from which $M_A = 57.14$. Eq. 25 is shorter than the one following Eq. 18b; and, as the computation progresses, the physical significance of each term is shown by recording it in Fig. 6. Mr. Spiker has shown another, but longer, solution for Fig. 6.

As minor points, Mr. Beskin criticizes the use of the symbol Δ for angular change due to bending, as well as the use of the term "tapered beams." The term "tapered beams" has an excellent precedent.[26] The symbol Δ was selected because, in a number of treatises, it represents the angle between the tangents to a curve, and it seemed appropriate to extend its use to traverse diagrams which are solved largely as problems in geometrical alinement.

The writer is grateful to Messrs. Goldberg and Weiss for their comments. As Mr. Weiss suggests, the moment-area constants are the basis of the elastic curve traverse. If they are delineated as a traverse, the flexure of the structure can be visualized and the deformations due to flexure become easier to use than when pictured as areas. Such delineation also gives an opportunity to diversify and extend the use of moment-area constants. The object of the paper was to take advantage of this opportunity.

Mr. Polivka commented that "* * * the author's claim that the method reduces by one half the work of computation * * * if applied generally, cannot be upheld." The writer's claim was worded as follows: "* * * for many problems, the algebraic work required will be only one half that required by the procedure taught in current texts." It may be added that slope deflection, as amplified by the elastic curve traverse, will never, for any problem, involve more algebraic work than slope deflection as taught in current college textbooks.

The detailed equation given by Mr. Polivka, which he consolidates into Eq. 21a, represents either derivation of a formula by an algebraic procedure involving simultaneous equations as described by A. Amirikian,[21] M. Am. Soc. C. E., or handbook work which the writer wishes to avoid.

An important point which should not be overlooked is that the formulas derived by Mr. Amirikian are for frames composed only of members with constant cross sections. The introduction of tapering members would make no important changes in solutions which use the elastic curve traverse but it would have a serious effect on Mr. Amirikian's algebraic use of slope deflection.

The excellent and constructive work of Mr. McGee in presenting the complete detail of the solution for an unsymmetrical two-story frame with fixed-base columns is helpful. Following this step-by-step solution should clarify most of the situations which can arise in the general use of the method. As shown by Fig. 4(c), this problem involves eight unknown moments. The number of geometrical unknowns is six, comprising four joint rotations and two story deflections. Only two unknowns (θ_1 and θ_2) appear in Fig. 4(d), the others having been eliminated in an easy manner so that only two simultaneous equations are needed for the solution.

Mr. Eremin offers suggestions regarding the extension of the method to structures having members of variable section. It is for such structures and also for such complicated conditions as "semirigid" joints that the use of an elastic curve traverse best shows its advantage.[7]

[26] "Tapered Structural Members: An Analytical Treatment," by Walter H. Weiskopf and John W. Pickworth, *Transactions*, Am. Soc. C. E., Vol. 102 (1937), p. 1.

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STORAGE AND THE UNIT HYDROGRAPH

By C. O. Clark,[1] Jun. Am. Soc. C. E.

With Discussion by Messrs. James S. Sweet, Otto H. Meyer, L. K. Sherman, Gordon R. Williams, Ray K. Linsley, Jr., E. F. Brater, L. C. Crawford, Robert E. Kennedy, Victor H. Cochrane, Franklin F. Snyder, Howard M. Turner, Don Johnstone, and C. O. Clark.

Synopsis

During the period since 1930, study of the flow records of many and various types of streams by the engineers of the United States has resulted in an improved concept and understanding of the physical factors which influence runoff and the flood-producing capacity of streams. Two fundamental tools have emerged from these studies, the unit hydrograph and methods of flood routing (means of modifying a hydrograph by the effects of valley storage), both of which have been the subject of many papers.

The purpose of this paper is to clarify the inherent relationship between these fundamental tools and to show how this relationship may be used to derive accurate unit hydrographs for very short periods of initial runoff which accurately reflect the influence of shape of drainage area upon the shape of the hydrograph, allow the segregation of elements of the hydrograph attributable to particular components of the drainage area, and permit the definition of the calculation procedure to a degree which reduces the dependence of accurate stream flow calculations on intangible factors of personal judgment.

After showing that the constant time units characteristic of unit hydrographs are the characteristics induced by storage capacity of the streams and that storage capacity and discharge capacity are each a limiting factor on the other, the paper illustrates the incorporation of these characteristics in hydrograph calculations.

Definitions

Throughout this paper, the terms "unit hydrograph" and "unitgraph" are used interchangeably to mean a regimen of stream flow for a unit quantity of uniformly distributed runoff originating at a uniform rate during a specified

Note.—Published in November, 1943, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

[1] Hydr. Engr., U. S. Engr. Office, Winchester, Va.

unit period of time. The time elements of the regimen are independent of the quantity of runoff and the discharge quantities are directly proportional to the quantity of runoff. Included are both the unit hydrograph of L. K. Sherman,[2] M. Am. Soc. C. E., and the distribution graph of Merrill Bernard,[3] M. Am. Soc. C. E. The terms "storage" and "valley storage" are applied to the entire volume of water stored between any two designated points along a river and include both the volume in the channel and on the flood plain. The same terms, expressed in time units, are used to designate the average ratio of storage volume to discharge capacity of the river reach.

The following definition of accuracy will apply to the writer's use of the term in subsequent paragraphs, and at the same time will illustrate some of the problems and aims of incorporating accuracy in stream flow calculations:

> An engineer should learn early to distinguish between the forms of accuracy which sometimes are called absolute, real, and relative. Absolute accuracy is the mathematical precision of correctly manipulated numbers, as used by engineers in computing payments to the cent from earthwork surveys by cross sections or soundings. Real accuracy is an evaluation of the closeness with which a measurement or computation approximates the truth, a quality inherent in the meaning of the engineering term "precision." Relative accuracy is an evaluation of the closeness with which a measurement or computation approximates another measurement or computation, preferably where both determinations are comparable, and frequently is expressed as a ratio or the difference between them, without necessarily giving an absolute value of either. The elevation of one bench mark with respect to another is a value that may have great relative accuracy without any absolute accuracy. The terms with which others designate these qualities of accuracy may differ, but the essential difference between them is acknowledged widely.

In the fields of evaluation and planning of river developments, relative accuracy is attainable at low cost, usually by thoroughly planned systematic analysis; and it is sufficient in the selection of the better of alternative plans. Real accuracy, as in the evaluation of the selected plan of construction, high-order surveys, or field observation and measurement of natural phenomena, such as river stage and discharge, is an elusive value pursued at great cost, with each additional fraction of attainment sometimes costing more than all prior to it. Absolute accuracy in most engineering offices is the product of a calculating machine, useful as a tool in attainment of relative accuracy regarding influences of small variables, but a source of colossal waste of time if confused with real or relative accuracy. One cannot start with an hypothesis having a real accuracy of 80% and develop therefrom conclusions of 95% real accuracy.

By any unmodified references to accuracy which may be made in the body of the paper which follows, the writer means only relative accuracy and the degree to which a computational determination by simplified procedure approximates the result of more laborious, accepted means of computation. No claims of unusual real accuracy are intended.

Investigators entering the field of flood flow analysis will do well to maintain an open mind on the subject of real accuracy, since the difficulties of field observation and measurement under severe flood conditions make it necessary to concede the possibility of errors as large as 20% (or more) in some items of published data. Such possibilities are indicated in many published records by the discussion preceding the tabulations, and in others by subsequent revisions when more reliable data are obtained.

[2] "Stream Flow from Rainfall by the Unit Hydrograph Method," by Leroy K. Sherman, *Engineering News-Record*, Vol. 108, 1932, pp. 501–505.

[3] "An Approach to Determinate Stream Flow," by Merrill Bernard, *Transactions*, Am. Soc. C. E., Vol. 100 (1935), p. 347.

The attainment of a high order of relative accuracy in computation is expedited by concise, precise definition of procedure and the desired result. As an aid to the more rapid attainment of relative accuracy in the computation of flood hydrographs, the paper seeks to define the determination of the unit hydrograph of surface and subsurface runoff in terms of two, large, basic factors —the shape of the watershed from which runoff must come and the storage through which it must come.

The writer does not claim that the determinations will exceed in real accuracy the many excellent ones which have been obtained by trial and error, tried, tested, modified, and retried until they reproduce, well, the hydrographs of record. He does believe that the first determination of the unit hydrographs will prove satisfactory for use without modification in more cases than those determined by any of several other methods with which he is familiar.

Introduction

Rapid advances in the understanding of floodtime stream flow date from the Report of the Committee on Floods of the Boston Society of Civil Engineers in which it was recognized that, for storms of equal duration, the period of flood runoff was a constant not dependent upon the total volume of runoff and that the maximum rate of discharge was, therefore, a direct function of the total runoff. "For an instantaneous storm, the peak flow will vary directly with the maximum width of the drainage area, all other conditions remaining the same * * * unit peak flows will vary directly with the velocity of flow, all other conditions remaining the same" and "pondage tends to reduce the peak flow in the direct ratio that the volume of pondage bears to the total flood run-off." In these conclusions[4] lie the bases for many current concepts of the unit hydrograph, time-area concentration curves, and valley storage modification of flood flow.

A refinement of theory by Mr. Sherman[2] confirms the conclusion that the flood period is a constant and that the peak flow is proportional to the total volume of runoff, and extends the theory to show that, for a given unit quantity of runoff, uniform over the drainage area, originating in a small unit interval of time, the length of the hydrograph is a constant; that all ordinates will be proportional to the total runoff; and that, by the application of this so-called "unit hydrograph" to successive short periods of uniform runoff throughout the flood-producing storm period, the resulting hydrograph of stream flow could be reproduced with reasonable accuracy. This theory combines sufficient verification in many watersheds with convenience of use to a degree which has

[4] Report of the Committee on Floods of the Boston Society of Civil Engineers, *Journal*, Boston Soc. of Civ. Engrs., September, 1930.

made it acceptable to many in the engineering profession and deserving of the status of an original contribution to fundamental hydrologic theory.

By the limitation of its definition, as pointed out by Mr. Sherman, the unit hydrograph is correct and applicable only to areas from which the runoff for a given period of time is uniform and therefore to drainage areas for which the rainfall is essentially uniform over the entire basin. The extension of the theory to the areas of such size that this condition is not strictly true is a matter of practical expediency, but one which introduces fundamental discrepancies between theory and fact which are not a fault of the theory but of its application. In its original conception and application, the unit period was taken as one day, and the agreement of computed and observed discharges within that period of time was considered to be confirmation of the theory. The accuracy of the determination of the unit hydrograph was limited by the practical requirement of finding an isolated flood hydrograph resulting from a uniform unit rainfall. This latter requirement, while not a serious limitation on the use of the method for periods of one day, provides an almost insurmountable obstacle to the derivation of the unitgraph for very small drainage areas and very short unit periods of time.

A number of writers[5,6] have shown that the theory is equally applicable to very small drainage areas when the unit period used is sufficiently short to be in proper proportion to the total hydrograph.

In determinations of unit hydrographs for short periods of time, practical difficulties of finding uniform, short-period rainfall, which produced large enough stream flow to be analyzed accurately, make it necessary to determine unitgraphs from multiple periods of runoff. However, such a procedure is indeterminate, as there are always more equations than unknowns and a multiple of possible solutions.[7] For example, in determining a seven-period unitgraph from a flood resulting from three periods of runoff, it is possible to determine thirty-six unit hydrographs which, applied to the three items of runoff, will reproduce the composite hydrograph exactly. The approximate solutions are even more numerous. Consequently, the range of possible solutions by independent investigators is too large—far greater than the range of inaccuracy of stream discharge records. Most methods, graphical and mathematical, of selection of the most probable of these unitgraphs underestimate the capacity of a stream to reach a high instantaneous peak from short, intense rainfall and ignore tendencies toward double peaks which some streams exhibit following intense, short-period rainfall.

The range of possible unitgraph determinations can be reduced by correlating recognized discharge concepts with the physical limitations of valley storage and with the time elements which necessarily result from storage and discharge relations. The correlation can be utilized to develop, from time-area concentration curves of specific drainage areas, unit hydrographs with a small

[5] "Relation between Rainfall and Run-off from Small Urban Areas," by W. W. Horner and F. L. Flynt, *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 140.

[6] "The Unit Hydrograph Principle Applied to Small Water-Sheds," by E. F. Brater, *ibid.*, Vol. 105 (1940), p. 1154.

[7] "Engineering Procedure as Applied to Flood Control Reservoirs with Reference to the Muskingum Flood Control Project, Appendix IV," The Engineer School, Fort Belvoir, Va., 1936.

range of determination variability, independent of assumptions regarding runoff distribution in the flood-producing storm and reflecting influences of drainage area shape and stream pattern. The streams of Virginia and North Carolina, with a wide range of basin shapes, stream slopes, and patterns, present admirable opportunities to prove or disprove some of the theories advanced.

In some respects, the degree of proof or disproof is limited by the quality of records and the means of their interpretation. Although recording stream gages have been installed on many of the streams, few of them are rated to a discharge more than one half of the maximum stage of automatic record. For the very flashy streams it is almost impossible for measuring parties to reach the streams before they have fallen well below peak stage. The greatest limitations on the check of accuracy of the theories, however, are the lack of recording rainfall gages and contour maps. Because of rainfall data limitations, determinations of the time of precipitation without a possible error of 3 to 6 hours were seldom possible. If the reproduced hydrographs agreed with the observed within these limits of time, the theories were regarded as confirmed. In all but a few cases, the agreement was much closer than the allowable error.

The constant time elements which have been indicated as essential to the theory of the unit hydrograph are dictated by valley storage relationships. For that reason, some of the vital elements of valley storage concepts are reviewed.

Valley Storage

One of the principal characteristics of the passage of a flood wave down an open channel is the time elapsing between passage at one point and at some downstream point—frequently referred to as the time of travel. The concept of this time of travel as a function of the velocity of the water and therefore influenced by the slope of the river, channel friction, and other items which influence the velocity of flow in the river, is deeply rooted in the hydrology of streams. However, time of travel is more closely related to the variable storage capacity of the open channel; dimensionally, time is the ratio of storage to discharge. Flow in pipes is influenced by all the elements of slope, friction factor, etc., which influence velocity of water in an open channel; but there is no time of travel in a closed pipe, which is full at the beginning of the passage of the wave, where an increase in the velocity and discharge at the upstream end is accompanied almost simultaneously by an increase in velocity and a duplication of the waves at the exit end of the pipe. Time of travel is almost nonexistent in rigid pipes, such as cast iron and concrete, and becomes increasingly apparent in pipes which are elastic enough to have some variable capacity for storage, such as rubber hose and the arteries of the human body.

Most methods of taking account of the storage in a river valley involve the solution, either graphically or mathematically, of the storage equation:

$$\frac{I_1 + I_2}{2} t - \frac{O_1 + O_2}{2} t = S_2 - S_1 \dots\dots\dots\dots\dots\dots (1)$$

in which I is the total inflow rate and O the outflow rate, both in cubic feet per second; S is the total volume in storage in the reach in (cubic feet per second)-hours (12 (cu ft per sec)-hr = 1 acre-ft); and the subscripts 1 and 2 refer to the beginning and the end of the period respectively.[8] This is the arithmetic expression of the storage equation. The calculus equation for the same condition is:

$$dS = (I - O)\,dt \qquad (2a)$$

or

$$\frac{dS}{dt} = I - O \qquad (2b)$$

which is merely a mathematical way of saying that the rate of change of storage is equal to the difference between the inflow and the outflow rates. In Eqs. 2, t is the length of a period of time, in hours.

In general, the quantity of water stored in a short open channel increases with an increase in the flow in the channel, or:

$$S = C\,Q^n \qquad (3)$$

in which Q is the flow in the channel, n is a positive exponent, and C is a constant for the particular channel. Thus the storage in a river is a function of the flow and varies directly with some power of it.

In a short reach of river it may make little difference whether storage is considered as a function of inflow discharge or outflow discharge since there is little difference between them. In long reaches of the length considered practical for flood routing studies or of the length over which appreciable backwater effects may extend in floodtime, there is a great difference.

Methods of accounting for the influence of storage on discharge originally were used in analyzing flood reduction effects of reservoirs, where storage was a function of outflow. The accounting of storage between successive level pool surfaces whose elevation was controlled at the dam was a satisfactory approximation of the total storage. The amount of storage above the slack-water pool level and under the backwater curve was ignored as truly of small consequence.

The backwater storage, however small, existed because of the concurrent existence of inflow to the reservoir. Reduction of the inflow rate necessarily would be accompanied by a decrease in storage and an increase of outflow rate. Rapid inflow shutdown necessarily would be accompanied by a release of the backwater storage and a consequent increase in the slack-water storage or in the outflow discharge or in both. The released volume of water would be free to roll, as a pressure wave, to the lower end of the reservoir. Others have compared the movement to the rotation of saturated soil in a landslide. In either case, the lower end of the reservoir would have to rise slightly and the outflow discharge would have to increase.

The extension of concepts of storage modification to sloping channels and natural streams extended the practise of correlating storage with outflow. In

[8] "Flood Routing," by Edward J. Rutter, Quinton B. Graves, and Franklin F. Snyder, *Transactions*, Am. Soc. C. E., Vol. 104 (1939), p. 278.

some river reaches, the correlation with inflow is as close as the correlation with outflow. Some current concepts of the action of valley storage in reducing flood peak discharge are faulty because they are in terms of the original, limited theory. The storage which may be treated as a function of outflow reduces flood peak outflow discharge, but the portion which is a function of inflow increases outflow, so that the net decrease or increase of peak discharge depends on which storage portion is the greater.

The idea that there is some valley storage which does not decrease flood peaks is rather new. The nature of the actions is illustrated in Fig. 1, which

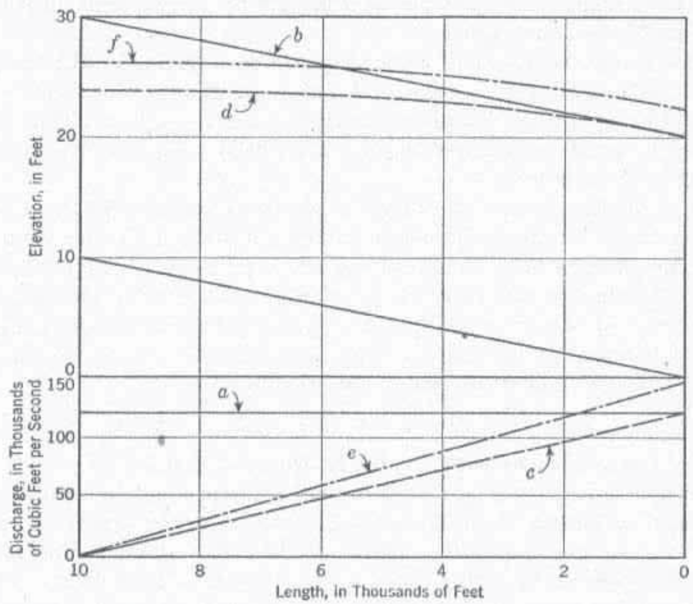

FIG. 1.—EFFECT OF DECREASING INFLOW ON FLOOD PROFILE

shows water-surface profiles for a uniform, rectangular channel 500 ft wide, 20 ft deep, and with a fall of 10 ft in its length of 10,000 ft. Line b shows the profile for steady flow of 116,000 cu ft per sec throughout the channel length. Assume that the steady flow into the channel at the upper end is reduced quickly as by the closure of gates on a reservoir or the crevassing of a levee. With discharge reduced at the upper end of the reach and maintained at the lower end by storage release, the distribution of discharge in the reach would be similar to line c to which profile d would apply. The change from profile b to profile d involves the release from storage of 260 acre-ft during the period of inflow reduction and requires the outflow discharge to increase to accommodate such a release. However, as the outflow increases, the discharge distribution in the channel approaches profile e and the water-surface profile approaches that indicated by line f. Storage under profile f is the same as under profile b.

A shift of water-surface profile from lines b to f takes place in a natural channel by mass rotation transmitted by means of pressure waves of small amplitude which surge back and forth through the reach, ceasing when the stable profile for the new discharge distribution has been attained. In natural channels under normal flow conditions, the reductions of inflow rates are gradual with the greater changes of discharge distribution occurring near peak stage. The surge of small amplitude transition waves is clearly apparent as a broad trace in many autographic records of stage. Following discharge change, the profile transition takes place in a very short time. The writer has a record of a natural flow reduction, caused by the crevassing of a large levee, in which the effect of the break in transforming the water-surface profile was transmitted 17 miles in as many minutes, whereas a normal flood wave transit of the same distance requires about 15 hours.

In the example of Fig. 1, if the water-surface elevation at the lower end is held constant by opening the gates of a dam, a material increase of outflow rate would be required. To prevent any increase in outflow discharge, a rise of about 3 ft would be required to store the initial volume as it shifted to the lower end of the reservoir.

In an unregulated reservoir of large capacity compared with the inflow to it, the storage may be solely a function of the outflow. In a long and uniform channel, the storage may be almost equally a function of the inflow and the outflow. All during the early stage of rapid increase in discharge in dry channels, such as occurs naturally in the arroyos of the Southwest, the storage is solely a function of the inflow. Thus, Q in Eq. 3 is to be considered as a weighted average value of the inflow and the outflow:

$$Q = x\,I + (1 - x)\,O \qquad (4)$$

in which x is a coefficient with a value between 0.0 and 1.0.

According to Eq. 3, storage tends to increase with some power of the stream flow. There seems to be no theoretical reason why the power could not be either greater or less than unity. If it were less than unity, the peaks of large floods would occur earlier than those of small floods; if the power were greater than unity, the reverse would be true. If the power is unity, storage and discharge are in constant direct proportion, and time elements are constant regardless of flood magnitude.

Storage in a river channel does not necessarily increase directly with the discharge, but studies of the variation in a large number of streams indicate that this is the case for most smaller streams and is a condition closely approximated over large ranges of discharge in many large streams. That is, the exponent is 1.0 for small streams, and for many rivers of as much as 10,000 sq miles of drainage area it is less than 1.3. Even for the latter streams, over a large range the true storage may be closely approximated by storage directly proportional to discharge.

Storage which is directly proportional to discharge does not interfere with the accuracy of the unit hydrograph theory, as its characteristically constant time elements are compatible with, and necessary to, the similarly constant time elements of the unit hydrograph. Wherever the unitgraph theory is applicable, or wherever the time elements of flood passage are essentially constant, the storage is proportional to discharge.

There are a great many mathematical and graphical solutions for the storage equations, both for conditions where the storage is considered to vary directly with the discharge and where it is considered to vary directly with some exponential power of the discharge. All but a few of these solutions are for the special case in which the storage is a function of the outflow only and, therefore, are applicable only to routing floods through a very large reservoir. Use of these special solutions for natural channels, or for small reservoirs in which backwater storage is appreciable, leads to undesirable optimism regarding reductions of discharge by storage use.

