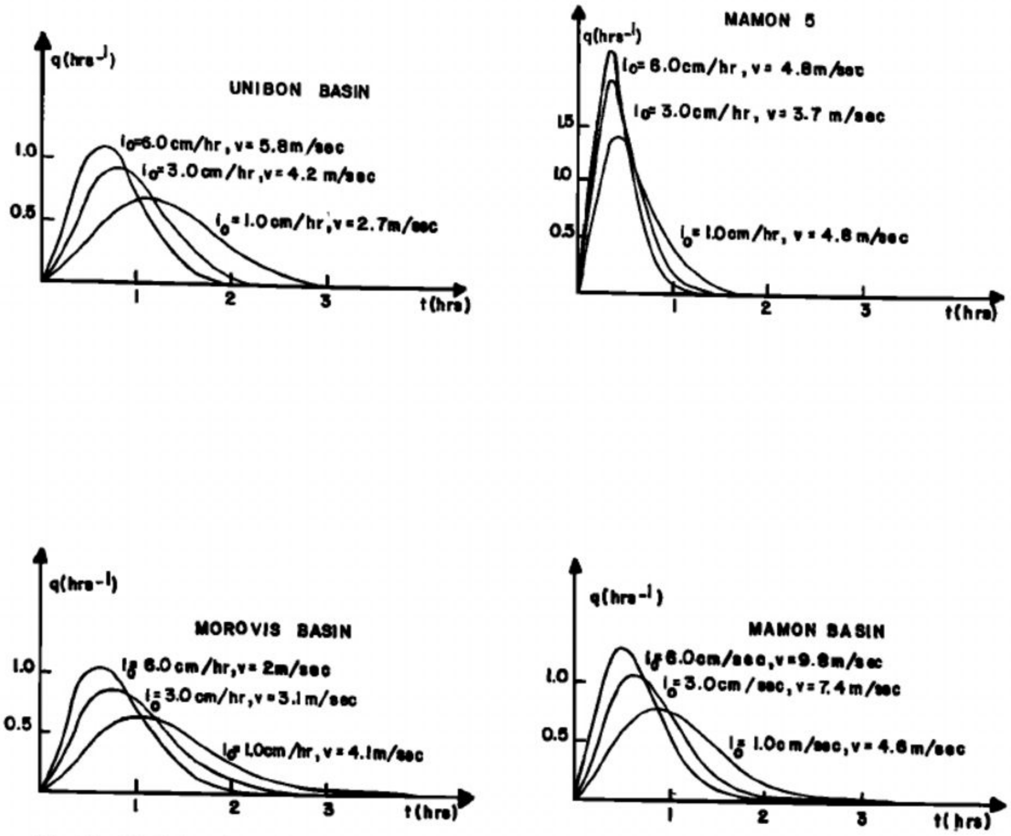

Fig. 16. Variations in the incremental IUH of the four basins as a function of the storm intensities.