The scope of this paper is limited to the general conditions in which the storage is considered to vary directly as the discharge, but where the discharge is a weighted value of the inflow and the outflow. Mathematically, these statements resolve to the three equations previously given:

$$dS = (I - O)\,dt \qquad (2a)$$

$$Q = x\,I + (1 - x)\,O \qquad (4)$$

$$S = K\,Q \qquad (5)$$

from which it is evident that:

$$K = \frac{dS}{dQ} = \frac{(I - O)\,dt}{x\,dI + (1 - x)\,dO} \qquad (6a)$$

and

$$O = I - K\,x\,\frac{dI}{dt} - K\,(1 - x)\,\frac{dO}{dt} \qquad (6b)$$

Eq. 6b can be solved arithmetically for successive short periods of time, T in length, by designating the flows at the beginning of the short period as O_1 and I_1 and those at the end by O_2 and I_2, thus:

$$\frac{O_1 + O_2}{2} = \frac{I_1 + I_2}{2} - K\,x\,\frac{I_2 - I_1}{T} - K\,(1 - x)\,\frac{O_2 - O_1}{T} \qquad (6c)$$

from which the following simple equation evolves:

$$O_2 = C_0\,I_2 + C_1\,I_1 + C_2\,O_1 \qquad (6d)$$

In Eqs. 6:

$$C_0 = -\,\frac{K\,x - 0.5\,T}{K - K\,x + 0.5\,T} \qquad (7a)$$

$$C_1 = +\,\frac{K\,x + 0.5\,T}{K - K\,x + 0.5\,T} \qquad (7b)$$

$$C_2 = \frac{K - K\,x - 0.5\,T}{K - K\,x + 0.5\,T} \qquad (7c)$$

and

$$C_0 + C_1 + C_2 = 1.00 \qquad (7d)$$

This is the Muskingum method of flood routing developed by T. S. Burns and F. B. Harkness, Members, Am. Soc. C. E., and G. T. McCarthy,[7] Assoc. M. Am. Soc. C. E.

To illustrate the action of linear storage upon wave forms, two mathematical curves have been routed through various quantities of storage by the Muskingum method. Fig. 2 illustrates the modification of a cosine-type curve by storage represented by a value of K of four time units, with various values

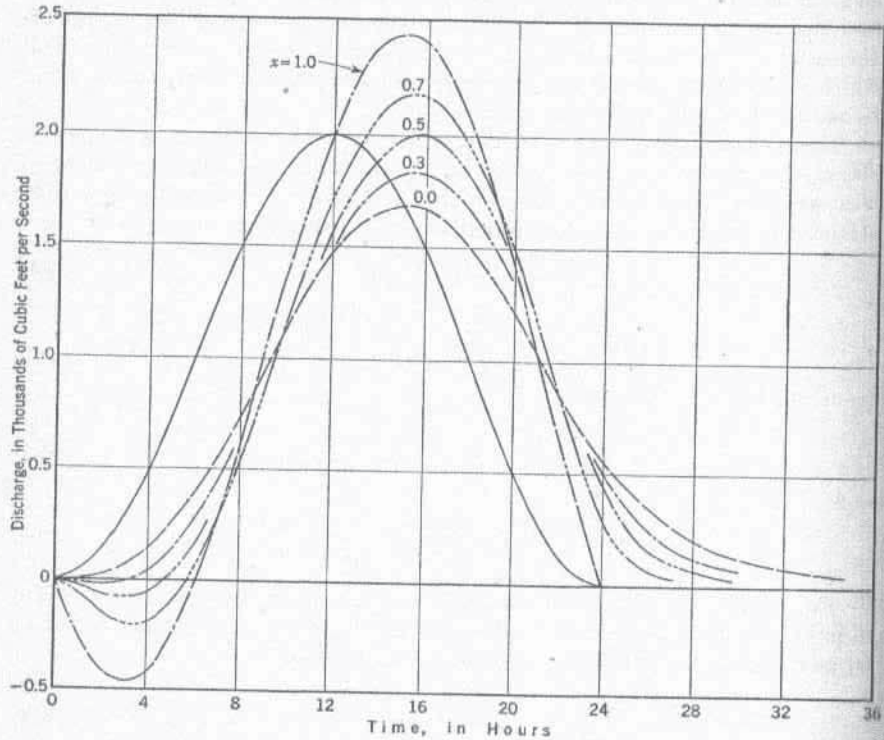

Fig. 2.—Modification of Cosine Type of Wave by Storage

of x from 0.0 to 1.0. For this well-rounded initial wave form, the fundamental effect of the storage modification is to translate the entire wave approximately four time units, and to change the magnitude of the peak. The change of magnitude seems to be more a function of the value of x than of K, because when $x = 0.5$, the modified wave appears to be a close duplication of the initial wave, except for the parts of the curve before initial outflow and after final inflow.

It may be desirable to explain the unorthodox appearance of the modified wave forms (Fig. 2) between the initial inflow and initial outflow. The indicated negative values are partly erroneous and partly correct. A slight error arises in the theory assumption that the average discharge in the reach of river is indicated by a weighted value of the discharge at the ends and, therefore, that the storage prism between successive positions of the rising water surface may be approximated by two wedges of storage extending the length of the reach, one a function of inflow and the other a function of outflow. Consider-

ing the real bulbous shape of an advancing wave front in a near-dry channel, the true storage in the reach is less than the full-length wedge during the first part of the initial inflow. During this period, the actual outflow discharge is more than computed. At the instant the wave front reaches the lower end, the actual storage in the reach is somewhat more than that of a full-length wedge. At this time and for a short time after, the actual discharge at the lower end of the reach is less than indicated by the theory and the actual rise is later than that calculated. The inequalities between the theory and fact of wedge storage approximation are ironed out by the time the rates of flow increase at both ends are equal. Since the discharge at the lower end is closely indicated at this time, the true rise at the downstream end is more rapid than computed. The maximum total storage volume in the reach, represented by the area between the inflow and outflow hydrographs, is the same at this time for actual and computed modifications.

In the case of a wave passing into a partly full channel, into which flow is also coming from local streams, much less of the indicated negative discharge is to be regarded as error. The negative values indicate the capacity of the advancing wave not only to store its own waters, but to force the storage of the incoming local flows through the induced backwater effect. Only when the algebraic sum of all components of flow (including the negative values of the routed inflow wave) is less than the flow prior to the flood can the sum be discounted as error. Even then, the appearance of large negative values is an indication of a period of hydraulic instability which in extreme cases results in the formation of a hydraulic bore.

The negative values play an important part in the accuracy of the general storage theory and are to be discounted judiciously. The writer follows the practise of retaining the computed negative values, discounting them only in the drawing of the final hydrograph.

Fig. 3(a) shows the effect of storage that is equally a function of inflow and outflow; that is, $x = 0.5$ on a wave form with an initially very sharp peak. The first wave has been routed through 2 hours of storage in each of five successive reaches. In this case as in the case of the cosine curve, the translation effected by storage is approximately equal to the value of K; but some reduction of peak discharge is effected. However, the reductive influence of the storage decreases as the initial wave approximates the stable, well-rounded form.

The actual progressive modification by storage which is almost equally a function of inflow and outflow rates, illustrated in Fig. 3(a), has about the same effect as the bodily transposition of the initial wave form combined with a final modification of the transposed wave by a small amount of reservoir storage. The difference between the progressive calculation of storage effect, which is probably closer to truth, and the more easily computed transposition and single storage modification calculation is illustrated in Fig. 3(b), in which the wave form (a) has been progressively modified by passage through five reaches, each having 2 hours of storage (2 (cu ft per sec)-hr per cu ft per sec of discharge) to become wave form (b). The approximation wave form (c) was derived by bodily transposition of wave form (a) by 9 hours and routing it through reser-

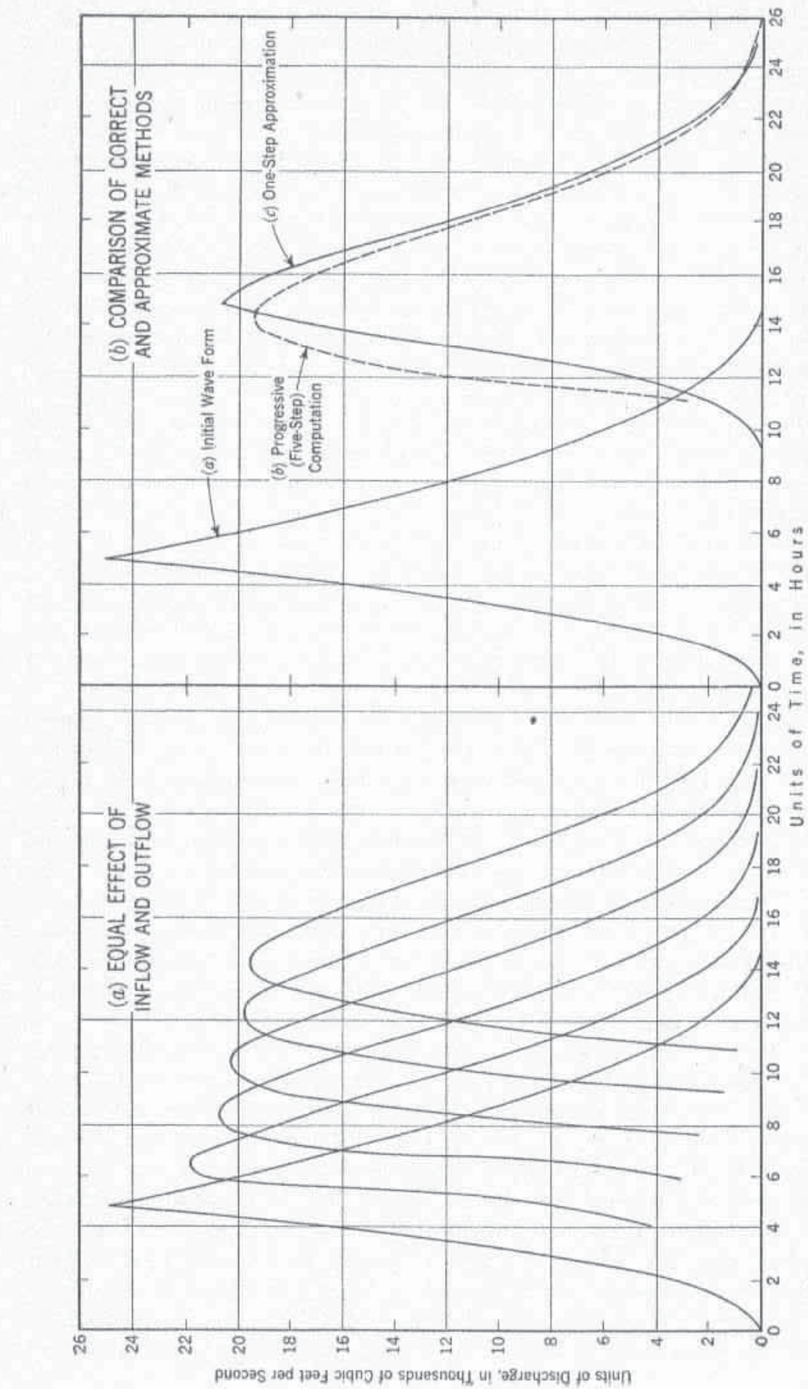

FIG. 3.—MODIFICATION OF A SHARP-CRESTED WAVE BY STORAGE

voir-type storage ($x = 0.0$) of 1 hour. The agreement between curves (b) and (c), far from perfect, is within the degree of accuracy usually attainable in runoff calculations.

This approximate form of storage calculation is used subsequently in the derivation of unit hydrographs; a time-area concentration curve of the drainage area (a composite of all of the transposed, unmodified wave elements of the watershed) is modified by a small amount of storage to form the basic unit-graph. The procedure will be explained in detail.

For many natural streams, the value of x, in Eq. 4, varies from 0.4 to 0.5, but the presence of a large flood plain carrying little flow near the lower end of a reach will cause the storage to be more of a function of outflow than inflow, and will reduce the relative influence of inflow, x, to 0.3 or lower. The back-water effect of a large incoming tributary may act in a similar manner. Conversely, if the large storage is near the upper end of the reach, the value of x may exceed 0.5 and result in an increase of crest discharge rather than the usually assumed decrease (the hydrographs in Fig. 12, introduced subsequently, indicate such an influence).

An understanding of the mechanical process of flow of water through a reach of river or a reservoir in light of the Muskingum routing theory will provide a clearer concept of what can and cannot be accomplished by valley storage. It also will show that, contrary to popular concept:

(a) Valley storage does not always decrease flood peaks; and

(b) The reservoir pool above a dam may not reduce flood discharges. In fact, if the pool level at the dam is held constant by gate control, the storage becomes the function of inflow solely, and the peak outflow may be larger than the peak inflow. Furthermore, such control, by reducing the effective storage, may decrease the time of travel through the reservoir to almost nothing, with a beneficial or detrimental effect as the case may be.

SIGNIFICANT STORAGE RELATIONSHIPS

The fundamental storage equation for storage which varies directly with a weighted average of the inflow and the outflow is (see Eq. 5b):

$$O + (1 - x)\, K \frac{dO}{dt} = I - K\, x \frac{dI}{dt} \quad \text{(8)}$$

Although the Muskingum equations are easier to use in routing procedure, a knowledge of the following relationships which exist at certain points in the hydrographs of inflow and outflow is of great value in determining the quality and quantity of the storage effect in the reach. The symbols, $\frac{dI}{dt}$ and $\frac{dO}{dt}$, are used to represent the slope of the inflow and outflow hydrographs. When the slope of inflow and outflow hydrographs is equal:

$$\frac{dI}{dt} = \frac{dO}{dt} \quad \text{(9a)}$$

and

$$I - O = K \frac{dO}{dt} \quad \text{(9b)}$$

When inflow and outflow are equal:

$$I = O \qquad (10a)$$

and

$$\frac{dO}{dt} = -K \frac{x}{1-x}\frac{dI}{dt} \qquad (10b)$$

At peak inflow:

$$O = I - K\frac{dO}{dt}(1-x) \qquad (11a)$$

At peak outflow:

$$O = I - Kx\frac{dI}{dt} \qquad (11b)$$

When inflow has ceased:

$$O = -\frac{K\,dO}{dt}(1-x) \qquad (12a)$$

and

$$K = -\frac{O}{\frac{dO}{dt}(1-x)} \qquad (12b)$$

Volume of runoff remaining in storage is

$$K\,O = -\frac{O^2}{\frac{dO}{dt}(1-x)} \qquad (13)$$

Valley Storage and the Unit Hydrograph

A few years ago, the unit hydrograph was thought to be inapplicable to streams having a great amount of valley storage. However, when valley storage is defined as "all of the volume displaced by water during the passage of a flood wave, including the volume in the river channel," it is evident that no flood wave is unaffected by valley storage.

From a different point of view, the nature of the effect is clear. The essential difference between nonsteady flow in open channels and in pipes is in the time that elapses in the former only between the change of inflow rate and the change of outflow rate. The essential difference between an open channel and a pipe is the variable capacity of the former for storage as compared with the constant capacity of the latter. The time element and the storage capacity are related, for the ratio of storage capacity to discharge capacity is time; that is, $\frac{S}{Q} = T$.

The unit hydrograph theory assumes that the ordinates of any flood hydrograph originating from uniform runoff in the unit period of time will be directly proportional to the quantity of runoff, but that the abscissas, or time elements, will be constant and, therefore, unaffected by magnitude of discharge. Since the time elements are a function of the ratio of storage to discharge but not of discharge alone, it follows that the ratio of storage to discharge must be a constant throughout the entire range of discharge to which the unit hydrograph is to be applied. In other words, the storage-discharge curve for all reaches of river above the point to which the unit hydrograph is to be used must be

straight lines when plotted on rectangular coordinate graph paper. This is the simplest of the storage discharge relationships for which methods of solution have been devised—the basis of the Muskingum method outlined by Mr. McCarthy.[9] An understanding of the manner in which it is possible for this type of storage to modify a flood wave will provide an understanding of several significant relationships which must exist in any actual unit hydrograph if the theory is applicable to the drainage area being studied.

The Hydrograph as an Index of Storage

Any flood hydrograph is the resultant of all of the factors that have created it. Although no exact determination of the effect of any factor is possible without the segregation of the results of other influences, it is possible to determine the effect between certain limits. The accuracy of the determination will depend on the degree to which other influences can be segregated or discounted as insignificant.

The analysis of the hydrograph, as an index of previous storage effect, is based on the hypothesis that, after inflow to the channel has ceased, all water which will eventually become runoff, but which has not yet passed the gage, is in storage (axiomatic), and that such storage varies directly with the discharge (approximate, approaching truth in very small impervious watersheds). After such a time, the ratio of rate of change of discharge to total discharge reaches its greatest negative value, and remains constant thereafter:

$$\frac{\frac{dO}{dt}}{O} = -\frac{1}{K(1-x)} \qquad (14)$$

If all storage in a stream above a gage has been equally a function of inflow and outflow (that is, if $x = 0.5$), the greatest ratio of decrease of discharge to total discharge will correspond closely to a K-value of the time elapsing since the cessation of inflow. For the same amount of total storage, entirely a function of outflow, the ratio would have only half as large a magnitude, indicating a lesser slope of the depletion curve.

The falling legs of a large number of hydrographs of flood discharge for streams in major drainage basins from New Hampshire to North Carolina exhibit a ratio of decrease of discharge to total discharge which, for a value $x = 0.5$, indicates a value of K closely approximating the time elapsing between termination of runoff producing rainfall and the point of most rapid relative decrease in discharge. The fact that the hydrographs fall so rapidly is highly indicative that the storage affecting the hydrographs is of the balanced variety, and, therefore, chiefly effective in the translation of waves.

The writer has indicated that the effect of such storage may be approximated by (a) the bodily transposition of the runoff wave element from its source to the gage, and (b) the routing of the element through some amount of storage less than the total with a value of $x = 0$. The summation of all of the transposed elements for an entire drainage area is the same as a time-area concentration

[9] "The Unit Hydrograph and Flood Routing," by Gerald T. McCarthy, unpublished manuscript presented at the Conference of the North Atlantic Div., Corps of Engrs., U. S. Army, New London, Conn., June 24, 1938; printed by the U. S. Engr. Office, Providence, R. I.

curve whose base length is the time between cessation of runoff and the time of most rapid relative decrease in discharge. The value of K selected for the routing of the time-area concentration curve is one corresponding to the ratio of discharge decrease to total discharge for an observed hydrograph (Eq. 12b with $x = 0.0$).

The application of this method to a theoretical drainage area of rectangular shape with uniform instantaneous rainfall over the entire drainage basin is shown in Fig. 4(a). If the rainfall were to remain constant for a unit period

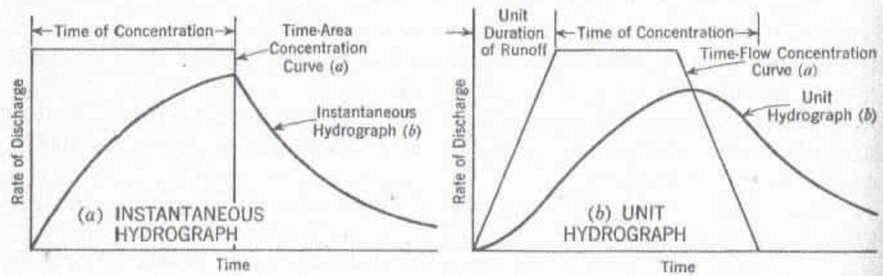

FIG. 4.—HYDROGRAPHS FOR A RECTANGULAR DRAINAGE AREA

of time, as is predicated in the theory of unit hydrographs, the flow concentration curve which was the time-area diagram (curve (a), Fig. 4(a)) is modified to become the time-flow diagram (curve (a), Fig. 4(b)) which, routed through the same storage, becomes curve (b), Fig. 4(b). The advantage of an instantaneous hydrograph is that it can be derived from the fundamental characteristics of the basin and then used with any length of unit period to determine the unit hydrograph, in which any ordinate of discharge is the average ordinate of the instantaneous hydrograph for the unit time period previous to time desired. Where the curvature of the instantaneous hydrograph is not excessive, the ordinate can be determined by averaging ordinates of that graph separated by the unit time period desired.

DETERMINATION OF THE INSTANTANEOUS HYDROGRAPH FROM STREAM FLOW RECORDS

Where adequate planimetric maps of a stream are provided in which recorder rainfall and discharge records are available, the elements to be determined are the base length of the time-area concentration curve, or the time of concentration, the time-area curve, and the magnitude of the unbalanced storage effect or the amount of valley storage (solely a function of discharge) by which the time-area curve is to be modified.

Referring to Eq. 12b, it will be seen that, when the inflow to the storage reach has ceased, the ratio of decrease of discharge to total discharge is a constant; or, as the Muskingum flood routing equation expresses it, the outflow at the end of any period of time bears a constant ratio, C_2, to the outflow at the beginning of the period of time.

Plotting the value of K which is indicated by the rate of decrease of discharge throughout the falling hydrograph, the indicated value should decrease

to a minimum at the time that inflow ceased and remain constant thereafter. The time between cessation of runoff-producing rainfall and the minimum indicated value of K is the writer's concept of time of concentration and is so used in the remainder of this paper.

Taking the time of concentration derived in this manner as the time base length, a time-area concentration curve for the drainage area above the gage is prepared. This curve is used as the inflow to the river channel and is routed through an amount of storage represented by the indicated minimum K-value and an x-value of zero.

EXTENSION OF STORAGE THEORY TO GROUND-WATER FLOW

In practise, the indicated value of K does not remain constant after reaching its minimum value. The inflow from ground water continues and does not

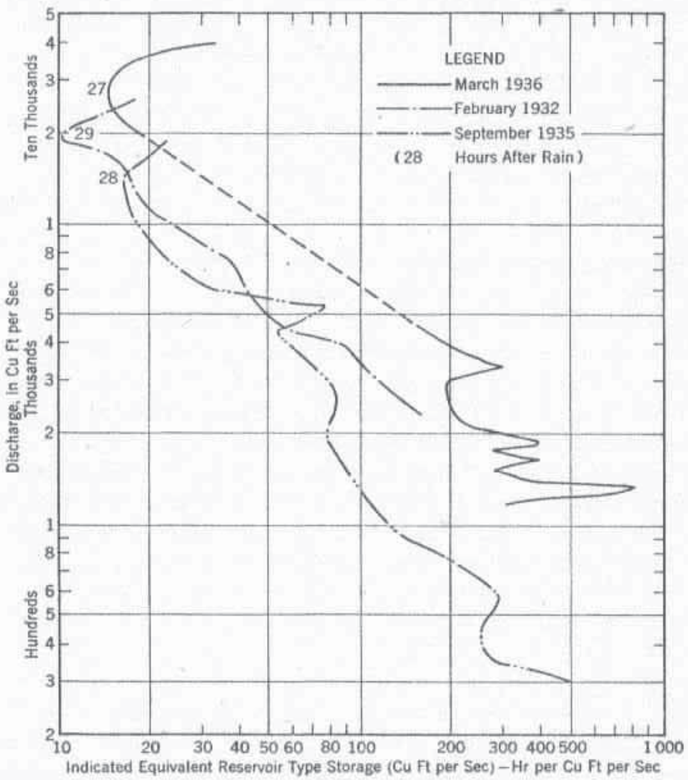

FIG. 5.—DETERMINATION OF EFFECTIVE STORAGE FROM RECESSIONAL HYDROGRAPH; JAMES RIVER AT LICK RUN

decrease as rapidly as the storage-controlled surface runoff. Therefore, it becomes an increasingly larger percentage of the total discharge and increases the indicated value of K.

Actual curves of the indicated value of K are plotted in Fig. 5 against the discharge rate for the major part of the falling leg of several hydrographs at a stream gaging station at Lick Run, Va., on the James River (drainage area = 1,369 sq miles). The indicated value of K decreases to a certain minimum value which is essentially constant for all hydrographs and then increases from that point as the discharge decreases, becoming very large as the discharge becomes very small.

Application of this theory of runoff would imply a definition of surface runoff as being that quantity of total runoff which, after the cessation of inflow, is discharged in direct proportion to the amount of flow remaining to run off. Ground-water flow would include all other flow.

Since ground water is detained by storage also, it would be convenient for hydrologists if that storage were found to be in direct proportion to the rate of discharge. In analyzing the discharge hydrograph of any stream, it is rare to find one which, after reaching an abnormally high peak, continues to fall without further rainfall until it has reached an abnormally low flow. However, this was the case following the flood of September, 1935, on the James River. In that hydrograph, the discharge at Lick Run increased from 271 cu ft per sec to 19,400 cu ft per sec and then fell steadily to 258 cu ft per sec, the lowest discharge of the year. During the falling period, the rainfall was so small and scattered that it is doubtful if it contributed in any way to the stream flow. The indicated K-curve for this hydrograph is among those shown in Fig. 5.

After studying a large number of hydrographs extending into periods of low flow, both for this station and for several other stations in the James River basin, an approximate value of K of 200 hours was selected as an average representation of the ground-water storage effect in that watershed. For this station and for three others of good record, five hydrographs for each station were separated into the two volumes which could be represented by the channel storage modification outlined and the subsurface (ground-water) flow released from storage of 200 hours. No hydrographs were found in which the subsurface part was less than 30% of the total runoff, and there were no major flood hydrographs in which it exceeded 42%. Using a value of K of 200 hours and of $x = 0.0$, another unitgraph for subsurface flow was derived. Combination of 70% of the surface unitgraph and 30% of the subsurface unitgraph provided an excellent practical unitgraph for this drainage area, which included both surface and subsurface flow.

Thus in the derivation of a unitgraph for instantaneous origin of runoff, by the method proposed by the writer, a unitgraph of surface runoff, derived by routing the time-area concentration curve through an amount of unbalanced storage as outlined herein, is combined with a unitgraph of subsurface runoff determined by routing the same histogram through an appropriate amount of subsurface storage. What the ratio of these components will be in the final hydrograph will depend on the pertinent fact of how much of the water comes from surface runoff and how much from subsurface runoff in the particular watershed. For the streams analyzed along the eastern Appalachian slope, a 70 : 30 ratio seemed quite satisfactory. (If the additional work is justified, separate unitgraphs of surface and subsurface runoff may be utilized. With such use, no assumption of a fixed ratio is necessary.)

Although such a division is arbitrary, it is not as unsatisfactory as the current practise of attributing to ground-water discharge all flow beneath a straight line drawn from a point before the rise to an arbitrary and undefinable point on the descending hydrograph. The volume of flow so neglected frequently contains more than 20% of the total runoff. Any determination of runoff from subsequent storms, to be used in a unitgraph so derived, must be made in the light of such a derivation, with the result that, in times of great floods, the runoff percentage used for the flood-producing precipitation may be less than the average percentage runoff for the month corrected for prior and subsequent ground water.

Derivation of the Instantaneous and Unit Hydrographs

For the purpose of illustrating the method of derivation of unitgraphs, the Appomattox River at Petersburg, Va., was selected. There are several reasons why the drainage basin is not a good location for the application of the unitgraph principle; but, because of its peculiar shape, it is particularly adapted to the illustration of the benefits inherent in the method of unitgraph derivation proposed. Because it was not used in any way in the derivation of any of the data or principles on which this method is predicated and because the writer

TABLE 1.—Derivation of Instantaneous Hydrograph and 12-Hr Unitgraph on Appomattox River Near Petersburg

Time, in days	Time-area curve (%)	Routing Calculation[a]			Surface flow = 0.70 × Col. 5	Sub-surface flow 0.30[b]	Instantaneous Hydrograph		12-hr unit hydrograph (cu ft per sec[e])
		$C_0 I_2 + C_1 I_1$ = 0.56 × Col. 2	$C_2 O_1$ = 0.44 × Col. 5	Routed time-area curve = Col. 3 + Col. 4			% per 12 hr[c]	Cu ft per sec[c]	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	....	....
0.5	1.8	1.01	0.0	1.01	0.70	0.03	0.73	500	250
1.0	3.8	2.13	0.44	2.57	1.80	0.10	1.90	1,300	900
1.5	6.9	3.86	1.13	4.99	3.50	0.22	3.72	2,550	1,920
2.0	10.8	6.05	2.19	8.24	5.75	0.40	6.15	4,220	3,380
2.5	19.1	10.70	3.63	14.33	10.00	0.72	10.72	7,360	5,790
3.0	7.6	4.25	6.32	10.57	7.40	0.81	8.21	5,630	6,490
3.5	6.5	3.64	4.64	8.28	5.80	0.88	6.68	4,580	5,100
4.0	5.5	3.08	3.64	6.72	4.70	0.93	5.63	3,870	4,230
4.5	9.0	5.04	2.96	8.00	5.60	1.03	6.63	4,550	4,210
5.0	14.0	7.83	3.52	11.35	7.80	1.22	9.02	6,180	5,360
5.5	9.5	5.32	4.98	10.30	7.20	1.32	8.52	5,850	6,010
6.0	5.5	3.08	4.53	7.61	5.33	1.34	6.67	4,580	5,210
6.5	....	....	3.35	3.35	2.35	1.25	3.60	2,470	3,520
7.0	....	....	1.47	1.47	1.03	1.18	2.21	1,510	1,990
7.5	....	....	0.65	0.65	0.46	1.11	1.57	1,110	1,310
8.0	....	....	0.29	0.29	0.20	1.04	1.24	850	980
8.5	....	....	0.13	0.13	0.09	0.98	1.07	735	800
9.0	....	....	0.06	0.06	0.04	0.92	0.96	660	700
9.5	....	....	0.03	0.03	0.02	0.86	0.88	600	630
10.00	....	....	....	....	....	0.81	0.81	560	580[d]

[a] Cols. 3, 4, and 5 are routing of Col. 2 with $K = 15$ hr, $x = 0.0$. [b] Col. 7 equals 0.30 × Col. 2 routed with $K = 200$ hr, $x = 0.0$. [c] Instantaneous rate of discharge. [d] 116,000 (cu ft per sec)-hr, or 13%, remaining to run off. Successive values of unitgraph hereafter are 0.942 × preceding value.

data or principles on which this method is predicated and because the writer had done no work on this stream previous to the present calculation, this basin was used. The calculations in Table 1 show the first computations made and neither the original data, the first interpretations of these data, the calculations, nor the plotting, have been changed or altered in any way to make the computed hydrograph agree more closely with the observed hydrograph. (Only a

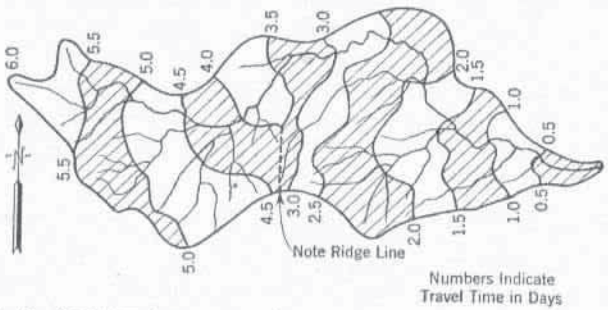

FIG. 6.—DRAINAGE AREA OF THE APPOMATTOX RIVER ABOVE PETERSBURG

slight change in estimated time distribution of runoff is necessary to improve the agreement.) This stream has a flatter slope and a longer time of concentration than any other stream of comparable drainage area in its immediate vicinity and therefore is adapted to illustrating some of the faults as well as the good points of the application. Fig. 6 shows the drainage area of the Appomattox River above Petersburg and the manner in which it was divided to provide the data for the time-area concentration curve; Fig. 7 is the profile

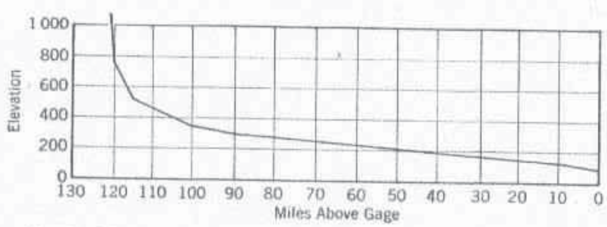

FIG. 7.—PROFILE OF THE APPOMATTOX RIVER ABOVE PETERSBURG

of the river above the gaging station as taken from the best available U. S. Geological Survey topographic sheets; and Fig. 8, curve (a), shows, in bar diagram form, the time-area concentration curve converted to flow for 1 in. of runoff; and curves (b) and (c) show the time-area curve after routing through 9 hours and 200 hours, respectively, of storage. A combination of 70% of curve (b) and 30% of curve (c) is the instantaneous hydrograph, curve (d), for the Appomattox River at Petersburg. Fig. 9 shows the application of the 12-hr unitgraph derived from the instantaneous hydrograph to the flood of April, 1937. This is the second largest flood for which there is any available record at Petersburg and one of the largest floods that has occurred in the past 100 years. (The largest flood, August, 1940, caused by protracted rainfall, did not exhibit a double peak but is equally well reproduced with the same unitgraph.)

The following statements of fact must be taken into consideration in judging the degree of accuracy with which the computed record follows the observed record:

1. The area is too large for the proper application of a single unitgraph (drainage area, 1,335 sq miles). Because of the direction of the stream in comparison with the general east and west travel of storm fronts in this vicinity and because of the long time of concentration, a difference in rainfall both in the

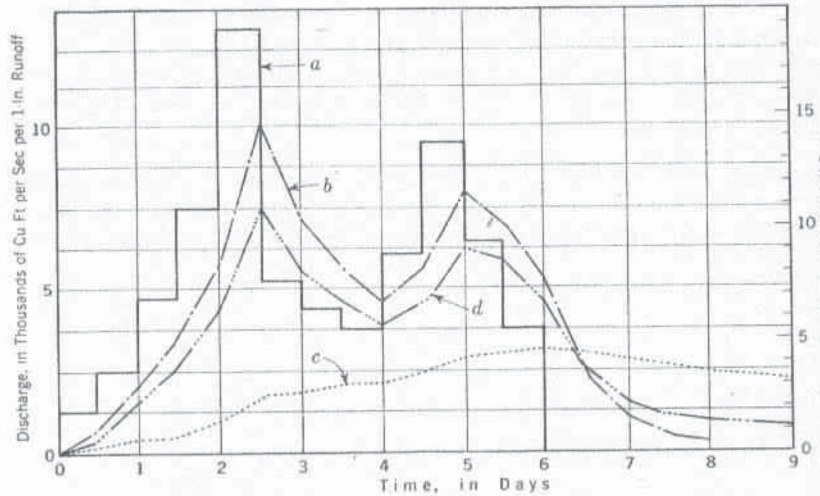

FIG. 8.—TIME-AREA CONCENTRATION CURVE, APPOMATTOX RIVER

quantity and in time on the east part and west part of the drainage area may be considerable as it was in this case. Much better results could be obtained by using separate unitgraphs at the same gaging point for the upper and the lower drainage areas.

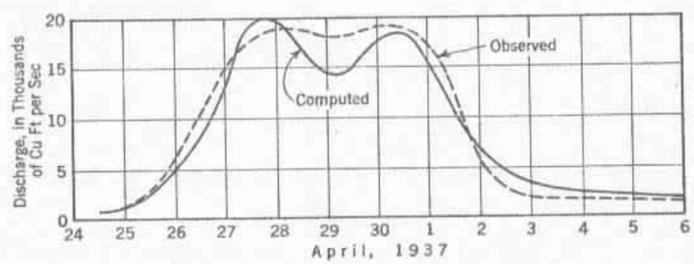

FIG. 9.—APPLICATION OF THE 12-HR UNITGRAPH, DERIVED FROM THE INSTANTANEOUS HYDROGRAPH, TO THE FLOOD OF APRIL, 1937

2. A more than usual degree of doubt as to the actual quantity of total precipitation is present in view of the small number of gaging stations (only one in the drainage basin), the nearest recording rainfall stations being some miles outside. The station in the center of the upper drainage area recorded 5.18 in.

of rain as compared with 2.58 in. recorded at Hopewell, Va., a station several miles east of Petersburg.

3. Observed discharges are from a single-stage discharge curve without slope correction. The gage is on a very flat stream and only 2.5 miles above a power dam. Although the gage is reported to be out of the backwater area in times of low water, the control for high water is thought to be the dam, in which case there is probably a large slope correction to discharges at this gage. If so, the observed stage hydrograph could not reflect the marked discharge decrease on April 29. At peak stage, 1 ft on the stage discharge curve is 3,000 cu ft per sec. This, and other features of the reported record, are sufficiently in doubt to cover the variation between observed and computed hydrographs.

4. On the other hand, the method may be no more accurate than is indicated by the agreement of the curves.

A comparison between the instantaneous unitgraph for the Appomattox River near Petersburg (drainage area, 1,335 sq miles; average slope of the river profile, 2.9 ft per mile; length above gage, 122 miles; and concentration time,

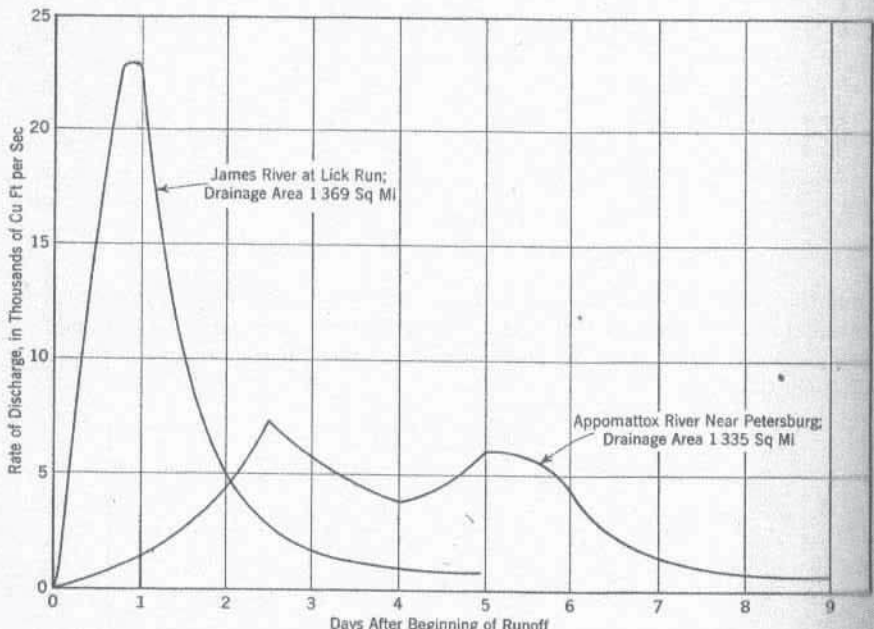

FIG. 10.—INSTANTANEOUS HYDROGRAPHS, APPOMATTOX RIVER NEAR PETERSBURG AND JAMES RIVER AT LICK RUN

144 hours) and for the James River at Lick Run (drainage area, 1,369 sq miles; average slope 17.3 ft per mile; length, 78 miles; and concentration time, 28 hours) is shown in Fig. 10. Fig. 11(*a*) shows a similar comparison between the instantaneous hydrographs of the Smith River at Bassetts, Va. (drainage area, 265 sq miles; average slope, 16.6 ft per mile; length, 37 miles; and concentration

time, 7 hours) and for the Meherrin River at Emporia, Va. (drainage area, 750 sq miles; average slope, 2.4 ft per mile; length, 86 miles; and concentration time, 78 hours). Fig. 11(*b*) shows a comparison of observed and computed flood hydrographs at Bassetts for the flood of August 18, 1939, and a similar comparison at Emporia for the flood of March 6, 1932.

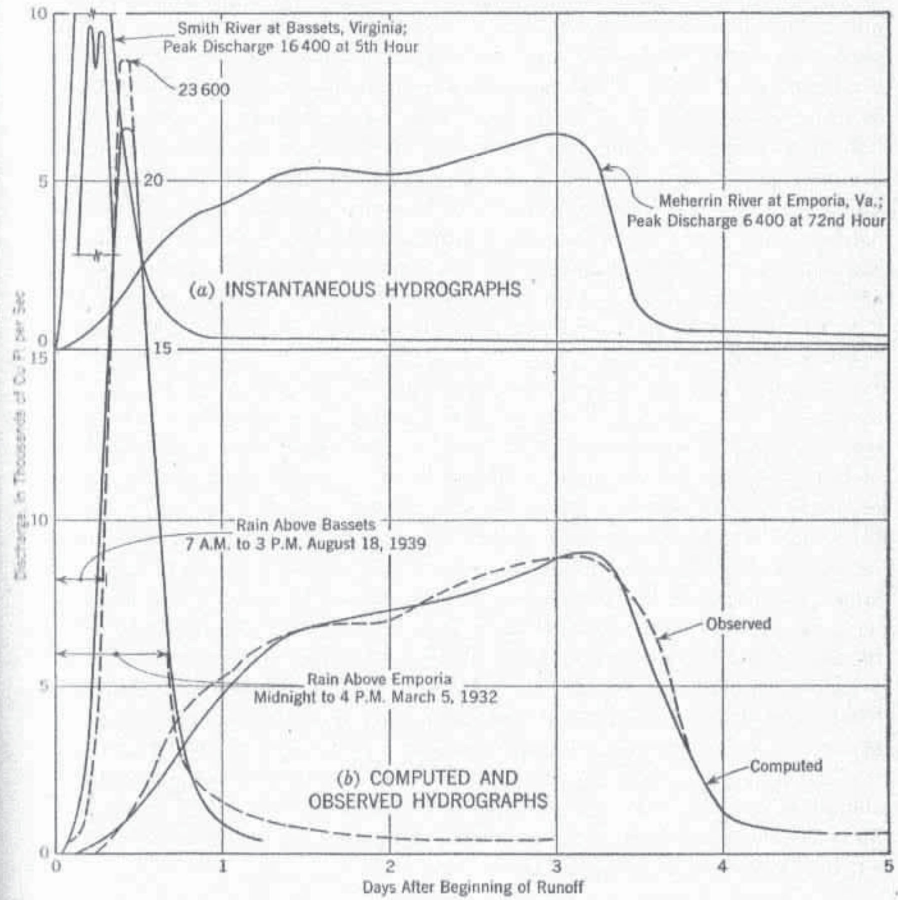

FIG. 11.—HYDROGRAPHS FOR THE SMITH RIVER AT BASSETTS AND THE MEHERRIN RIVER AT EMPORIA

DETERMINATION OF THE INSTANTANEOUS HYDROGRAPH FOR STREAMS WITHOUT FLOW RECORDS

The determination of any kind of unit hydrograph for a stream without records is a hazardous task. In most cases, it will be possible to install river stage recorders temporarily, obtain a record of rise and fall, compute a rating curve, and determine the elements of time of concentration, K and x, with a fair degree of accuracy. Since the record is utilized chiefly to determine the time of concentration, the proposed derivation of a unit hydrograph from a record of a small flood is not subject to the errors incurred by use of similar records in unit hydrograph derivations as originally proposed by Mr. Sherman. However, it is necessary occasionally to approximate the flow characteristics of a stream without records. For such attempts, a correlation of time of concentration or storage capacity with river slope would be adequate.

Attempting to correlate the storage per mile of river expressed as hours per mile (the reciprocal of velocity) with average slope of the stream above the gage, the writer found, as would be expected, that this item increased with the decrease of slope. The average storage in hours per mile was found to be numerically equal to the reciprocal of the square root of the slope in feet per mile from source to gage, when the slope is taken as that of a straight-line profile with the same area below it as the actual profile. For some streams the variation from the average value was large—sometimes as much as 100%. The storage was larger for streams in alluvial beds than for those in rock gorges, although the range in slope for which studies were made overlapped. The average storage above two or more gages on the same stream, although varying with the slope, was more nearly constant than indicated by the aforementioned average criterion.

How much of the variation in storage is to be attributed to the natural causes and how much to limits of data cannot be ascertained. Most of the stream profiles were determined from U. S. Geological Survey sheets of 100-ft contour interval, although some of the flatter streams crossed only two such contours in their entire length. For steeper streams, crossing enough contours to reduce the possible error in slope determination, the time of concentration became so small that the possible error of 6 hours in time of precipitation was a large percentage of the total time. Perhaps someone working in an area for which good maps were available and which contain recording rainfall gages may be able to find a more precise relationship. Also, the writer was unable to find a usable definition of average river slope that could evaluate the influence of local concentration of extreme slope as a truly small influence on flow-time of the entire stream, compared with well distributed fall of the same amount.

The amount of the storage by which the time-area concentration curve should be modified will approximate 10% to 25% of the time of concentration, with the smaller values being used with the longer times of concentration and extremely narrow drainage areas.

Unit Hydrographs for Large Drainage Areas

By definition the application of the unit hydrograph is limited to areas over which the amount of runoff originating in any unit period may be considered as uniform within the required accuracy of the solution. For the flashy streams of the South Atlantic Coast, this is a drastic limitation, because they drain areas whose principal dimension is west to east, whereas the usual rainfall isohyets run north to south, and complete coverage of a major drainage basin by any storm is the exception rather than the rule. Because the streams are flashy, 6-hr unit periods are too long for component drainage areas, but the

shortest practical period for entire principal river systems. Under such conditions, the difference between the theory and fact of uniform origin of runoff may be large even for drainage areas of 500 sq miles or less.