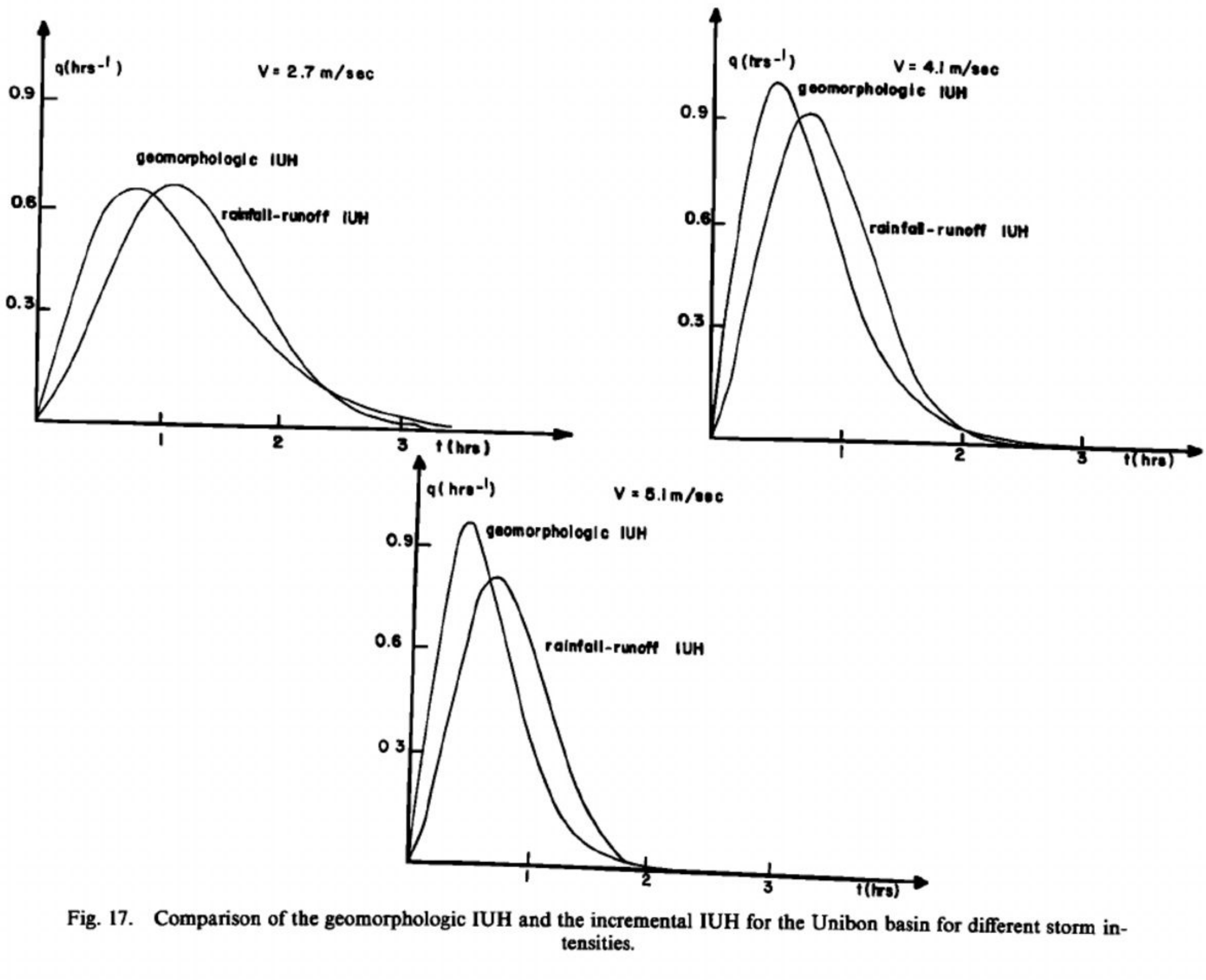

Fig. 17. Comparison of the geomorphologic IUH and the incremental IUH for the Unibon basin for different storm intensities.