For these streams, the storage-discharge relationship in all reaches of many of the rivers may be approximated closely by a straight line throughout the range of flood discharge. This being the case, the Muskingum method of flood routing is applicable to the translation and storage modification of flood waves. Where the Muskingum method is applicable, the unit hydrograph theory may be used, by deriving the unit hydrograph for the component drain-

TABLE 2.—Unit Hydrograph of James River at Richmond in 1,000 Cu Ft per Sec per 1-In. Uniform Runoff in 6 Hours

Time, in days[a]	James River at Lick Run, Va.	Craig Creek at Parr, Va.	Catawba Creek at Fincastle, Va.	Local at Buchanan, Va.	North River at Buena Vista, Va.	Local at Holcombs Rock, Va.	Local at Bent Creek, Va.	Buffalo River at mouth	Rockfish River at mouth	Local at Scottsville, Va.	Hardware River at mouth	Slate River near Arvonia, Va.	Rivanna River at Palmyra, Va.	Willis River at Flanagan Mills, Va.	Local at Cartersville, Va.	Local at Richmond, Va.	James River at Richmond, Va.	Time, in days[a]
Area[b]	1,369	331	104	280	660	506	421	417	245	238	135	235	675	247	379	515	6,757	Area[b]
0.25	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.82	0.82	0.25
0.50	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	5.94	5.94	0.50
0.75	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3.65	13.12	16.77	0.75
1.00	0	0	0	0	0	0	0	0	0	2.24	0	0	0	0	8.92	11.06	22.22	1.00
1.25	0	0	0	0	0	0	0	0	0	4.42	0	0	0	0	8.12	5.43	17.97	1.25
1.50	0	0	0	0	0	0	4.73	3.30	0	4.57	0.39	0.44	0	0.34	5.62	2.50	21.89	1.50
1.75	0	0	0	0	0	0.89	7.16	6.54	2.01	3.88	1.59	1.78	2.58	0.73	3.53	1.23	31.92	1.75
2.00	0	0	0	1.75	0	4.56	7.30	7.24	3.58	2.94	2.34	3.34	8.26	1.17	2.18	0.72	45.38	2.00
2.25	0	0	0.37	3.47	3.55	6.96	6.22	6.53	3.91	2.07	2.21	3.89	10.38	1.55	1.36	0.51	52.98	2.25
2.50	1.58	0	1.18	4.14	7.19	7.48	4.78	5.18	3.46	1.41	1.70	3.36	9.74	1.61	0.90	0.44	54.15	2.50
2.75	6.89	1.97	1.63	3.95	9.03	6.75	3.45	3.82	2.72	0.96	1.21	2.51	8.00	1.63	0.61	0.38	55.51	2.75
3.00	11.83	3.56	1.68	3.30	8.87	5.44	2.40	2.70	1.98	0.68	0.81	1.74	6.07	1.88	0.46	0.37	53.77	3.00
3.25	14.64	4.37	1.49	2.56	7.86	4.12	1.64	1.88	1.41	0.49	0.53	1.15	4.40	2.14	0.38	0.35	49.41	3.25
3.50	15.29	4.37	1.19	1.89	6.44	3.02	1.13	1.30	0.98	0.37	0.36	0.77	3.11	2.22	0.33	0.35	43.12	3.50
3.75	14.25	3.87	0.90	1.37	4.97	2.18	0.81	0.93	0.67	0.29	0.25	0.43	2.21	2.06	0.30	0.33	35.82	3.75
4.00	12.39	3.13	0.64	0.97	3.71	1.55	0.60	0.69	0.47	0.25	0.18	0.32	1.60	1.69	0.28	0.32	28.79	4.00
4.25	10.24	2.39	0.56	0.69	2.69	1.12	0.44	0.53	0.35	0.21	0.14	0.26	1.18	1.25	0.27	0.31	22.63	4.25
4.50	8.19	1.78	0.38	0.51	1.98	0.84	0.37	0.44	0.28	0.19	0.12	0.21	0.90	0.88	0.26	0.30	17.63	4.50
4.75	6.30	1.32	0.26	0.38	1.45	0.65	0.33	0.37	0.24	0.17	0.10	0.19	0.71	0.62	0.24	0.29	13.62	4.75
5.00	5.00	0.99	0.20	0.31	1.12	0.52	0.30	0.33	0.22	0.16	0.09	0.17	0.59	0.45	0.24	0.29	10.98	5.00
5.25	3.96	0.75	0.16	0.27	0.90	0.45	0.27	0.31	0.20	0.15	0.09	0.16	0.52	0.34	0.23	0.28	9.04	5.25
5.50	3.09	0.57	0.12	0.25	0.75	0.40	0.27	0.29	0.18	0.14	0.09	0.16	0.47	0.26	0.22	0.27	7.53	5.50
5.75	2.40	0.48	0.08	0.22	0.66	0.38	0.26	0.28	0.17	0.14	0.09	0.16	0.44	0.22	0.21	0.26	6.45	5.75
6.00[c]	1.88	0.41	0.06	0.21	0.59	0.35	0.25	0.26	0.15	0.14	0.09	0.16	0.42	0.19	0.21	0.25	5.62	6.00[c]

[a] After beginning of runoff. [b] Drainage area above station in square miles. [c] Values beyond 6 days omitted.

age areas of such size as to approximate the uniformity criterion and by routing the unit hydrographs themselves to the point for which a composite unit hydrograph is desired. The use of the proper amount and distribution of runoff with each routed component unitgraph will reproduce the component of runoff from the drainage area in question, and the sum of all components will be the desired composite flood wave. The degree of accuracy will be the same as though flood hydrographs had been computed for all tributary gages and then routed to the point for which the hydrograph is desired. In flood control work, the advantage of being able to identify individual components of a hydrograph is obvious.

For forecast work, it is desirable not only to have a complete breakdown of the drainage area into small parts but also to have a table of summations of these component unit hydrographs in various applicable combinations. If, for any particular storm pattern, practical uniformity of runoff origin over several component drainage areas exists, considerable time may be saved by using the combination graphs rather than the separate components.

Table 2 is a typical arrangement of component unit hydrographs prepared for the James River at Richmond, Va. Illustrating the nonuniformity of occurrence of runoff typical of South Atlantic rivers, the largest known flood (May, 1771) at this city is reported to have occurred without sufficient rain in the city to settle the dust.

Determination of Approximate Routing Constants

There are several ways of determining the storage constants for use in the routing of the unitgraph components. If the inflow between the tributary gage and the outflow is small or if it can be determined with enough accuracy to give proper weight to its influence, the amount of storage effective between the tributary gage and the desired outflow point can be determined from the relationships of Eqs. 9 to 12; or, where the hydrographs are of similar shape,

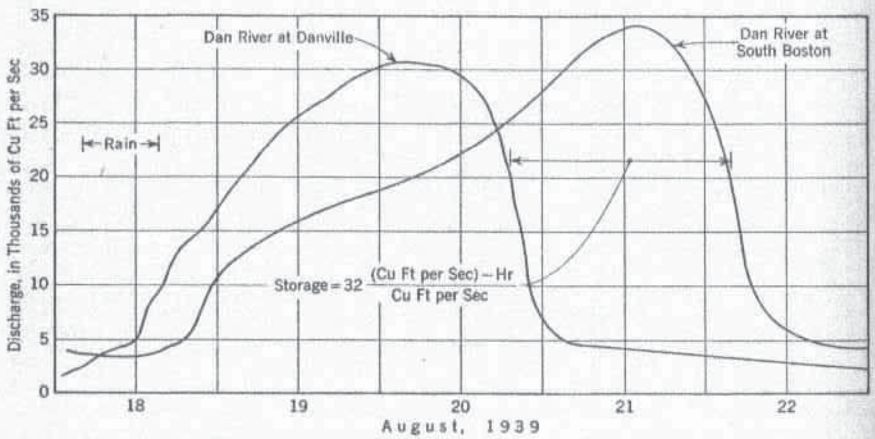

Fig. 12.—Approximate Determination of Storage from Hydrographs

as is frequently the case, by measuring the displacement between the falling leg of the inflow and outflow hydrographs. Thus, in Fig. 12, the storage effect between Danville, Va., and South Boston, Va., on the Dan River, is 32 hours. Although there are 680 sq miles of intervening drainage area, the discharge from that area had decreased to insignificant proportions prior to the time which is used as a criterion.

The most accurate method of determining the storage-discharge relation is still from the accumulated differential between inflow and outflow, as outlined by Mr. McCarthy.[9]

Summary

The paper has attempted to show that:

1. The constant time elements essential to the theory of the unit hydrograph, and common in many streams, are attributable to fundamental characteristics of the watersheds which maintain the storage capacities of the stream channels in constant direct proportion to the flows in them.

2. Such a relationship between storage capacity and discharge capacity produces certain characteristics in the rates of rise and fall of the hydrograph which are calculable by a simple established method.

3. These characteristics can be incorporated in a unit hydrograph, without the necessity of making or utilizing any quantitative determination of runoff distribution, in a few steps defined as follows:

(*a*) From a record of flood discharge, determine the time at which the rate of discharge decrease is greatest in relation to the prevailing discharge;
(*b*) The time elapsed between the end of runoff-producing rainfall and the time so determined is the base length (time) of a time-area concentration curve derived by dividing the watershed a long its water courses;
(*c*) The time-area concentration curve is routed as if through a reservoir with storage equal to K times the outflow when K is the time indicated by Eq. 12*b* at the time determined as in step (*a*) when $x = 0.0$;
(*d*) The result expressed in cubic feet per second per inch of runoff depth on the watershed at unit intervals of time is the instantaneous hydrograph; and
(*e*) The instantaneous hydrograph is converted to a unit hydrograph for any desired period of time by subdividing the instantaneous graph into periods of the desired unit length and averaging the ordinates over the preceding periods of time. Thus, in a 6-hr unit hydrograph, the rates of discharge at the end of 6, 12, and 18 hours, respectively, are the average ordinates to the instantaneous hydrograph in the 6-hr periods preceding the sixth, twelfth, and eighteenth hours.

4. A similar procedure may be applied to the determination of a unit hydrograph of subsurface flow, and the two unitgraphs may be combined in the average proportion applicable to the watershed, or used separately if desired.

5. The derivation of the unit hydrograph in this manner has some advantages:

(*a*) The procedure is definable so that all hydrographs derived by different individuals will be identical;
(*b*) The determination is independent of any knowledge of runoff distribution, except the time of ending, and therefore not subject to the errors due to indeterminability of runoff distribution which are factors in other methods known to the author;
(*c*) The influences of shape of drainage area and the capacity to produce large peak flows from concentrated runoff—both factors which are obscured by protracted runoff in other methods of derivation—are well indicated. The method also can be used to indicate the change in hydrograph form which is likely if a natural reach of river is replaced by a reservoir pool; and
(*d*) The unit hydrograph quantities are instantaneous rates of discharge at the time specified, not average values during the preceding unit period as in Mr. Sherman's original method, and the resulting hydro-

graph is properly drawn through all computed points, rather than as a bar diagram, and peaking of the hydrograph is no longer a matter of personal judgment. Hydrographs computed from the same distribution of runoff with 6-hr unitgraphs will agree with those by 1-hr graphs.

6. The derivation of the unit hydrograph in this manner has some disadvantages:

(a) Hydrographs are likely to fall a little too low on the receding leg between the point of maximum recession rate and the point where subsurface flow becomes predominant;
(b) When applied to too large drainage areas, the hydrograph may have too slow a rise and too rapid a fall, as a result of using the same storage factor for near and remote elements. This fault is corrected easily by subdividing the drainage area and computing separate unitgraphs for the components and combining them in the manner indicated in Table 2;
(c) Although this method reflects influence of shape of drainage area and capacity to produce high peaks well—better than any other one known to the author—it is possible that such influences are exaggerated slightly.

DISCUSSION

JAMES S. SWEET,[10] ASSOC. M. AM. SOC. C. E.—The relationship between flow and storage in open channels has been demonstrated in this paper. The analysis and the suggested procedures of the flow routing are a definite contribution to flood control design estimates. However, in forecasting work, where time of preparation and issuance of a forecast must necessarily be reduced to a minimum, the procedure must be simplified to effect a quick answer, without sacrificing the limits of relative accuracy. Mr. Clark's method involves time-consuming, laborious computations, and therefore is not adaptable for this type of work. On the James River where stage relationships between the gages along the entire length of the river are not always well defined, due to the nonuniformity of the distribution of the storms over the basin, it is best to use the method of the summation and routing of the discharges. Discharges in the upper reaches of the main stem and the tributaries should be ascertained either through the direct reports from the stations or through the simple rainfall-stage relations and should be routed to a downstream point on the main stem. Adjustments should be made to synchronize the arrival of the discharges at the desired point simultaneously. The discharges from the intervening ungaged areas should be adjusted, taking into consideration the ratio of index areas and rainfall over them and the area coefficient that reflects runoff characteristics and channel storage of the area. Determined empirically, these coefficients for various parts of the basin range from 0.6 to 0.7. Thus, discharge of the James River at Buchanan may be expressed as follows:

$$Q_B = Q_L + Q_P + Q_F + Q_U \qquad (15)$$

in which Q_B = discharge of the James River at Buchanan; Q_L = discharge of the James River at Lick Run, 9 hours previously; Q_P = discharge of Craig Creek at Parr, 7 hours previously; Q_F = discharge of Catawba Creek at Fincastle, 5 hours previously; and Q_U = discharge from the ungaged area (280 sq miles) above Buchanan.

For uniform storms, using the discharge at Lick Run as an index for the ungaged area discharge, Eq. 15 is further simplified:

$$Q_B = 1.123\, Q_L + Q_P + Q_F \qquad (16)$$

Discharges, including peak discharges, can be determined in a similar manner at all desired points along the main stem and converted into stages. With the equations prepared in a simple form such as Eqs. 15 and 16, actual computations take very little time after the headwater discharges become available to the forecaster.

It is regretted that Mr. Clark is introducing a new concept of time of concentration, thus adding another to the multitude of existing definitions. Not in the hope of relieving the confusion, but rather to add another choice for the future selection of an appropriate definition, the writer suggests that con-

[10] Regional Hydrologic Engr., U. S. Weather Bureau, Flushing, N. Y.

sideration be given to the definition of concentration time as time elapsed between the beginning of the effective rainfall (rainfall after the initial loss is satisfied) and the time of the peak rise of the stream.

OTTO H. MEYER,[11] Assoc. M. AM. Soc. C. E.—A new tool for synthesizing runoff from rainfall is presented in this paper. It is a good and useful tool, as is shown by the excellent agreement of reconstituted hydrographs with natural hydrographs. Although this method will not replace other accepted techniques of synthesis entirely, it deserves to be used in a large proportion of cases because of both its simplicity and accuracy.

The author does well to emphasize relative accuracy. Estimates have often been condemned for lack of accuracy when they were in fact the best available.

Some conclusions, in the paper, such as increase in outflow following decrease in inflow, mass rotation of water-surface profiles, and negative outflows, are apparently deduced from empirical and approximate formulas. These conclusions should be examined critically. Also, the statement that K is constant (and the derivation of formulas based on that condition) involves rationalizing from an empirical assumption. The value of K can often be assumed constant, with good "relative accuracy" and in many cases with entirely satisfactory results; yet this value usually varies somewhat with S and Q; in fact, it is often so variable as to be unusable. The writer has developed numerous storage-outflow curves for streams in the Sacramento and San Joaquin valleys of California, and for good routing results these curved considerably; in several cases, there were sharp reversals of curvature. The principal effect of using a constant K, instead of a variable K, will probably be the shifting of high peaks in time relative to lower flows.

Mr. Clark mentions storage as "balanced" or "unbalanced." These two types of storage appear to refer to "reach" and "reservoir" storage, respectively.

The division of runoff into surface and ground-water flow, by the criterion that the runoff in direct proportion to storage is surface runoff, is arbitrary. This theory is permissible only for convenience or when no other means of division is practicable. An important part of the runoff other than surface runoff never reaches the true ground-water table, but flows quite near the surface, and responds promptly to rainfall. The writer has found an apparent correlation between this volume in subsurface storage and the infiltration from rainfall; that is, the subsurface storage determines not only the outflow but also the surface runoff factor. Thus, a part of the rainfall may be routed "through the ground"; but as it travels underground a short distance only (at most several hundred yards) the outflow should be added to the "net" pluviograph and synthesized into the hydrograph along with the direct surface runoff. Of course, the inflow into subsurface storage is not total rainfall less direct surface runoff, as evaporation must also be considered; likewise, some of the subsurface storage is lost to the true ground water, and some is lost in transpiration.

[11] Maj., Field Artillery, U. S. Army, Corvallis, Ore.

The author constructs a time-area concentration curve, routes it through storage, and averages the outflow over a unit time to form a unitgraph. If the time-area concentration curve were averaged over a unit period, routing of the resulting graph might produce the same unitgraph. To extend this idea further, a summation of the products of a time-area concentration curve with successive parts of a net pluviograph could be routed through the storage with about the same labor as by the author's procedure. Although the suggested method would have the advantage of permitting use of a storage-outflow relation involving a variable K, it would involve more routing. However, there are less laborious procedures than the Muskingum method, such as "lag routing,"[12] or the use of a flood routing machine.

The application of Mr. Clark's technique is not necessarily limited to the Muskingum method of routing, but can be used with any of about a half-dozen methods in current use.

L. K. SHERMAN,[13] M. AM. Soc. C. E.—The definite, but complex, relations that exist between the hydrograph of surface runoff, the procedure of flood routing, and the law of proportionality of certain hydrographs, are presented in this paper.

All the objectives noted in the "Synopsis" have been fulfilled, with one very important exception. That exception is the author's statement, "to derive accurate unit hydrographs." No unit hydrographs or percentage distribution graphs have been derived in this paper.

Col. 10, Table 1, is labeled "12-hr unit hydrograph (cu ft per sec)." This caption is very misleading to a reader. It is not a unit hydrograph. The definition of a unit hydrograph, accepted and used since 1932, is: The hydrograph of surface runoff from a given basin due to a 1-in. depth of net rainfall, applied at a uniform rate in a unit of time. The ordinates of flow of this unit hydrograph (expressed in cubic feet per second) and the ordinates of flow of an observed hydrograph, due to a net rain of X inches in the same unit time, are proportional to their respective volumes of total surface runoff.

Net rain means rainfall minus infiltration and other losses.

The distribution graph is the unit hydrograph with ordinates expressed in percentage of the total volume of the observed hydrograph. The percentage distribution graph is commonly used in practice.

The proportionality of ordinates holds true for any two hydrographs of surface runoff on the same basin, provided the net rains in each case hold a direct relation of intensities, and follows identical sequence of downpours.

Let it be required to apply the foregoing criteria to this Appomattox River basin with a drainage area of 1,335 sq miles. If 1 in. of net rain fell on this basin, and if the rain followed identical times and similar intensities as that of April, 1937 (see Fig. 9), it would produce a hydrograph with the same time base as that observed in Fig. 9. The ordinates of runoff would be proportionate to the ordinates in Fig. 9, and the volume of surface runoff would be 1,335 in.-miles or 36,000 (cu ft per sec)-days or 72,000 (cu ft per sec)-half-days. The total surface runoff (by scaling ordinates in Fig. 9) in the flood of April, 1937, is

[12] "Simplified Flood Routing," by Otto H. Meyer, *Civil Engineering*, May, 1941, p. 306.

[13] Cons. Engr., Chicago, Ill.

215,000 (cu ft per sec)-half-days. The base flow (also estimated from Fig. 9) was, approximately, $\frac{1,000 + 2,000}{2} \times 19$ half-days or 29,000 (cu ft per sec)-half-days. Then $216,000 - 29,000 = 187,000$ (cu ft per sec)-half-days as the total volume of surface runoff due to the April, 1937, storm.

A similar storm producing 1 in. of runoff from this basin was 72,000 (cu ft per sec)-half-days.

The average depth of net rain, measured from the surface runoff hydrograph, is $187/72 = 2.6$ in. of average depth over the Appomattox basin in the flood of April, 1937.

The comparable figures given in Table 1 are: Runoff for 1 in. of net rain = 60,000 (cu ft per sec)-half-days. This is the total of Col. 10. This amount is 87% of the total runoff. Therefore, 100% of 1 in. of net rain is 69,000 (cu ft per sec)-half-days. Accordingly, the depth of net rain utilizing the author's figure is $187/69 = 2.7$ in.

This agreement by the author's elaborate routing analysis is an ingenious and possibly very useful accomplishment. It has no value as a derivative of the ratio of 1 in. of net rain to the total runoff from a particular storm. That is found readily in a few minutes from an observed hydrograph and a known size of drainage area. Col. 10, Table 1, is the runoff of 1 in. of net rain over the basin; but it certainly is not 1 in. applied over the basin in one unit of time. Therefore, it is not a unit hydrograph or a distribution graph. It certainly cannot be used to derive the hydrograph of runoff from the Appomattox basin under a storm of entirely different pattern. That is what the unit hydrograph is for. The same criticism applies to all the data in Table 2.

The author states (see heading, "Derivation of the Instantaneous and Unit Hydrographs") that another storm "is equally well reproduced with the same [so-called] unitgraph." He has not demonstrated the truth of this statement.

However, it would seem possible to utilize the methodology of Table 1. In this case the data furnished no record of the amounts of unit-time rainfalls on the basin. The total net rainfall averaged over the basin is simply and readily found as 2.6 in. or 2.7 in. How much of this fell in each of how many half-day periods is unknown. If this were furnished or found, a unit hydrograph could be derived. The unit hydrograph could be derived just as well if the 12-hr rainfalls for the period, April 25–29, were known—by the methods described by Mr. McCarthy[9] or the writer.[14] In deriving a unit hydrograph, the first step is to find the volume or volumes of net rainfall that fell in unit periods of time. These are the volumes that produce the hydrograph of surface runoff.

In the author's example of April, 1937, in Fig. 9, the total runoff was found to be 2.7 in., which is the sum of several unit-time downpours; but, in this case, the number or relative amounts of the unit volumes which total 2.7 in. is not known.

The bar pattern, Fig. 8, curve (a), indicates that the author has discovered that the 2.7-in. runoff was due to 12 unit periods of rain with ordinates pro-

[14] "Physics of the Earth," National Research Council, Div. of Physical Sciences, Pt. 9—"Hydrology," by O. E. Meinzer, 1942, p. 521.

portional, or equal, to this bar pattern. Mr. Clark does not describe how he found this bar pattern. The number of blocks agrees with the number of time units in Table 1, Col. 2. The ordinates or heights of these blocks agree fairly well with Col. 2 averages.

The total percentage of Col. 2, however, is not 100%, or even 87%, to correspond with the total of Col. 8, Table 1.

It is hoped that, in his closure, the author will consider this feature of Table 1 and the block pattern (a) of Fig. 8. He may have found a way of determining the unit volumes of net rainfall which created the observed hydrograph, without prior data on rainfall. Astounding as this statement may seem, there is nothing fundamentally impossible about it. The hydrograph of surface runoff reflects, not only the physical characteristics of the basin, but also the pattern or sequence of the creating net rainfall. In other words, with an observed hydrograph of surface runoff, it is possible to derive the unknown unit rains and thus the unit hydrograph. With the unit hydrograph and the observed rainfall, the corresponding hydrograph of runoff is found readily.

If the author presents further consideration of this feature, it is also hoped that he will follow the orthodox procedure of eliminating base flow. The problem is sufficiently complex without including it. In this connection, attention is called to the work of Robert E. Horton, M. Am. Soc. C. E., which includes routing and the derivation of unit hydrographs.[15]

The writer does not agree with the author's statement that 1,000 sq miles is a large basin upon which to apply the unit hydrograph or distribution graph. The distribution graph has been frequently and satisfactorily used for basins of 5,000, 10,000, and even 30,000 sq miles. The theory of the unit hydrograph assumes a uniform rate of rain for a relatively small fraction of the time of runoff. The volume in this unit time is the same volume as the nonuniform rain in this unit time. Each unit volume changes with the storm intensity. If the unit times are taken small enough, the runoff hydrograph will be exactly reproduced.

In practise, whether to use small areas and apply routing, or use large areas without routing, depends altogether upon the available data including the number of stream gages in the basin.

Gordon R. Williams,[16] Assoc. M. Am. Soc. C. E.—Numerous positive statements are made by the author which controvert accepted concepts and experience in the field of river hydraulics. These statements are not substantiated in the paper by experimental evidence or observations of actual conditions.

The most radical theory deals with "pressure" or momentum waves moving downstream through reservoirs and natural river reaches. The author claims that, if the inflow to a reservoir or river reach is decreased abruptly, the slope storage in the upper end of the reservoir or reach moves downstream as a pressure wave and increases the stage and discharge at the lower end of the pool

[15] "Virtual Channel-Inflow Graphs," by Robert E. Horton, *Transactions*, Section of Hydrology, Am. Geophysical Union, Pt. III, 1941, p. 818 (4).

[16] Senior Engr., Office of the Chf. of Engrs., U. S. Army, Washington, D. C.

or reach. Laboratory experiments have been conducted in which "decrement waves" were produced in artificial channels.[17] Such waves move downstream in accordance with the equation,

$$u = v_1 + \sqrt{g\, d_1} \qquad (17)$$

in which u = wave velocity; g = acceleration due to gravity; d_1 = initial depth; and v_1 = initial velocity of flow. It is doubtful whether the natural inflow to a reservoir or river reach could decrease fast enough to create a true momentum wave. The contention that such a wave would cause an increase in stage and discharge at a downstream point was not substantiated by Robert E. Horton, M. Am. Soc. C. E., in his experiments. The author refers to an incident in which the effect of a levee break was transmitted downstream as a decrement wave at the rate of a mile a minute. The river would have had to be about 240 ft deep to transmit a momentum wave at the rate of 88 ft per sec.

A decrease in inflow to a river reach might be accompanied by an increase in elevation, but not in discharge, at the lower end of a reach in a river having an erodible bottom or in a river carrying a large bed load. In such a case the decrease in inflow would cause a flattening of slope and a loss of erosive power and carrying capacity, which in turn would result in a filling in of the channel and a possible increase in elevation. Such a phenomenon is basically different from the author's theory, but would give evidence that might lead to such a theory.

The statement is made (see heading, "Valley Storage") that the aforementioned phenomenon "takes place in a natural channel * * * by means of pressure waves of small amplitude which surge back and forth through the reach * * * ," and that "the surge * * * is clearly apparent as a broad trace in many autographic records of stage." This surge, which is evident on gaging station charts, is very common, particularly at high stages, and is merely a local disturbance at the station, due to turbulence in the stream. The surge is often most pronounced on swift, turbulent streams where the presence of shooting flow and even small waterfalls eliminates any possibility of pressure waves traveling up and down the stream. Surges in gaging station wells can often be reduced by modifications in the size and type of intake connection.

Ray K. Linsley, Jr.,[18] Jun. Am. Soc. C. E.—The author's discussion of storage, the unitgraph, and accuracy in engineering is a valuable contribution to hydrology.

The concept of storage which may act to increase outflow is familiar to all persons who have worked with reservoirs storing appreciable volumes of water under the backwater profile. The navigation pools of the Upper Mississippi and Ohio rivers are examples of this effect. To the writer's knowledge the author is the first to apply this concept to storage in natural streams. Obviously, under the proper conditions it can apply, but the discussion should be approached with some caution.

In Fig. 1 it will be noted that, as the water-surface profile changes from b to d, the slope of the water surface at 0 has been decreased. Since values of

[17] "Channel Waves Subject Chiefly to Momentum Control," by Robert E. Horton, *Bulletin TP-16*, SCS, U.S.D.A., May, 1938.

[18] Associate Hydrologic Engr., U. S. Weather Bureau, Sacramento, Calif.

velocity and hence $v^2/2\,g$ upstream from 0 have been lowered, the slope of the energy gradient is decreased even more than that of the water surface. Using the data of profiles a and b the writer computes $1.486/n$ in Manning's formula to be 54.3. Assuming s equal to the slope of the water surface from station 2 to 0, the discharge at 0 under profile d becomes 108,000 cu ft per sec, whereas for profile f it would be 130,000 cu ft per sec. By approximation the slope of the energy gradient at station 0 for profile f seems to be 0.00055 ft per ft which gives a discharge of only 108,000 cu ft per sec.

Although these discharge values are only approximate, they serve to demonstrate the close interrelation between stage, discharge, and storage which must be considered in any discussion of river hydraulics. Actually, in the example of Fig. 1, it appears that the change from profile b to f would absorb the storage released by the decreased inflow through a rise in stage but with a decrease in outflow.

The writer has never observed a natural stream in which the release of inflow-controlled storage acted to increase outflow. On numerous occasions the writer has observed the effect of levee breaks or the operation of diversion weirs. The usual reaction is a sudden sharp fall in stage at the downstream station which is transmitted, as the author indicates, with great rapidity. Generally, however, the fall in stage is small in comparison to the volume of water diverted. This is explained by the two factors—(1) loss in discharge capacity at the station by reason of the reduced slope and (2) the release of a volume of inflow-controlled storage. However, since a considerable portion of this inflow-controlled storage is withdrawn by the diversion, the effect of the lost discharge capacity seems to be the important factor.

In natural channels the rates of change of flow, especially of decrease of flow, are usually too slow to permit a sufficiently rapid release of inflow-controlled storage to produce an increased downstream discharge. Possibly the abrupt cessation of rainfall from an intense local thunderstorm over a small basin or sharp decrease in discharge from a reservoir might accomplish such a result.

It is possible that, for some reservoirs, the decrease in discharge capacity of the outflow weir, as a result of a decrease in velocity of approach, may account for all or a portion of the rise in stage at the dam when the inflow is reduced.

The general scheme of deriving a unit hydrograph by adjusting a time-area concentration curve for valley storage has been discussed by several writers. The method outlined by the author should permit the determination of unit hydrographs with a minimum of basic data, a minimum of disagreement between separate investigators, and a high degree of relative accuracy. It represents a considerable advance over earlier procedures.

The author makes the point that the ordinate of a unit hydrograph for any unit period will be equal to the average ordinate of the instantaneous hydrograph for the previous unit period. This is demonstrated easily since the ordinates of the time-flow concentration curve of Fig. 4(b) bear the same relationship to the time-area concentration curve of Fig. 4(a). Inasmuch as the routing method assumes storage to be directly proportional to flow, the relationship is not disturbed by any manipulation which follows. The relationship is not peculiar to instantaneous hydrographs, however. If two 6-hr unitgraphs are added with a 6-hr lag and the sum is divided by two, a 12-hr unitgraph results. The ordinate of the 12-hr graph at hour 12:00 is the sum of the 6-hr and 12-hr ordinates of the 6-hr graph divided by two, or the average ordinate of the 6-hr graph during the first twelve hours.

The author indicates an attempt to correlate storage with slope in order to devise a method for determining K for basins without adequate data. He found that the relation could be expressed approximately as: $\frac{K}{L} = \frac{1}{\sqrt{s}}$; or,

$$C_1 = \frac{K\sqrt{s}}{L} \quad \text{(18a)}$$

in which L is the length of the stream in miles. Eq. 18a considers only the distance along the main stream and the slope. It would seem that other factors of importance would be the storage in tributaries and the comparative magnitude of flood flows to which the streams are subjected. If drainage area were introduced into Eq. 18a, it should indicate the effect of these two factors. Then: $\frac{K}{L} = \frac{\sqrt{A}}{\sqrt{s}}$; or,

$$C_2 = \frac{K\sqrt{s}}{L\sqrt{A}} \quad \text{(18b)}$$

The writer has used the square root of A because peak flows tend to vary as the square root of the drainage area, and, in all probability, tributary storage also varies as some power of drainage area less than one.

TABLE 3.—COMPARISON BETWEEN EQS. 18a AND 18b

Stream	s (ft per mile)	L (miles)	K (hours)	A (sq miles)	C_1 (Eq. 18a)	C_2 (Eq. 18b)
Appomattox River near Petersburg, Va.	2.9	122	144	1,335	2.01	0.055
James River at Lick Run, Va.	17.3	78	28	1,369	1.49	0.040
Smith River at Bassetts, Va.	16.6	37	7	265	0.77	0.047
Meherrin River at Emporia, Va.	2.4	86	78	750	1.40	0.051
Middle Fork, Feather River at Bidwell Bar, Calif.	59	98	16	1,353	1.26	0.046
American River:						
North Fork, at Colfax, Calif.	80	50	8	308	1.43	0.082
North Fork, at Auburn, Calif.	80	56	10	619	1.60	0.064
South Fork, at Coloma, Calif.	91	60	12	635	1.90	0.075
Kings River:						
At Piedra, Calif.	72	86	9	1,694	0.89	0.036
Above North Fork, Calif.	132	60	7	956	1.34	0.043
San Joaquin River above Big Creek, Calif.	192	40	7	1,042	2.22	0.075

Table 3 affords a comparison between Eqs. 18a and 18b. The first four items are from data given by the author, and the last seven are based on data available to the writer. Unfortunately, determinations of K for streams of the Sierra Nevada drainage are complicated by the presence of a snow pack which effectively reduces the drainage area, length of channel, and time of concentration, in addition to the other problems indicated by the author. The criterion of Eq. 18b seems to be more nearly a constant than does the value of $K\sqrt{s}/L$ from Eq. 18a. Values of $K\sqrt{s}/L\sqrt{A}$ have somewhat more of tendency to be higher for the California streams than do the values of Eq. 18a. This may indicate that a variation must be expected from region to region, but more likely it indicates that some power other than the $\sqrt{s}$ should be used.

Inspection of Fig. 6 indicates that the author divided the drainage area on the basis of equal units of stream mileage. Either Eq. 18a or Eq. 18b could be used to determine the subareas and should give a time-area concentration graph which more closely approximates the true graph. Eq. 18a is the simplest to apply, and, since the problem is merely to divide a basin for which K is already known, should be nearly as satisfactory as Eq. 18b. The number of reaches will equal K/T, and the values of $d/\sqrt{s}$ (d is the length of the reach) for all reaches should be equal. The solution is reached easily by approximations if a family of curves showing the variation of $d/\sqrt{s}$ with d, and with s, is plotted. A first approximation to the proper value of $d/\sqrt{s}$ will be the value of $T\,L/K\sqrt{s}$, which, for the Appomattox River, is 6.0. A final value of 5.75 was determined by the writer, which gives ten reaches of 8.6 miles, one reach of 12 miles, and one of 24 miles.

E. F. BRATER,[19] JUN. AM. SOC. C. E.—The unit hydrograph is the most useful tool available for the purpose of converting a quantity of surface water, resulting from precipitation, to a river discharge hydrograph. Since the unit hydrograph concept was introduced, a number of different interpretations and methods of application have been evolved. A description of these various concepts and techniques should be included in technical literature. Such a study and discussions of it should help to unify and improve engineers' understanding of the unit hydrograph. This paper is a valuable step in that direction. However, the engineer who may be desirous of making use of this technique will hope that a more detailed description of the methodology, with examples of the numerical computations, will be included in the author's closing discussion. For instance, the "time-area concentration curve" is an important feature of the method, but the author has not shown how one may be obtained for a given watershed.

In his method of flood routing, Mr. Clark sets up the relationship between storage and discharge in Eqs. 3 and 4. In Eq. 4, x is a term that is allowed to vary from 0 to 1. When $x = 0$, the storage becomes proportional to the outflow from the channel reach; when $x = 1$, the storage must be proportional to the inflow; and, when $x = 0.5$, the storage becomes proportional to both inflow and outflow in equal weight. The third condition assumes trapezoidal or double-wedge storage. Mr. Clark indicates that for most streams x is nearly 0.5. However, he chooses to discuss at considerable length the importance of the case in which x is greater than 0.5. In this connection, the writer does not agree with the author.

In the first place, the case of $x = 1$ is a physical impossibility in a real reach of channel, since, if storage is proportional to inflow, storage must become zero at the same instant that inflow becomes zero. For this to be true, the final portions of flood water would have to travel the length of the reach in-

[19] Asst. Prof., Civ. Eng., Univ. of Michigan, Ann Arbor, Mich.

stantaneously. This paradox is illustrated by Fig. 2, in which, for $x = 1$, the inflow hydrograph and the routed or outflow hydrograph end at the same time. Thus, if such a graph were routed through successive reaches, the recession side of the graph could not advance in time. The result of several routings of the author's graph is a number of large plus and minus ordinates occurring at the end of the original hydrograph.

Mr. Clark is of the opinion that storage of this type does occur and that it may cause outflow rates from a given reach to be greater than the inflow rates even without the addition of inflow from the local drainage area. In the third paragraph following Eq. 3 he states:

> "Reduction of the inflow rate [where backwater exists] necessarily would be accompanied by a decrease in storage and an increase of outflow rate. Rapid inflow shutdown necessarily would be accompanied by a release of the backwater storage and a consequent increase in the slack-water storage or in the outflow discharge or in both."

As an illustration of the nature of this action, the author takes the case of uniform discharge in the rectangular channel shown in Fig. 1. He assumes that the uniform flow of 116,000 cu ft per sec is suddenly stopped at the upper end of the reach and concludes that this will cause an increase in the depth at the lower end because 260 acre-ft of water are released from storage at the upper end. The writer wishes to point out that, since there is no change in the conditions at the lower end of the reach, the normal discharge of 116,000 cu ft per sec will continue there until the effect of this sudden stoppage can be transmitted through the length of the channel. A recession type of wave will be started at the instant of shutoff which, neglecting energy losses, might have a velocity as great as $V_w = \sqrt{g\,d} + V = \sqrt{32.2 \times 20} + 11.6 = 37$ ft per sec. Therefore, the earliest possible time that the effect will reach the lower end is $\frac{10{,}000}{37} = 270$ sec after stopping inflow. During this time a volume of $\frac{270 \times 116{,}000}{43{,}560} = 720$ acre-ft will have passed out of the lower end of the reach. This is more than sufficient to make room for the 260 acre-ft which the author wishes to accommodate. There is in no sense a "release" of stored water, due to a decrease in the discharge at the upper end of the reach, either in a channel such as this or in the backwater above an impounding reservoir. Kinetic storage is held in place solely by the resistance to flow from point to point in the channel. It is already in complete equilibrium—that is, flowing as fast as possible with the energy available. A decrease in depth at the upper end of the channel can only decrease the energy and thus retard, rather than accelerate, flow.

L. C. Crawford,[20] Assoc. M. Am. Soc. C. E.—One phase of this interesting paper pertains to natural channel or valley storage and the relationship of this volume to discharge or stage. The channel-storage rating has become a valuable tool in flood routing, hydrograph analysis and synthesis, and infiltration studies. The author's comments on real and relative accuracy are

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very pertinent to unit hydrograph and channel-storage ratings which may be developed by indirect methods through the analysis of records of discharge during past major rises or capital floods.

The basic stage and discharge records, which are cooperatively collected by the United States Geological Survey and other agencies, give an opportunity for potential channel storage and related studies that probably has not been

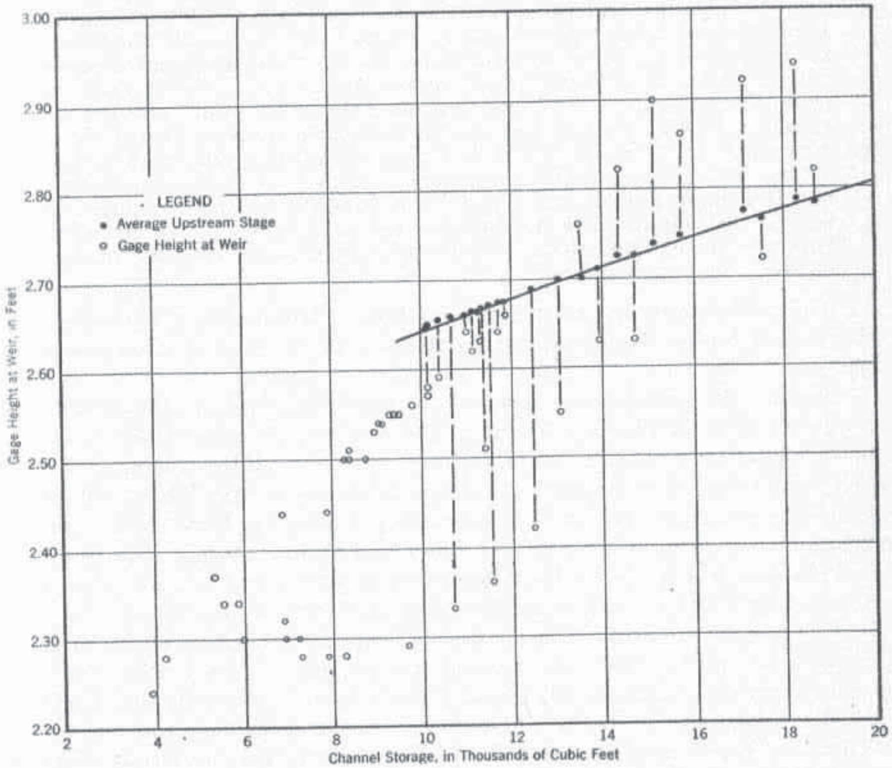

Fig. 13.—Channel-Storage Rating Curve for Difficult Run Near Fairfax, Va.

sufficiently emphasized. Indirect methods of storage determination have been developed and are being further extended through new concepts, such as Mr. Clark presents, and through the use of basic records of stream flow. Thus, it is possible to ascertain with considerable relative accuracy various relations pertaining to natural channel or valley storage that otherwise would be too costly and wholly impracticable without extensive cross-sectional surveys.

Practical experience with recession curves and storage relations have shown, however, that additional refinement in technique may be necessary and desirable. Such refinement is possible if sufficient hydrologic data are collected and correlated with stream flow records.

Fig. 13 shows determinations of gage height and storage in a small headwater area of about 1 sq mile on Difficult Run near Fairfax, Va. The method of channel-storage determinations was presented by O. E. Meinzer and other members of the United States Geological Survey,[21] in the following descriptive discussion:

> "The gaging station is equipped with a V-notch weir and a water-stage recorder. Measurements of channel storage were made by means of 134 secondary gages. The trunk-stream and each of the branches were divided into 100-foot segments, and at the middle of each segment a secondary gage was installed and a profile of the cross section of the channel was made. Each gage consists simply of a 2- by 2-inch stake driven into the bed of the stream so that the depth of water above the top of the stake can be measured with a scale. Secondary gages were installed and cross sections of the channel were made as far upstream on each of the four branches as any stream existed. The volume of water in each segment at any time was computed in cubic feet by multiplying the area of the cross section, in square feet, of the stream at the gaging station for that segment, by 100, and the channel storage in the entire stream system was computed as the sum of the volumes of water in all of the 134 segments. No account was taken, in the computations, of the dead storage below the level of the tops of the 2- by 2-inch stakes."

The observations and calculations of storage were made by V. C. Fishel of the United States Geological Survey in connection with an effluent seepage study.

Despite the probable precision in the determinations of storage volume, the relationship of the gage height at the weir to a storage volume is shown to be somewhat complicated and uncertain. However, if the computed storage is related to an average stage prevailing upstream as calculated from depth measurements from each of the 134 stakes, about two dozen of the storage determinations during appreciable outflow and channel storage may be alined as indicated in Fig. 13. This relationship is interesting in connection with Mr. Clark's discussion of the correlation of storage with discharge.

The measurements on Difficult Run indicate that the stage or discharge at the weir is a very rough index of stages and storage that may prevail in even such an extremely small basin. In addition, it is often presumed that the same relation exists between outflow and storage during rising stages as during the recession. Some methods recognize the existence of this hysteresis effect in connection with channel-storage studies but continue to assume that the net result is small or may be neglected. Some investigators in the field, nevertheless, recognize the inaccuracies in such an approach.

In any event, the study on Difficult Run demonstrates that more exact analysis is possible and probably desirable when the base data are suitable and sufficient. By relating mean upstream stages with channel-storage determinations, evaluations can be made of certain factors whose effect heretofore has been generally given only casual consideration. Moreover, computations involving the volume of channel storage during the rising side of a hydrograph can be made with some increased accuracy from such a storage curve as determined from the recession limb or by other methods.

The use of multiple gages for determining the channel-storage rating also affords a relatively simple method for larger basins. For example, several

[21] "The Channel-Storage Method of Determining Effluent Seepage," by O. E. Meinzer, R. C. Cady, R. M. Leggette, and V. C. Fishel, *Transactions*, Am. Geophysical Union, 1936, Pt. 2, pp. 415–416.

basins with areas greater than 3,000 sq miles have been examined to explore and demonstrate the procedure presented in this discussion. Fig. 14 shows the orientation of the channel-storage determinations as computed from recession curves for several major rises at the gaging station on the Wabash River at Logansport, Ind. The basin above Logansport comprises 3,760 sq miles and storage takes place in several tributaries with complicated inflow conditions.