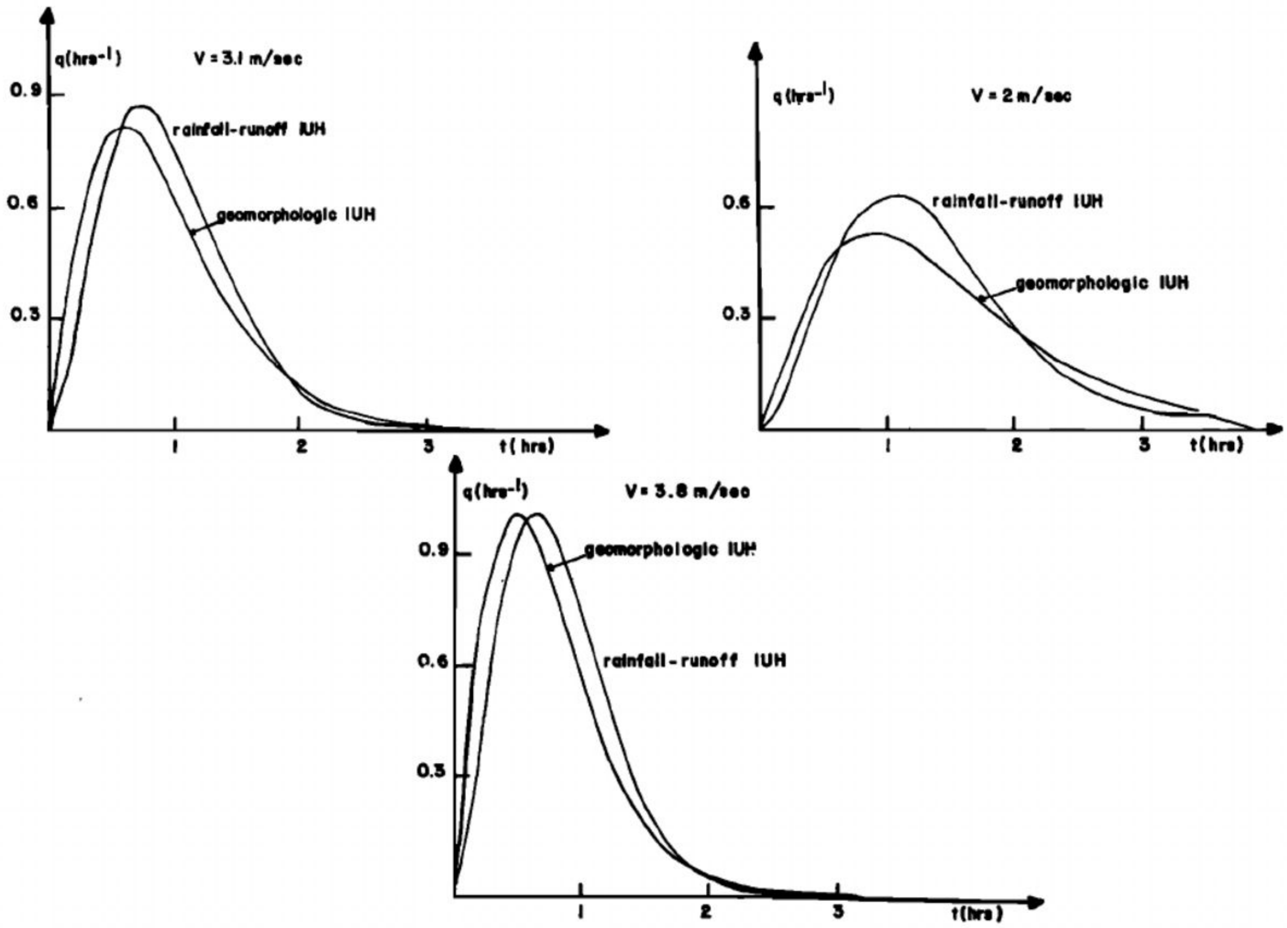

Fig. 18. Comparison of the geomorphologic IUH and the incremental IUH derived for the Morovis basin for different storm intensities.

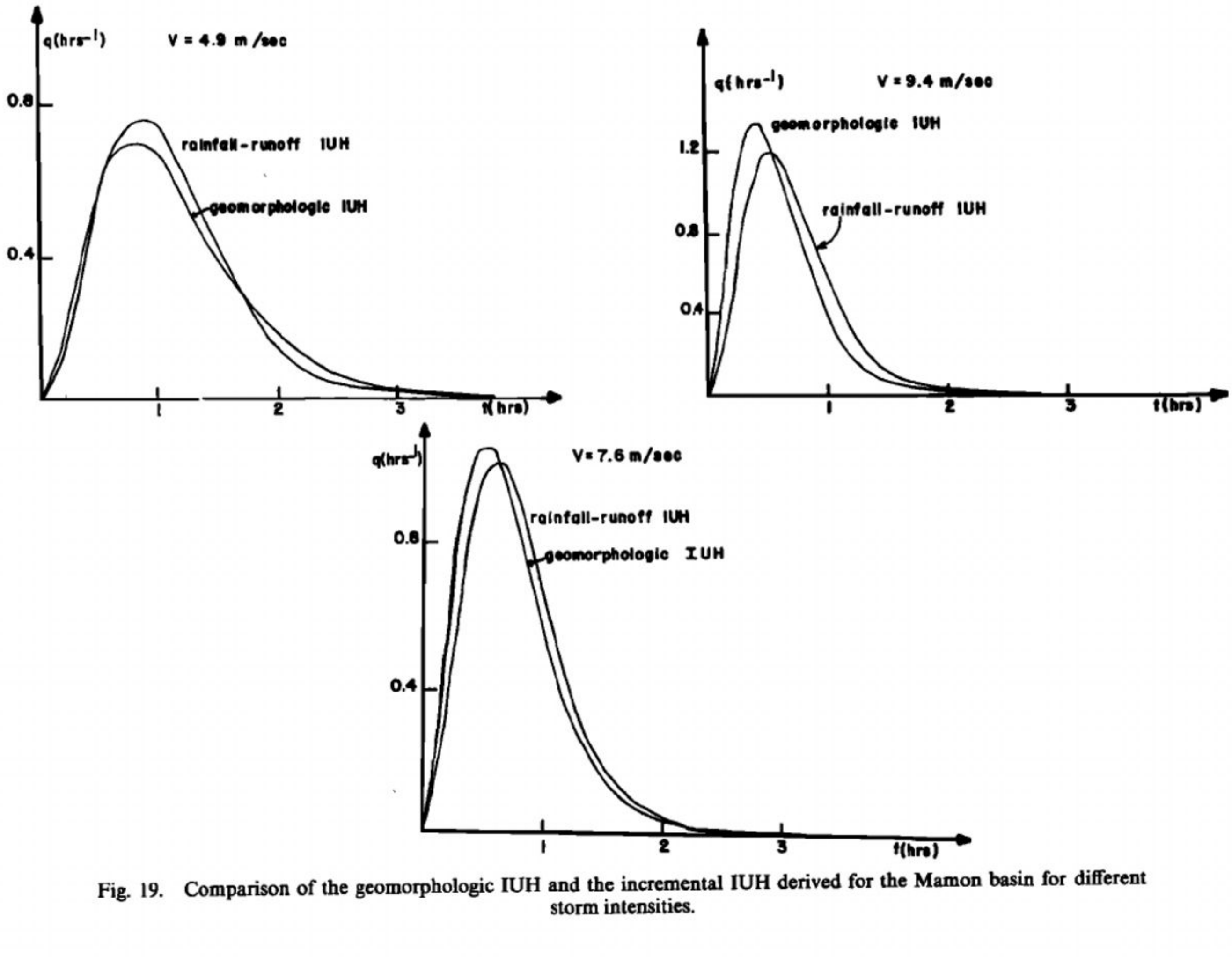

Fig. 19. Comparison of the geomorphologic IUH and the incremental IUH derived for the Mamon basin for different storm intensities.