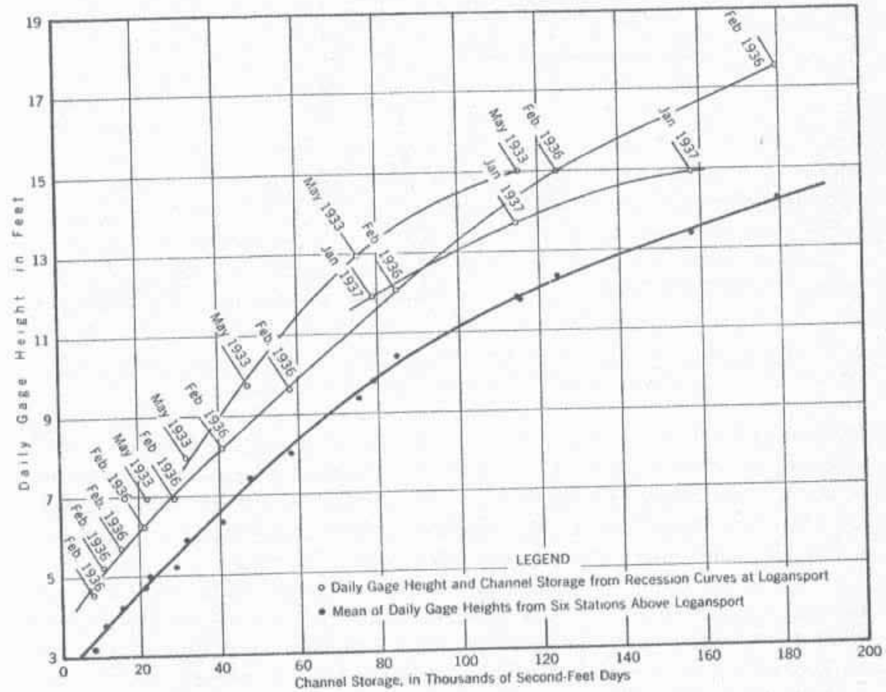

FIG. 14.—CHANNEL-STORAGE RATING CURVE FOR WABASH RIVER AT LOGANSPORT, IND.

Figs. 13 and 14 illustrate important limitations in the assumption of Mr. Clark and others (see heading, "Valley Storage") that "over a large range the true storage may be closely approximated by storage directly proportional to discharge." The method, utilizing an average height or weighted upstream prevailing stage as an index for storage, appears also to provide a somewhat convenient approach for channel-storage determinations during the rising, as well as the falling, stream stages.

The opinions expressed herein are those of the writer and not necessarily those of the organizations with which the writer is identified.

ROBERT E. KENNEDY,[22] M. AM. SOC. C. E.—At the risk of being more or less academic, the writer would like to comment on the old controversy over the basic assumption of the unit hydrograph.

In Fig. 15 the unit hydrograph, as labeled, is the runoff of a net rain of 1 in. over the watershed in a certain time unit—1 hr, 6 hr, or 12 hr—as may be

[22] Asst. Engr., U. S. Bureau of Reclamation, Denver, Colo.

chosen by the investigator. Then the basic assumption of the unit hydrograph idea is that, when a 2-in. net rain, for instance, falls on the watershed in the same length of time as the 1-in. net rain, the ordinates of the new hydrograph will be twice as high as those of the unit hydrograph, but the time of runoff will be the same for both storms. In Fig. 15 the hydrograph of a 2-in. net rain is

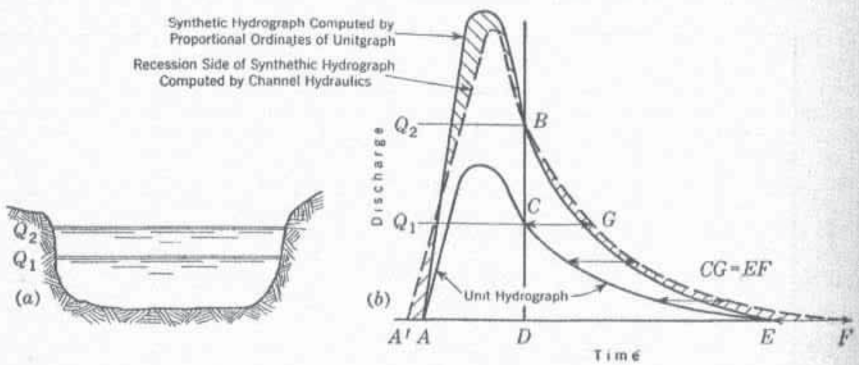

FIG. 15.—COMPARISON OF HYDROGRAPH COMPUTED BY UNITGRAPH METHOD WITH ONE COMPUTED BY CHANNEL HYDRAULICS ON RECESSION SIDE

labeled "Synthetic Hydrograph Computed by Proportional Ordinates of Unitgraph." The time length of both hydrographs, according to the basic definition of the unit hydrograph, is the line AE, for practical purposes. It is not claimed that the base lengths are exactly identical, which opens the question as to how much difference might occur under extreme conditions.

A glance at the cross section of a stream channel in Fig. 15(*a*) shows that the channel storage of the two storms cannot be emptied in the same time. That would mean that the water stored in the channel at Q_2 and represented by point B on the hydrograph in Fig. 15(*b*) must run out in the same time as that stored at Q_1 in the channel and shown as point C on the smaller hydrograph, Fig. 15(*b*). This cannot occur because the water stored in the channel between Q_2 and Q_1 must go first, and that takes time. No matter how fast it may flow, the water cannot escape in "nothing flat"!

Just how much longer time the larger storage requires to empty is susceptible of mathematical treatment. The writer developed this relation and then found that his labor had been largely anticipated in mathematical analyses published by Robert E. Horton, M. Am. Soc. C. E., in 1936[23] and 1937.[24]

Any two floods can be so compared on the recession side of their respective hydrographs that the discharge at one point, for example, is twice that of the other. Let these two points be points of contraflexure. Two such stages are shown in Fig. 15(*b*) at point B on one hydrograph (dashed line) and point C on the smaller hydrograph below, ignoring for the present that the latter was used to illustrate the unit hydrograph.

[23] "Natural Stream Channel-Storage," by Robert E. Horton, *Transactions*, Am. Geophysical Union, 1936, Pt. II, pp. 406–416.

[24] "Natural Stream Channel-Storage" (Second Paper), by Robert E. Horton, *ibid.*, 1937, Pt. II, pp. 440–456.

The difference in time required by the channel to empty itself of these two floods from this point of contraflexure is computed as $n^{0.20} - 1$ when n is the ratio of the ordinates of the two stages at this point. In Fig. 15, $n = 2$; and $2^{0.20} - 1 = 0.14$. Lines EF and CG, Fig. 15(*b*), are each 14% of line DE.

When the discharge of the larger flood is three times the smaller one at the point of contraflexure the larger flood requires 25% longer time to empty from that point on the hydrograph. If the larger flood discharge is ten times the smaller flood discharge, the larger flood would take 58% longer to empty. However, in practise the last featheredge of the storm water flow is so diluted with ground water or base flow that the end of the hydrograph is entirely indeterminate.

To complete the dashed-line hydrograph, resort was made to the non-mathematical assumption that the larger storm would take as much longer to fill the channel system as it does to empty it when compared to the time required for the smaller storm, so A'A was made 14% of AD. The rising leg of the dashed-line hydrograph was made parallel to that of the smaller flood.

The two areas shown crosshatched in one direction were made equal to the one area crosshatched in the other direction because the two hydrographs bounding these crosshatched areas represent the runoff from the same storm.

In conclusion it appears from the completed figure that the multiple ordinate concept of the unitgraph is not strictly correct because, when two such hydrographs as shown are superimposed, at only one point is the ordinate of the larger one exactly twice that of the smaller one. No matter how much shifting was done, all the ordinates of the larger or dashed-line hydrograph could not be twice those of the smaller one.

Nevertheless the unit hydrograph is a most useful tool in hydrology.

VICTOR H. COCHRANE,[25] M. AM. SOC. C. E.—The relationship between the unit hydrograph and the storage capacity of the drainage system, as well as a simple and practical procedure for calculating hydrographs, is presented in this excellent paper. The author rightly states that the determination of a hydrograph depends upon two, large, basic factors—the shape of the watershed and the storage through which the runoff must come. The influence of these two factors is largely accounted for by the use of what the author calls the time-area concentration curve, and both the accuracy and simplicity of the method are due to this device.

Storage is related to time, and may be expressed in time units. The paper contains a discussion of storage concepts, and some rather surprising conclusions are drawn, but it does not appear that the author's procedure is dependent upon the validity of these conclusions. If the processes by which a hydrograph is built up are viewed from the standpoint of time instead of storage, a different, and perhaps more easily understood, explanation of the basis of the author's method is derived.

If a drainage basin is divided into a number of areas, or zones, such that their downstream boundaries are at equal time intervals of flow from the point of measurement, a bar diagram of these areas would be the time-area

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diagram for the watershed, the base of each bar being the constant time interval for flow through the zone. This diagram, when converted to flow in cubic feet per second for 1 in. of runoff in one period, becomes the time-area concentration curve.

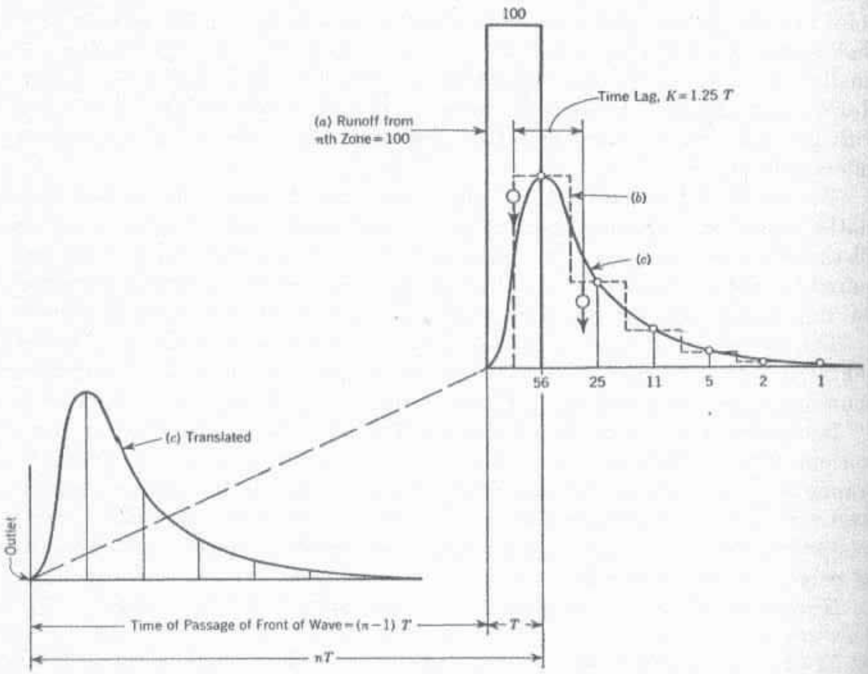


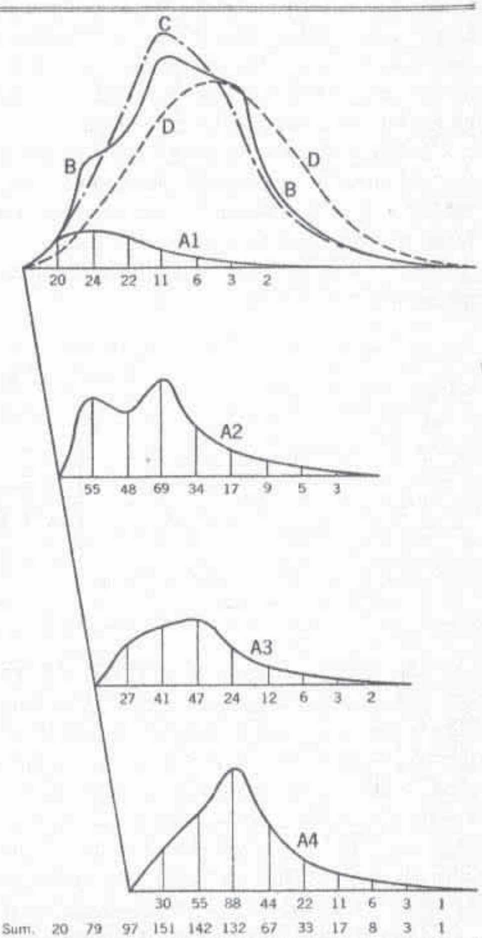
FIG. 16.—TYPICAL ZONE HYDROGRAPH

If all the runoff-producing rainfall in a unit period of time could be delivered instantaneously at the outlet (zero storage), the hydrograph would be a single bar with one time unit as the base and a height equal to the combined height of all the bars in the time-area concentration curve. If the net rainfall in each zone could be concentrated instantaneously at the zone outlet, and if the zone concentrations should then flow to the point of measurement at channel velocity and without further modification, the time-area curve would be the hydrograph of flow at the outlet. This latter process accounts for the greater part of the effect of time (or storage), and the time-area diagram resembles the actual hydrograph.

However, the zone contributions do undergo modification within the limits of the zone and during the passage downstream. In Fig. 16 the bar diagram represents the runoff originating in the nth zone above the outlet during the unit time T. By the time it reaches the outlet it is modified to a typical outflow hydrograph, such as curve (c), which is shown in relation to the precipitation and in the translated position. In this connection the author makes the valuable suggestion that the modification (which is in addition to that implicit in the time-area concentration curve) be determined by means of the Muskingum equations with x equal to zero and with K, the time lag in hours between center of mass of net zone rainfall and center of mass of zone runoff, equal to some small amount, such as $1.25\,T$. The modified hydrograph becomes the bar diagram (b), and if a smooth curve is drawn through the ordinates of (b) it will be the zone hydrograph (c).

TABLE 4.—COMPUTATION OF HYDROGRAPH; $K = 1.5\,T$

Zone Number	Area of Zone, in Square Miles	Net Rainfall: Depth, in Inches	Net Rainfall: Miles² Inches	Distribution to Periods: 1	2	3
1	40	2.2	88	40	28	20
2	100	2.4	240	110	40	90
3	60	2.7	162	54	54	54
4	100	2.6	260	60	80	120
Totals	300		750			



The author's use of a constant value for K seems to be based on the assumption that most of the modification occurs within the limits of the zone. This is a reasonable basis for practical use, for it will be found that the precise shape of the zone hydrograph is of little consequence. However, it is not difficult to make use of variable values of K. The summation of the zone hydrographs for a one-period rain may be used as a unit hydrograph, but it is better to complete each zone hydrograph for its entire runoff and then combine the flows.

The hydrograph is sensitive to the shape of the time-area diagram and to the number of periods of rainfall. Variations in rainfall intensity and distribution, infiltration losses, and other factors may be taken into account zone by zone. The method is simple and flexible. It is based on the same conception of constant time elements that underlies the unit hydrograph theory, but it is more adaptable to a variety of conditions. The hydrograph may be computed for an approximate time interval and then corrected to fit the proper time base. When the time-area diagram and other constants are properly determined, the hydrograph due to any type of storm can be calculated with satisfactory limits of approximation. Conversely, any kind of storm can be employed in deriving the constants.

Table 4 shows the suggested arrangement of data. The calculations are for a four-zone watershed having an area of 300 sq miles. The zones vary considerably in size, and a three-period rainfall is irregularly distributed both with respect to zones and time periods. Constant K equals $1.5\,T$, so that $C_0 + C_1 = C_2 = 0.50$. The modification of runoff for zone 1 is computed as shown in Table 5.

TABLE 5.—COMPUTATION FOR THE MODIFICATION OF RUNOFF, ZONE 1, TABLE 4

Line No.	Description	Runoff in Mile²-Inches							
		40	28	20	..	..	..	..	Remainder
1	Runoff, in mile²-inches × 0.50	20	14	10	..	..	..	..	..
2	Line 3 (advanced one period) × 0.50	..	10	12	..	..	..	..	..
3	Time-period ordinates	20	24	22	11	5.5	2.75	1.38	1.37
4	Recorded in Table 4, and shown graphically	20	24	22	11	6	3	2	..

The zone hydrographs, A1, A2, A3, and A4, are combined by adding the computed ordinates, resulting in the irregular hydrograph B (see Table 4). The relatively small amount of precipitation in zone 2 during the middle time period is the cause of the dip in A2 and the corresponding flattening of B. The sharp peak in A4 results in a bulge in B. If the same total net rainfall were uniform over the entire drainage area over the three time periods, the resulting hydrograph, C, is much less irregular, but the peak is changed only slightly. The same total rainfall uniformly distributed over five periods would produce hydrograph D, having a considerably lower peak.

If the surface and subsurface flows are separated, the foregoing method may be applied in the case of the Appomattox River (see heading, "Derivation of the Instantaneous and Unit Hydrographs"). Assuming a uniformly distributed net rainfall of 2.35 in. (3,140 mile²-inches), occurring in the three periods ending at noon of April 26, and a time lag (storage) of 15 hours, then adding subsurface flows amounting to 0.57 in. in 10 days in accordance with curve (c), Fig. 8, the results agree substantially with the computed hydrograph shown in Fig. 9.

In the "statement of fact" No. 1, Mr. Clark states that there may have been a considerable variation in rainfall from east to west both in quantity and time. The writer assumed a variation of from 1.7 in. to 3.1 in., with a weighted average of 2.35 in. over the entire basin, and found that, on the whole, the computed curve did not fit the observed hydrograph any better than did the author's hydrograph, Fig. 8. The effect of varying K from a minimum of 9 to a maximum of 20 was found to be negligibly small. It would seem that the time-area diagram is the most important factor in the computation. In all cases there was too much difference between the peaks and the valley between, as compared with the recorded flows. It is likely that the record is at fault, as suggested in the "statement of fact" No. 3, or that the largest zone areas have been somewhat overestimated.

FRANKLIN F. SNYDER,[26] ASSOC. M. AM. SOC. C. E.—Papers dealing with unit hydrograph and flood routing procedures have been quite numerous in recent years and the author is to be complimented for his efforts to clarify the inherent relationship between these hydrologic tools. However, the extent to which that objective has been accomplished is modified by the introduction of a number of confusing and unsupported statements about certain hydrologic phenomena.

In the "Introduction," the author, referring to an article,[7] states that "For example, in determining a seven-period unitgraph from a flood resulting from three periods of runoff, it is possible to determine thirty-six unit hydrographs which, applied to the three items of runoff, will reproduce the composite hydrograph exactly." Theoretically, if the unit hydrograph procedure were exact, there would be one unit hydrograph that would reproduce the composite hydrograph exactly. For illustration, one can assume a unit hydrograph and compute a composite hydrograph. There exists, therefore, a unit hydrograph which will reproduce the composite hydrograph exactly. In actual practice there is no unit hydrograph that will reproduce observed composite hydrographs exactly. However, if the composite hydrograph is not the result of a too complex storm, numerous methods are available for determining a unit hydrograph that will reproduce the observed composite hydrograph with a satisfactory degree of accuracy. The example[7] cited by the author illustrates this theory, as the most probable distribution-graph values obtained by "least squares" for a composite storm do not reproduce exactly the observed hydrograph.

Fig. 3 shows the results of a study of the effect of storage that is equally a function of inflow and outflow on a wave form with an initially very sharp peak. The author has made a common mistake[27] in obtaining an outflow hydrograph different from the inflow hydrograph after assuming that the reach storage is equally a function of inflow and outflow ($x = 0.5$, or that storage varies as the sum of inflow plus outflow).

The fact that the outflow hydrograph should be identical with the inflow hydrograph except for a time shift is readily proved from the general storage equation and is evident from a consideration of the Muskingum formulas,

[26] Hydr. Engr., War Dept., Office of Chf. of Engrs., Washington, D. C.

[27] "A Direct Method of Flood Routing," by C. O. Wisler and E. F. Brater, *Transactions*, Am. Soc. C. E., Vol. 107 (1942), p. 1519.

Eqs. 6*d*, 7*a*, 7*b*, and 7*c*. Another result is obtained when certain basic requirements fundamental to most routing procedures are violated:[8] (1) The routing interval, T, must be small enough to define, satisfactorily, variations in rate of flow; and (2) the reach should be of such length that the time of travel is approximately equal to T. In the example under discussion a 2-hr value of time of travel, K, was assumed by the author for an imaginary reach of a river. The value of T that was used to derive the hydrographs in Fig. 3 is not given, but, according to requirement (2), T should be 2 hours. Then, with x equal to 0.5, $C_0 = 0$, $C_1 = 1.0$, $C_2 = 0$, and $O_2 = I_1$—that is, outflow at the end of the period equals inflow at the beginning. Requirement (1) was violated since a value of T equal to 2 hours is too large to define, adequately, the hydrograph in Fig. 3. However, O_2 will always equal I_1 from Eq. 6*d* as long as T is made equal to K.

On the same basis the outflow curve labeled $x = 0.5$ in Fig. 2 is in error and should be identical to the original curve except for a time shift of 4 hours. The negative values for the curve, $x = 0.5$, and for the other outflow curves, also, are definitely due to the selection of units and the limitations of the routing procedure, and to couple these values with physical phenomena either real or imaginary seems unwise.

Another erroneous result is often obtained when routing a hydrograph such as that for Danville in Fig. 12, through a reach with a value of K considerably greater than a reasonable value of T. The slow prolonged rise and rapid fall coupled with the assumptions of the Muskingum routing procedure result in a peak outflow rate higher than the inflow rate even though a value of x considerably less than 0.5 is assumed. Walter B. Langbein,[28] Assoc. M. Am. Soc. C. E., has suggested an adjustment of the Muskingum procedure to compensate for the inability of the weighted flow to represent storage in the reach adequately when either inflow or outflow is changing rapidly.

To justify the basic assumptions followed in developing the Muskingum routing procedure, the value of T should be made approximately equal to the value of K. If this is not done (and it is seldom possible in practical applications), the value of x is a function of the length of reach, or K, in addition to indicating the portion of the weighted flow that is derived from the inflow.

The attempt to prove that shifting a hydrograph in time and routing it through a small value of storage gives a hydrograph quite similar to that obtained with the Muskingum routing procedure using a value of $x = 0.5$ has resulted in a failure to make clear an important objective of the basic idea. Most flood routing procedures actually provide no time advantage for forecasting purposes—that is, the inflow data must be extended into the future just as far as it is desired to obtain values of outflow. The procedure proposed by the author—namely, shifting the inflow hydrograph in time and routing it through a predetermined amount of reservoir-type storage—does give a time advantage equal to the amount of the time shift.

The writer has been accumulating data in connection with such a routing assumption and, as would be expected from the ease with which routing procedures can be verified, has had little difficulty in obtaining satisfactory

[28] "Channel Storage and Unit Hydrograph Studies," by Walter B. Langbein, *Transactions*, Am. Geophysical Union, Pt. II, 1940, pp. 620–627.

results when routing observed hydrographs for various streams in the eastern United States.