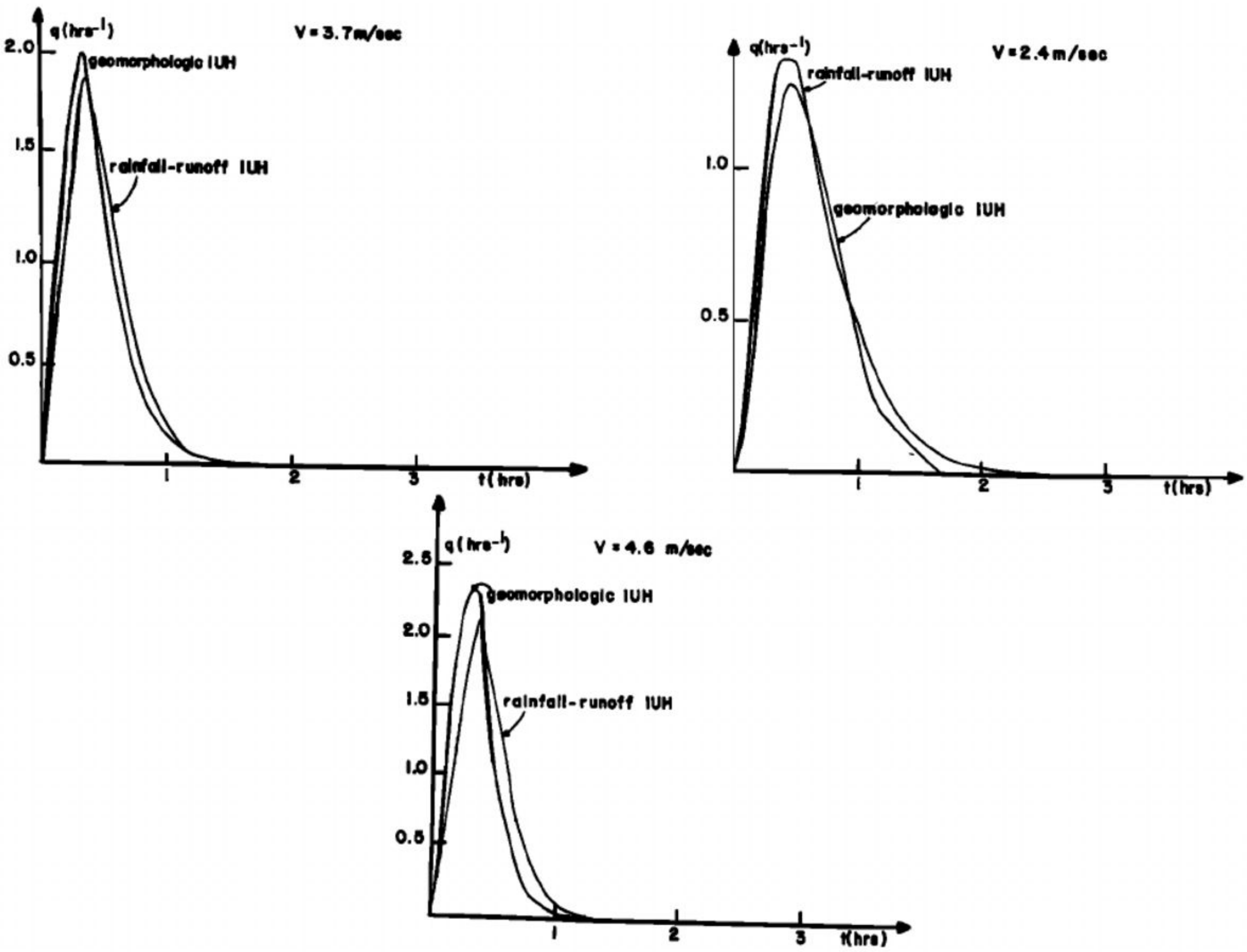

Fig. 20. Comparison of the geomophologic IUH and the incremental IUH derived for the Mamon 5 basin for different storm intensities.

serious errors when comparing IUH's derived from storms of different characteristics to those of the design storm. These nonlinear characteristics of the response function of a basin can be modeled with a linear scheme such as the IUH but with a velocity representative of the discharge expected at the basin outlet. The study of this velocity is developed by *Rodríguez-Iturbe et al.* [1979]. The variation of the velocity during the storm can be incorporated with a time variant IUH throughout each storm event, but this is not as important as the case of different storms with dissimilar rainfall intensities and durations.

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