In analyzing these records a procedure convenient for routing the transposed hydrograph through a certain amount of reservoir-type storage was developed. As stated by the author, where storage, S, and outflow, O, are uniquely defined by the elevation at the outlet such as in a reservoir ($x = 0.0$ in the Muskingum procedure), the ratio of successive increments of storage to increments of discharge, $\frac{\Delta S}{\Delta O}$, has the dimension of time and is represented in this discussion by the symbol T_s. If storage is in acre-feet and O in cubic feet per second, T_s is in units of days and equals (very nearly) $0.5 \frac{\Delta S}{\Delta O}$. Starting with a volumetric expression of the storage equation for some satisfactory routing period, the mean inflow, $\bar{I}$, minus the mean ouftlow, $\bar{O}$, equals the change in storage. For units of cubic feet per second for discharge, acre-feet for storage, and days for time:

$$\bar{I} - \bar{O} = \frac{\Delta S}{2\,t} \qquad (19)$$

With the usual assumption of straight-line variation of inflow and outflow during the routing period, t, and, since $\bar{O}$ then equals $\frac{\Delta O}{2} + O_1$, with the subscript 1 indicating the value of the variable at the beginning of a routing period:

$$\bar{I} - O_1 = \frac{\Delta O}{2} + \frac{\Delta S}{2\,t} = \Delta O \left(0.5 + 0.5 \frac{\Delta S}{t\,\Delta O}\right) \qquad (20)$$

Substituting T_s for $0.5 \frac{\Delta S}{\Delta O}$ and solving for ΔO gives

$$\Delta O = C\,(\bar{I} - O_1) \qquad (21)$$

in which

$$C = \frac{1}{0.5 + \frac{T_s}{t}} \qquad (22)$$

This solution of the storage equation lends itself readily to tabular computation and to the use of variable values of T_s by plotting the variation of C against O. When inflow is zero, ΔO equals $-C\,O_1$ and C equals $1 - C_r$ in which C_r is a recession coefficient of the outflow hydrograph equal to $\frac{O_2}{O_1}$.

Fig. 17 shows graphically some of the time relationships utilized in analyzing observed records. Local inflow has been separated from the outflow hydrograph so that the latter represents the total effect of the transposition, T_t, and the reservoir-type storage, T_s, on the hydrograph of inflow at the upper end of the reach. The total time shift between center of mass of inflow and outflow, T_T, is equal to the sum of T_t and T_s; that is,

$$T_T = T_t + T_s \qquad (23)$$

The value of T_T is also equal to the value of K in the Muskingum routing procedure provided the fundamental routing requirements previously discussed are not violated. However, if the inflow hydrograph, I, of Fig. 17 is routed using the Muskingum coefficients with $K = 24$ hours, $x = 0.40$, and $t = 6$ hours (too long a reach), the time lag between center of mass of inflow and outflow, T_m, will be 21 hours instead of 24 hours. A quite similar outflow hydrograph is obtained by this procedure, using $T_t = 10$, $T_s = 11$, and $T_T = 21$ ($T_m = 21$).

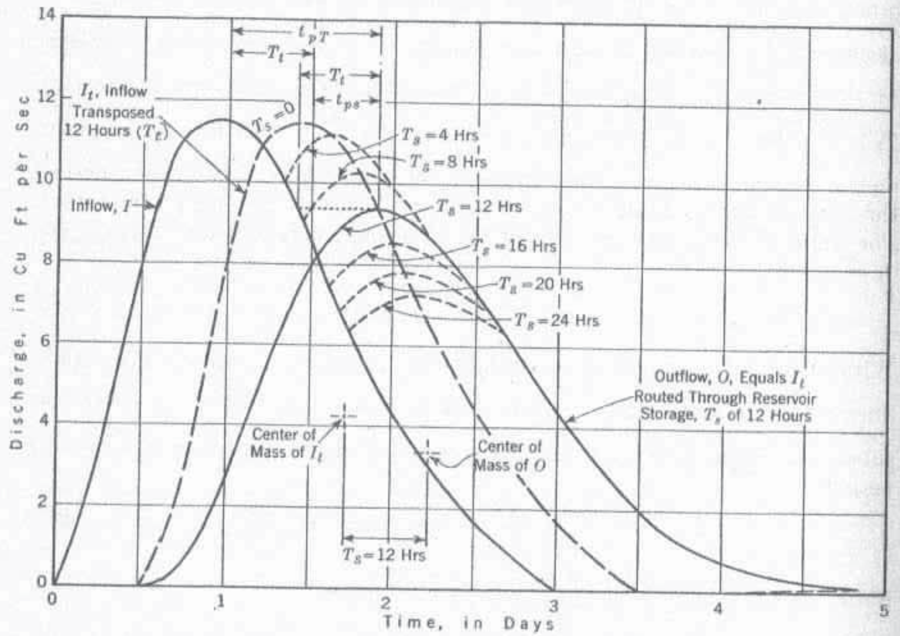


FIG. 17.—TIME AND STORAGE RELATIONSHIPS

The value of T_t in any particular case is available from inspection of the hydrograph and the following relationship is useful in estimating values of T_s: From Fig. 17, t_{pT} equals T_t plus t_{ps}, in which t_{pT} is the total time lag of the crest discharge, T_t is the crest lag due to the transposition, and t_{ps} is the lag of the crest discharge due to reservoir-type storage effect; therefore:

$$t_{pT} = T_t + t_{ps} \quad \text{(24)}$$

and, since T_s equals $T_T - T_t$,

$$T_s = T_T - t_{pT} + t_{ps} \quad \text{(25)}$$

The time between center of mass of the inflow and outflow hydrographs, T_T or T_m, is readily obtained by transferring the hydrograph shapes with carbon paper to a sheet of stiff paper, cutting out the hydrographs, and locating the centers of area.

In the beginning the writer had hoped that values of T_t and T_s could be assumed constant throughout a wide range of flow conditions, an assumption that is theoretically incorrect but one that is made in most routing procedures.

A variation of T_s can be handled without much difficulty but when both T_t and T_s vary so that their sum also varies, the problem becomes more difficult. As might be expected, observed hydrographs for many streams could be duplicated by assuming constant values of the time and storage factors. However, when a stream with a wide flood plain such as the Missouri River goes overbank, there is a variation of several hundred per cent in the factors.

Under the heading, "Valley Storage," the author states that "The reservoir pool above a dam may not reduce flood discharges." Most engineers will subscribe readily to this generality, and its acceptance is indicated by the fact that considerable attention is given to this problem in the design and operation of reservoirs for hydroelectric projects. The author also states that, "* * * if the pool level at the dam is held constant by gate control, * * * the peak outflow may be larger than the peak inflow." The writer could also agree with this statement if the words "natural peak" discharge at the dam site were substituted for the words "peak inflow"; but it will take considerable more evidence than the Muskingum routing hypothesis to convince him that the peak outflow can be greater than the peak inflow. As a matter of fact, because of backwater storage above a level reservoir pool, the level of the pool at the dam would have to be lowered somewhat to make the peak outflow of a flood as large as the peak inflow. The peak inflow in some cases is larger than the natural peak at the dam site would have been prior to construction of the project because of the elimination of some valley storage but principally because of the change in timing of the concentration of the runoff from the areas draining directly into the reservoir.

The author's extension of storage theory to ground-water flow is quite interesting. The use of a unit hydrograph for distributing ground-water runoff may be quite practicable, but the use of storage values determined from recession curves for deriving the rising part of the runoff hydrograph is subject to more criticism in the case of ground-water runoff than in the case of surface runoff, and the procedure is open to question even for surface runoff.

Other than the use of a constant factor for dividing total runoff into surface and ground-water runoff, the author's procedure probably would be satisfactory for many purposes. However, the writer takes exception to the inference that, in comparison, current practise in treating ground-water discharge is unsatisfactory. Reference is made to a paper[29] discussing this and other phases of runoff phenomena in which both direct runoff and ground-water runoff for the Schuylkill River above Pottstown, Pa. (drainage area, 1,147 sq miles) were computed storm by storm for a year of record using only observed values of precipitation and temperature. This article presents a graphical comparison of the computed ground-water discharge and the observed hydrograph.[30] The accuracy of the ground-water phase of the computations proved more than sufficient for any need of the writer in flood forecasting or hydrologic studies.

The author's use of the quantity, K, and the time of concentration in the development of unit hydrographs is confusing. Under the heading, "The

[29] "A Conception of Runoff-Phenomena," by Franklin F. Snyder, *Transactions*, Am. Geophysical Union, Pt. IV, 1939, pp. 725–738.

[30] *Ibid.*, Fig. 4, p. 728.

Hydrograph as an Index of Storage," it is stated that, at the time of most rapid relative rate of decrease of discharge, "The falling legs of a large number of hydrographs of flood discharge for streams in major drainage basins from New Hampshire to North Carolina exhibit a ratio of decrease of discharge to total discharge which, for a value of $x = 0.5$, indicates a value of K closely approximating the time elapsing between termination of runoff producing rainfall and the point of most rapid relative decrease in discharge." With $x = 0.0$ the values of K for these same recessions determined by Eq. 12*b* would be equal to one half the value of time of concentration ($0.5\ T_c$). In the "Summary" it is stated that the time-area concentration curve should be routed through reservoir storage "equal to K times the outflow when K is the time indicated by Eq. 12*b*" at the time of most rapid relative rate of decrease of discharge when $x = 0.0$; that is, $K = 0.5\ T_c$. However, the author states (see heading, "Determination of the Instantaneous Hydrograph for Streams Without Flow Records") "the amount of storage by which the time-area concentration curve should be modified will approximate 10% to 25% of the time of concentration * * *." In the example for the Appomattox River above Petersburg, a value of $K = 0.06\ T_c$ was used ($K = 9$ hours, $T_c = 144$ hours) whereas the average value of K for the James River at Lick Run as determined from the data in Fig. 5 was $0.5\ T_c$ ($K = 14$ hours, $T_c = 28$ hours). With a somewhat similar procedure, which is about the most common and oldest known rational method of determining synthetic hydrographs, the writer has obtained satisfactory unit hydrographs from time-area curves routed through storage values equal to the unit hydrograph "lag" defined as time from center of mass of effective rainfall to peak of the unit hydrograph. Values of "lag" are almost always greater than one half the time of concentration as defined by the author.

Howard M. Turner, M. Am. Soc. C. E., and Allen J. Burdoin,[31] using a similar procedure, obtained fairly good reproduction of observed hydrographs for the Connecticut River at Sunderland, Mass., by routing a time-area or inflow hydrograph through a reservoir storage for which K equals 59 hours with a value of time of concentration equal to 50 hours ($K = 1.2\ T_c$).

The principal purpose of the preceding discussion is to emphasize the fact that there are still two variables in the development of a synthetic unit hydrograph: A time element (time of concentration, T_c, in this paper), and a storage factor (expressed as a percentage of T_c by the author). The use of a slope factor has been introduced by the author as an aid in determining the time factor, but the storage factor has not been correlated directly with any physical features of the drainage basin.

From the writer's experience in attempting to correlate the two variable factors with physical features,[32] he is not too optimistic about the author's use of a slope factor to determine the time variable except in localized areas. The river slope adopted by the author is believed to be a more representative slope factor than the average over-all slope usually obtained by dividing the total

[31] "The Flood Hydrograph," by Howard M. Turner and Allen J. Burdoin, *Journal*, Boston Soc. of Civ. Engrs., Vol. XXVIII, No. 3, July, 1941, p. 232.

[32] "Synthetic Unit-Graphs," by Franklin F. Snyder, *Transactions*, Am. Geophysical Union, Pt. I, 1938, pp. 447–454.

fall by the length. However, the data presented in the paper indicate a range of values of C, in which $T_c = C \frac{L}{\sqrt{s}}$, from 0.6 for the Smith River at Bassetts to 2.0 for the Appomattox River near Petersburg. (A value of average slope, as defined by the author, of 11.6 ft per mile for a length of 37 miles was determined from available reports and used for the Smith River at Bassetts instead of the value of 16.6 given in the paper.) Although the writer believes the exponent of the slope factor should be smaller than 0.5, variation of the exponent of s in the expression $T_c = C \frac{L}{\sqrt{s}}$ would have little effect on the relative range of values of C which vary from 0.6 to 2.0 for the Virginia drainage basins discussed in the paper.

The suggestion of the author that the determination of the time and storage factors for streams without records should be based on actual measurements should be emphasized. A time factor such as "lag"[32] can be obtained for any river station where discharge is principally a function of stage, without installing any equipment, by observing the time of crest stage and comparing it with field observations or official records of the rainfall characteristics of the storm. A storage factor is not so easily determined but it is not subject to as great a range in value as the time factor and the time factor provides a partial index of the actual value of the storage factor.

Howard M. Turner,[33] M. Am. Soc. C. E.—The first part of this interesting paper is a discussion of the channel storage in a reach of a river. The author compares the storage in this case with that in a reservoir, indicating the difference in the behavior, due to the fact that the relation between outflow and storage is not clear on account of the slope. The author cites an assumed case showing how a sharp diminution in inflow "requires" an increase in outflow at the outlet end of the reach. The writer has never seen any such effect. There is a sudden decrease in inflow at the outlet of many water power plants at 5 o'clock or 6 o'clock every afternoon for months during the year when there is no waste. The writer has examined many hydrographs taken at gaging stations some distance downstream from such stations and has never seen any evidence of any rise attributable to the cause given by the author. He admits the possibility, but believes that the author's statement is much too categorical in stating that it is "required" under such circumstances. There may be a slightly rising outflow because the high discharge existing at the upper end has not yet become stabilized at the lower end—that is, water is still going into storage and this rise will continue for a period of time after the power station shuts down due to the time lag between the two points. The outflow then drops off in the regular recession curve as the storage drains off. The difficulty in the author's contention is his statement (see heading, "Valley Storage") that the additional storage must be released "during the period of inflow reduction." This increased storage that must flow out may be released over a longer period of time.

[33] Cons. Engr., Boston, Mass.; Lecturer on Water Power Eng., Harvard Univ., Cambridge, Mass.

The writer has analyzed several of these conditions and, by using the storage curve determined from the recession side of the hydrograph (that is, the storage as a function of the outflow), he has obtained very satisfactory agreements with an actual hydrograph by routing such power station discharges through this storage with time allowance for travel.[34] In two of the three cases considered, the hydrograph downstream did show higher flow than that upstream, but this was due to the flow from intervening drainage and the inaccuracy of computing the power station discharges from the kilowatt-hour output and the difference disappeared when the inflow hydrograph was adjusted to give the same total runoff. There was no evidence in these cases of any rise which the author states is required by sudden reduction of inflow to the reach. If the author has any actual examples showing this effect, it is hoped that he will present them as it is an extremely interesting and, as he states, a somewhat new conception.

The author then describes his distinction between storage as a function of the outflow and storage as a function of the inflow. The writer believes Mr. Clark has oversimplified this theory. Actually, storage is a function of neither inflow nor outflow alone, depending as it does on topography and the level of the water surface, which in turn are dependent upon slope. Therefore, any difference between the author's two concepts of storage is dependent on a difference in changing slope which with the topography of the banks affects the volume of storage. In this case a more exact statement would be that, in the case of the unsteady flow of a rising or falling river stage, the changing slope affects the storage.

The writer finds it difficult to accept some of the statements made in regard to the storage affected by the inflow. In Fig. 2 the author shows curves for routing a simple sine curve with different storages based entirely on inflow ($x = 1.0$) through to a curve based entirely on outflow ($x = 0.0$). In the case of the former the flow with storage is greater than without storage. Where $x = 0.5$, the hydrograph is very little changed. It is not until x equals less than 0.5 that the storage reduces the inflow, and not until $x = 0$ does the outflow hydrograph intersect the inflow hydrograph at the peak of the former, which is generally held as a basic requirement between inflow and outflow hydrographs.

As would naturally occur if these curves are correct, the curve of outflow with $x = 1.0$ intersects the curve of inflow at the peak of the former and thus the storage increases the peak discharge from the outlet of the reach by a very great amount. The writer gives $x = 0.3$ to 0.5 as the usual range of values. If this condition is valid, there will be many cases in which there will be little if any diminution of peak due to storage. The writer does not question the conclusions reached with the author's equations, but he has never seen any indication of such effects and therefore distrusts the original equation. He believes that it is based on too simple a function of the storage compared to that actually existing in any normal river channel.

The second part of the paper develops a method of constructing a unit hydrograph from an instantaneous hydrograph. In this case the author uses storage as a function of outflow alone ($x = 0$). The writer is not certain whether the author considers the condition to be somewhat different for a given reach than for the storage of the entire river system, since the storage used in this section of the paper does not seem to correspond with his previous statement regarding what he has found to be that on most rivers.

[34] "The Flood-Hydrograph and Valley-Storage," by Howard M. Turner, *Transactions*, Am. Geophysical Union, 1943, Pt. II, p. 613.

The author's process consists of using storage based on the actual flood hydrograph at the beginning of the recession curve and routing through it a time-area concentration curve which is in effect an inflow hydrograph built from an analysis of the time required for the water from the different parts of the drainage area to reach the outlet. In doing this the same storage relation is used for the rising stage of the hydrograph as for the falling stage.

The writer was much interested in this procedure as it is based on the same principles that were used by him and Mr. Burdoin.[31] Using their method, a rational inflow hydrograph was constructed from the duration of the rainfall, the concentration time was obtained from the deflection point on the recession curve of the hydrograph, and the total flood volume was routed through storage computed from the recession curve to obtain the outflow or flood hydrograph. The author extends these principles somewhat further adding the feature of the instantaneous hydrograph.

The author's method differs with regard to the storage which he determines from only one point on the hydrograph—the inflection point on the descending limb. This assumes a straight line from this point down to zero, an assumption which may not always be correct. (On the Appomattox River at Petersburg, one of the examples cited, the curve is not a straight line all the way down.) Mr. Clark draws an instantaneous inflow hydrograph for a given river by actually figuring the time-area relation instead of using, as the writer did, a general one based on shape. This makes a much more flexible arrangement than the writer's and gives a more accurate inflow hydrograph. Such accuracy may not always be necessary, however, when the rainfalls over the area for different storms are considered. (For example, the Appomattox River at Petersburg has had other floods which present different characteristics. The one of March, 1932, with a peak of 8,000 cu ft per sec and about the same length of base shows no depression at the top of the hydrograph.) The writer found that an assumption of a rectangular shape gave satisfactory results in many cases that varied considerably from a rectangle.

The author's method, however, would permit extending the time-area concentration inflow not only to cover the shape of a given area with assumed uniform rainfall but also to include varying rates of rainfall on different parts of this area.

The author presents a new treatment of the base flow which, in the particular case of the Appomattox River, gives much better results than would the customary method—that is, a straight line at the base of the hydrograph, which, if used, gives a computed hydrograph less close to the actual one than the author's. This hardly can be taken as a proof that the author's method is any better since the entire matter of the rainfall distribution is unknown, as the author states. (Rainfall at Hopewell amounting to 23% of the total occurred during the two days, April 28 and 29, in the middle of the flood.)

There are many unknowns in the behavior of the ground storage during a flood. Unanswered are such questions as how the water gets into the ground with a rapidly rising river, how far back the rise in ground-water level continues before the river begins to drop again and the storage flows back into the river, and how much of this drainage from storage in the ground may be due to normal ground-water flow backed up by the rising level and thus not originating from the flood rainfall. The outflow hydrograph will also be much affected by the assumption as to the amount of flood appearing as ground-water flow assumed by the author as 30% of the total. The author suggests that this percentage, taken from records of another river, must vary with floods of different magnitude. To use 30% for an instantaneous hydrograph for all floods may lead to error in the case of much greater or much smaller floods. The author should give some examples of the proportion of ground-water runoff which his method discloses in small floods compared to very large floods. The writer has found that with all these possible differences, the simple method of drawing a straight line across the base of the hydrograph is sufficiently accurate for general use in spite of its theoretical limitations.

The author is to be congratulated for devising an additional tool to use in the attempt to analyze the steps between the rainfall, inflow hydrograph, and the flood hydrograph.

Don Johnstone,[35] Assoc. M. Am. Soc. C. E.—The difficulties of winnowing out the grains of wheat in technical literature are certainly not decreased by inventing new meanings for old phrases. Engineering terms should mean the same thing today as yesterday. If new concepts are introduced, new terms should be invented. The author, in applying the term "unit hydrograph" or "unitgraph" to a hydrograph which includes ground-water flow, is misusing a basic definition. Similarly, in applying the term "concentration time" to the time interval "between cessation of runoff-producing rainfall and the minimum indicated value of K" he is tying another time-honored phrase to a new concept. Mr. Bernard's definition of concentration time ("a time interval at the end of which all parts of the watershed are contributing to the flow at the point of observation") gives that particular combination of words a job to do and should pre-empt it from other uses.[36] Other examples of criticizable terminology include the use of "runoff" for "rainfall" in the third and fifth paragraphs of the "Introduction" and in the third paragraph of the section headed "Valley Storage and the Unit Hydrograph," followed by its use in an entirely different sense in the second paragraph of the next section; and the use of "the channel" in that same section without indicating whether reference is to the main stream alone or to any "channel" in the watershed.

Passing to other matters, attention is invited to the statement in the "Introduction" that the determination of unitgraphs from "multiple periods of runoff [sic] * * * is indeterminate, as there are always more equations than unknowns and a multiple of possible solutions." Presumably the author meant "more unknowns than equations," but, waiving that point, the statement is still incorrect. By unitgraph theory, surface runoff in the xth period resulting from a storm of n periods duration is $c_1 f(x) + c_2 f(x-1) + \cdots + c_n f(x-n+1)$, in which $c_1, c_2, \cdots, c_n$ are the net rainfalls for the corresponding periods and $f(x), f(x-1), \cdots, f(x-n+1)$ are the ordinates of the unitgraph for the xth, $(x-1)$th, $\cdots$, $(x-n+1)$th periods. If flow measurements and rainfall measurements were precise, if the base flow (ground-water flow) were known, if the net rainfalls for all periods were determinable, and, finally, if the stream behaved exactly in accordance with unitgraph theory, it would be possible to derive the unitgraph from the hydrograph of a multiple-period storm by successive subtractions. Actually, successive subtractions yield nothing of value, as none of the four "if's" is exactly in accordance with fact and as the process involves the accumulation of error from step to step. The concept is set forth here, however, to demonstrate that there are actually as many equations available as there are unknowns (so far as unitgraph theory is concerned), and that there should therefore be no possibility of multiple solutions from a given set of data if a proper technique of derivation is applied. One such technique has been outlined by William T. Collins,[37] Jun. Am. Soc. C. E.

It follows (see "Introduction") that "the practical requirement of finding an isolated flood hydrograph resulting from a uniform unit rainfall," which the author states "provides an almost insurmountable obstacle to the derivation of the unitgraph for very small [?] drainage areas and very short unit periods [intervals] of time," is no longer a requirement at all. As a matter of fact, a protracted period of heavy rainfall is the ideal period from which to derive the unitgraph, because (*a*) rainfall is more likely to be nearly uniform over the watershed, (*b*) infiltration conditions are more nearly uniform throughout the period of rainfall, and (*c*) the fact that rainfall actually extended over the selected unit time interval is assured. To carry this argument further would be beyond the proper scope of discussion; the writer acknowledges that despite the incorrectness of the quoted statements there is still need for a method that will permit deriving unitgraphs without reference to rainfall records.

The argument under "Valley Storage," down to the introduction of Eq. 4, is confusing. Particularly welcome would be a justification of the statement that "reduction of the inflow rate necessarily would be accompanied by * * * an increase of outflow rate." Fig. 1 and its explanation are unconvincing, and the writer would like to see (here as elsewhere) proof in the form of actual recorded data rather than in the form of a rather casual theoretical analysis.

The argument on the extension of storage theory to ground-water flow appears to "boil down" to this: It would be convenient if unitgraph principles were applicable to ground-water flow; therefore they are applicable. Without further proof, the author proceeds to apply them. There is something about neatly arranged tables of computations that seems to consecrate the result, and it is likely that computing a number of ground-water flow hydrographs by this method would confirm many a budding hydrologist in the belief that the implied relationships had been proved. Actually, the "refinement" attained by treating ground-water flow in this manner (if it be, in fact, a

[35] Asst. Prof., Civ. Eng., Ohio State Univ., Columbus, Ohio (on military leave).

[36] "An Approach to Determinate Steam Flow," by Merrill Bernard, *Transactions*, Am. Soc. C. E., Vol. 100 (1935), p. 350.

[37] "Runoff Distribution Graphs from Precipitation Occurring in More than One Time Unit," by William T. Collins, *Civil Engineering*, September, 1939, p. 559.

refinement at all) seems scarcely worth the effort; 30% of curve (c) in Fig. 8 could be replaced by the customary sloping straight line and the change in curve (d) would scarcely be noticeable.

An even more basic criticism, however, is the inclusion of ground-water flow, by whatever means computed, in the unitgraph. By including it, the range of usefulness of the unitgraph is seriously impaired. Its major function is not—as emphasis in many papers would lead one to believe—to confirm itself repeatedly by reproducing second-rate floods with more or less accuracy. Rather, it is a tool to indicate what will probably happen when a really first-rate rainfall comes along; and it should be left to do that job unhampered by the introduction of ground flow factors that will likely be inapplicable at the big moment. Suppose the maximum storm hits the watershed when the ground is saturated—or frozen. A true unitgraph can handle that situation (compare the "pluviagraph" of Mr. Bernard[38]); a composite graph of surface and sub-surface runoff cannot handle it—and it errs on the side of danger.

Passing to the author's example, there is much difficulty in following the detail. Table 1 shows $K = 15$, whereas the text states that $K = 9$. Also, the relationship between the plotted curves of Fig. 8 and the data of Table 1 is obscure. Should not curve (d) be the plotting of Col. 9?

However, the major shortcoming of the example is the failure to explain the construction of Fig. 6. This is the heart of the entire matter—the drawing of lines on a map in such a manner as to define areas that make their contributions to "the channel" in successive intervals of time. How is it done? What factors are taken into account? Why do the area boundaries cross the streams without bending stream-upward, when logic suggests that they should take the form of exaggerated contour lines? (See, for example, Mr. Bernard's statement:[36] "* * * the efficiency of the main channel, through higher velocities, places headwater areas much closer to the outlet in terms of time than extensive areas nearer but less advantageously located.") With a knowledge of how to chart these areas, the operator finds the remainder of the technique to be a matter of measuring areas by planimeter and of running a calculating machine; without it, he is at a loss where to start.

The justification for applying the same value of K to the contributions of all areas is not apparent. It would appear to be reasonable only on the assumption that storage conditions are the same in all areas and that no storage exists in the main channel; but obviously the author does not make any such assumption. Rather, he takes for the value of K the value existing at the moment when all surface runoff is presumed to have reached "the channel." It follows that K for the downstream areas should be smaller, and for the upstream areas larger, than the average.

Since delineating the areas on the map is subject to a wide range of personal choice, and since the use of a single value of K is open to question, the writer suggests that a somewhat different approach might yield equally satisfactory results with less effort. Using "concentration time" as defined by Mr. Bernard, the base length of any hydrograph of surface flow (including the unitgraph as defined by Mr. Sherman) is equal to the duration of runoff-producing rainfall plus the concentration time. This statement includes the instantaneous unitgraph, for which the base length is simply the concentration time itself. Concentration time, by the Bernard definition, can be determined from an inspection of hydrographs, with reasonable accuracy, as readily as concentration time by the Clark definition. Having determined it, divide the watershed map into the corresponding number of time-interval areas and determine the percentage area of each. The tabulation of these percentages, without any further operations, is in itself the instantaneous distribution graph of surface flow for the watershed.

[38] "An Approach to Determinate Stream Flow," by Merrill Bernard, *Transactions*, Am. Soc. C. E., Vol. 100 (1935), p. 358.

It cannot be said that the effect of storage is overlooked in this proposed method, for it is reflected in the fact that concentration time (the Bernard definition) takes in the entire period of time required for a particle of water to travel from the hydraulically-most-remote point to the point of observation. Basically, the analysis seems as sound as that of the author's more complex procedure. It would be interesting to determine a unitgraph for the Appomattox River by this proposed method and test it, also, for the flood of April, 1937. Unfortunately, the necessary basic data are not available to the writer. Also, an adequate technique for dividing the watershed map presumably remains to be developed. This should be a fruitful field for further inquiry.

In making such a test, it would be necessary to assume a "reasonable" ground-water flow, and the writer would be content with a straight line joining the point on the observed hydrograph at which rainfall begins, with the point at which surface runoff ceases. Fig. 9 indicates the latter to be at the line marked "April 3."

C. O. Clark,[39] Jun. Am. Soc. C. E.—The discussions represent much thought and labor at a time when most of the writers are actively engaged in entirely different matters. The expressions which corroborate the paper are gratifying and greatly appreciated. The divergent views outline some fertile fields for further investigation of problems which are still largely dependent on judgment and should stir stimulating and constructive thinking. Some of the queries and counterproposals presented lie beyond the ability of the writer to answer; some have been answered by other discussers; and a few are reviewed in the closing discussion. All are sincerely appreciated.

Mr. Sweet pertinently emphasizes that the labor involved in unit hydrograph and flood-routing techniques is excessive for routine river forecasting. Reduction of this labor by the development of simplified empirical coefficients is necessary and highly desirable. However, the simplification might be based upon a large number of hypothetical floods computed by appropriate methods as well as upon records of past floods. As Mr. Sweet states regarding the James River, nonuniformity of storm distribution may produce variable stage relationships between the gages along the river. If records of only a few large floods are available, it is easy to assume that relationships are fixed, only to be embarrassed when nature proves otherwise. However, unit hydrograph study of hypothetical adverse distributions of storm rainfall will reveal many of the variable possibilities which may serve as guides for good forecasts rather than

[39] Hydr. Engr., U. S. Engr. Office, Winchester, Va.

as alibis for poor ones. The James River, with its two halves on either side of the Blue Ridge Mountains in Virginia, offers opportunities for illuminating study to any one who believes that nonuniformity of runoff distribution on an area is not significant, or that gage-to-gage stage relationships are simple or constant.

Major Meyer clarifies several points in the paper, and appropriately warns of situations in which K is not a constant. Unit hydrograph theories and constant time elements of flow are convenient oversimplifications of a most complex form of open channel flow. They are by no means universally applicable. The paper treats only the elements common to unit hydrograph theory and to the theories of flood routing based upon constant values of K. "Lag routing," to which both Major Meyer and Mr. Snyder refer, is a very practical and expeditious method of flood routing. Fig. 3 and its supporting discussions demonstrate the essential agreement between this method and the much more laborious application of the Muskingum technique. The construction of the time-area concentration curve of Fig. 8 and its routing modification conform essentially to "lag routing" procedure.

Lag routing is an empirical approximation of some solutions of the storage equation; it is not itself a solution of the storage equation for any fixed relationship between storage capacity and discharge. It is an expedient method for use in a natural state of river development, as in flood forecasting or in computation of past floods. However, the writer would be unable to determine, with it, the effect of improvements which change the storage capacity along a river, such as extensive levee systems—or particularly those improvements which change the relative influence of inflow upon storage, such as a series of dams.

Thus, for solving routing problems which involve a change in storage-discharge relationships, the writer believes that the storage equation will be more reliable than empirical "lag routing." As Major Meyer writes, there are several methods other than the Muskingum method for the solution of Eqs. 2, 4, and 5.

Mr. Sherman—the creator of the unit hydrograph—reviews the underlying theory of this excellent hydrologic tool with masterliness. Possibly through misinterpretation of the bar pattern of Fig. 8 and through the assumption that curve a is a net rainfall pattern of twelve unit periods rather than a representation of shape of drainage area, Mr. Sherman concludes that the resulting derived hydrograph was not a unit hydrograph. In Fig. 8, a bar pattern representing net rainfall would be of infinitesimal duration at zero time. In Fig. 9, rainfall was principally confined to that on April 25 and that early on April 26.

An apology is made to Mr. Sherman and to others who found difficulty in checking the computations in Table 1, particularly the conversion of Col. 8 to Col. 9. The conversion at a rate of 69,000 (cu ft per sec)-half-days per in. is a rounded value appropriate to a drainage area of 1,300 sq miles, rather than to one of 1,335 sq miles. The former area is that for which the first derivation on this stream, determined for a dam site above the gage, was prepared. Intent upon presenting an unretouched, original, routine solution, the writer

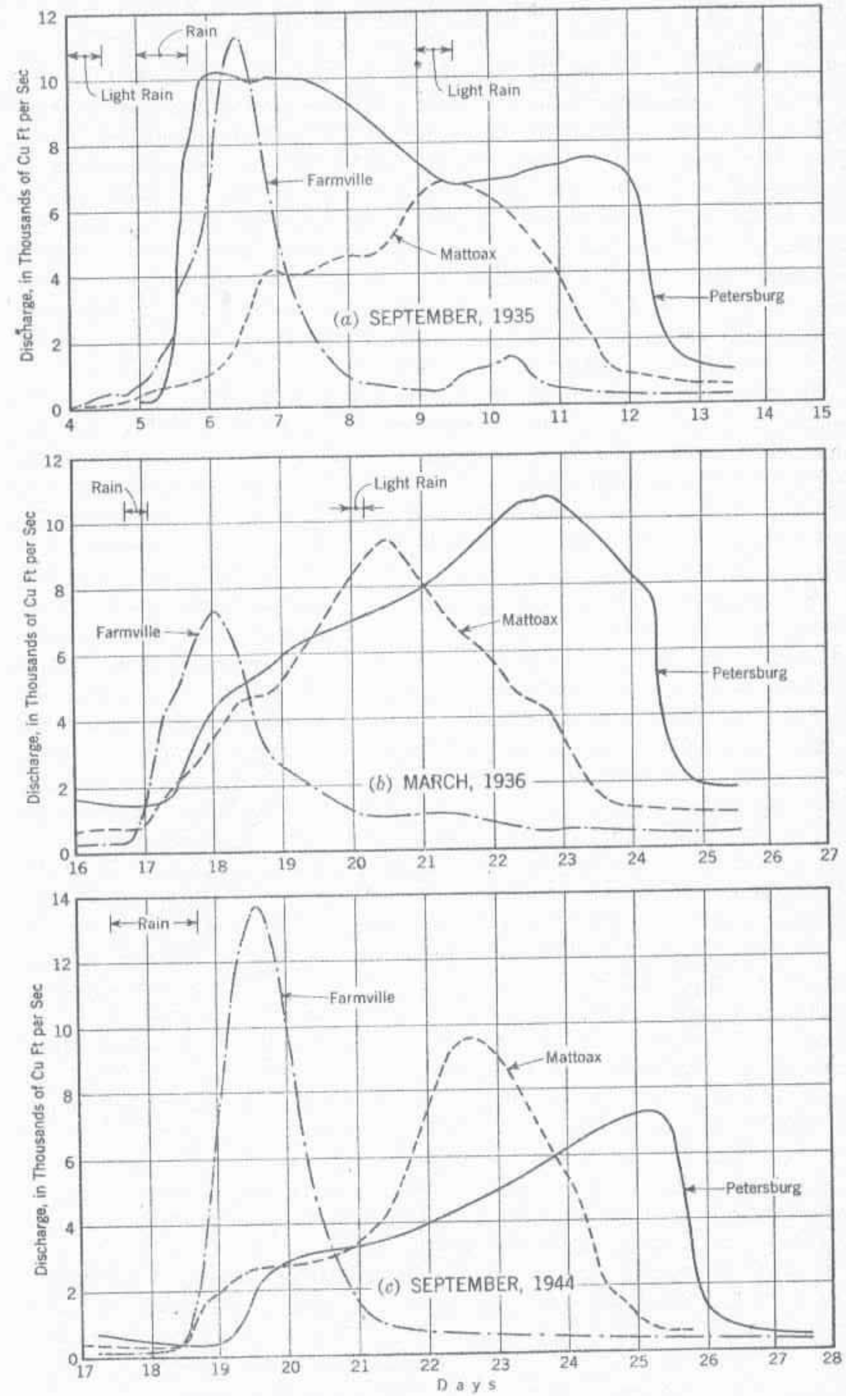


FIG. 18.—HYDROGRAPHS OF THE APPOMATTOX RIVER

failed to notice and properly explain the small difference in drainage area. The routing calculation illustrated is for 15 hours, as stated in Table 1, rather than for 9 hours, as stated in the text. Subsequent investigations indicated the smaller value to be more suitable. There are also some slight errors in Table 1, all well within the range of acceptable routine office practise, and there is much room for improvement in the analysis. The entire original analysis was prepared in 4 hours and was not then intended as a part of a published paper. The presentation of these data was intentional. The real value of a hydrologic tool is much better judged by the rough-hewn original product with its faults, than by the prettier refinements which most investigators possess and ultimately adopt for use.

Mr. Sherman comments on the size of watersheds to which unitgraphs may be applied. He correctly states that size is not a barrier to the adequate treatment of nonuniformity of rainfall with respect to time. On the other hand, nonuniformity with respect to areal distribution of the rainfall creates variations such as those shown in Fig. 18. Each of these hydrographs resulted from a rainfall brief enough to be considered a unit rain. The relative size of hydrographs at the three stations (gaging, respectively, 23%, 55%, and 100% of the Appomattox River above Petersburg) and rainfall records indicate heavier rainfall on one end or the other in each case. Of the hydrographs at Petersburg, none would be suitable for derivation of a unitgraph by any method which assumed that the hydrograph resulted from a uniform distribution of runoff. In the proposed method, which makes no such assumption, an equally suitable unitgraph could be derived from each of the records. However, neither of the hydrographs at Petersburg could be reproduced well by use of a single unit hydrograph, because, although a reproduction would represent flow from uniformly distributed runoff, the record does not. Subdivision of the area and the use of three component unitgraphs like those in Table 2 produce better results. The methodology as extended by Mr. Turner, incorporating both area and runoff depth in the time-area curve, gives much better results.

The possibilities of storage acting to increase outflow drew varied comment from Messrs. Meyer, Williams, Linsley, Brater, Snyder, Turner, and Johnstone, ranging from "controvert[ing] accepted concepts and experience" and "erroneous" to "familiar to all persons who have worked with reservoirs storing appreciable volumes of water under the backwater profile." The writer agrees heartily with those who suggest "These conclusions should be examined critically." The possibility should not be passed over lightly, however, as failure to recognize the possibility leads directly to overoptimism about the effectiveness of certain types of flood control dams, and to underestimation of the importance of avoiding fixed-pool operations at power and navigation dams and unnecessarily rapid gate closures.

The writer's comments about the inflow-controlled storage were intended to be qualitatively explanatory (rather than quantitatively assertive) of the statement (see heading, "Valley Storage"), "Valley storage does not always decrease flood peaks * * *." The excellent discussions reviewing the case against such a viewpoint appear to require some presentation of the data supporting an affirmative case.

The writer's use of a uniform channel in Fig. 1 for the qualitative explanation was ill chosen from a quantitative viewpoint. The uniform channel was selected because of the simple, familiar form of the two backwater profiles. Obviously, however, a uniform rectangular channel would not have an x-value in excess of 0.5. Therefore, Professor Brater's quantitative criticism is well founded. A quantitative demonstration might consider that the rectangular channel possesses a non-water-carrying flood plain several times as wide as the channel at the upper end, converging to the channel limits at the lower end. By appropriate selection of the flood plain width at the upper end of such a channel (which would be quite similar to the channels of many natural streams carrying relatively small flows in their extensive overbank areas), the volume of stored water under the backwater curve could easily be made to exceed 720 acre-ft, which is the volume Professor Brater's analysis indicates as the limiting volume that could be passed without surge.

Channels—the variable cross sections of which effectively converge in the direction of flow—might be expected to possess storage capacities influenced more by inflow than by outflow. Such channels might include:

(a) Channels converging in variable depth (such as low-slope channels approaching a steeper channel and thus flowing from a zone of great range in stage to one of small range in stage and at the same time possessing flood plains of about equal width, or such as reservoirs maintaining fixed levels at the dam with pronounced fluctuation at their upper ends, or such as reaches ending in tidal estuaries); and

(b) Channels converging in width but of relatively constant range in flood rise.

In the first class may be found certain South Atlantic streams which cross the Piedmont Plateau flowing toward the Fall Line. The Fall Line is a cascade zone which lies between the Piedmont Plateau and the Atlantic Coastal Plain. For example, the Appomattox River, whose drainage area above Petersburg was shown in Fig. 6, lies on the Piedmont Plateau. Two gaging stations are located above the Petersburg gage: Mattoax, Va., with a drainage area of 729 sq miles, is on the 2.0-day isochrone of Fig. 6, and Farmville, Va., with a drainage area of 306 sq miles, is on the 4.3-day isochrone of Fig. 6.

The typical flood rise at Mattoax in feet is much larger than that at either Farmville or Petersburg although flood plain widths are comparable for the three stations. The variable cross section of flood flow can be considered diverging in the direction of flow from Farmville to Mattoax and converging from Mattoax to Petersburg. Hydrographs of flow at these three stations resulting from rainfall of short duration are shown in Fig. 18. (Note that the later portions of the hydrographs at Mattoax and Petersburg must be essentially the flow component from above Farmville.) To occupy such a position in the hydrographs shown, it appears that the flood wave might have undergone flattening from Farmville to Mattoax and then have been built up from Mattoax to Petersburg. A more provable indication of the type of storage influence represented by x-values greater than 0.5 is the increase in recessional slope of the Petersburg hydrograph as compared with the Mattoax hydrograph.

In the second class might be short reaches of many rivers, as most flood plains alternately widen and narrow. Many rivers have long reaches through which flood waves flow without appreciable reduction.[40] It seems a little more rational to interpret such unreduced passage as the result of alternate slight increases and decreases in peak rather than as a passage of an unvaried peak. A record of a wave passage having slightly more increase than decrease was shown in Fig. 12, in which the hydrograph of flow at Danville appears to have built up slightly and shortened to become the later portion of the hydrograph at South Boston. The discharge from the intervening drainage area would have been insignificant on the twenty-first and twenty-second days.

Most rationalizations about the possibility of storage operating to build up peak flow turn upon the question: "Granting that a volumetric differential exists between the equilibrium profiles of flow applicable to flow distributions before and after a change in inflow, can the change from one profile to another take place rapidly enough to cause a surge at the downstream end?" Fig. 19

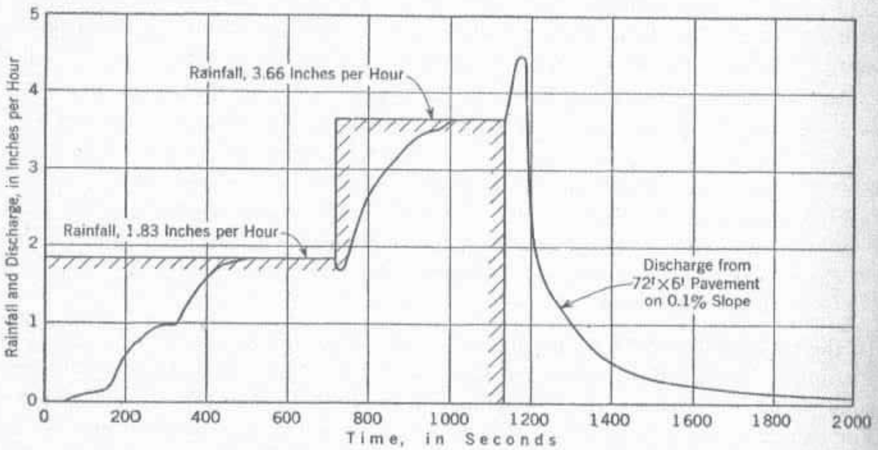


FIG. 19.—HYDROGRAPHS FROM EXPERIMENTS BY C. F. IZZARD AND M. T. AUGUSTINE

shows the experimental hydrographs which C. F. Izzard, Assoc. M. Am. Soc. C. E., and M. T. Augustine obtained by sprinkling a pavement.[41] In this case, the change from one flow distribution profile to another did take place rapidly enough to cause a surge. How fast does the change take place in rivers? How fast does storage volume react to a change in flow?

Several discussers mention the assumed instantaneous reaction of storage to a change in flow (which is characteristic of most flood-routing techniques and favorable to reduction of peak flow in the method discussed when x is less than 0.5) as the error which leads to the author's conclusion. The formula for reaction time presented by Messrs. Williams and Brater is quite favorably regarded by many, whereas Mr. Snyder appears to regard any value less than K as inadequate. Several intermediate viewpoints have some adherents. The time elements indicated by Eq. 17, first discussed by Mr. Williams, vary

[40] "The Floods of March 1936," *Water-Supply Paper No. 800*, U. S. Geological Survey, pp. 89 and 90 (last two hydrographs on each page).

[41] "Preliminary Report on Analysis of Runoff Resulting from Simulated Rainfall on a Paved Plot," by C. F. Izzard and M. T. Augustine, *Transactions*, Am. Geophysical Union, Vol. 24, 1943, Pt. I, p. 500.

from very long values for a shallow depth at the beginning of a flood to crest-depth values which are very short in relation to the time elements of mean flood-wave travel. Thus, in the reach of river to which Mr. Williams applied the formula, the channel depth was more than 50 ft at the time of crevassing. For this depth, the formula discussed would indicate less than 40 min. (Neither the methods of observing stage, nor those of defining the time of crevassing of a levee are sufficiently reliable to distinguish between the time of transit as originally stated and that given by the formula.) This value would be insignificant in flood routing over a reach involving flood-wave travel time of 15 hours.

The writer does not claim that the upgrading of flood crests by storage action has been proved, but merely offers the explanation and the examples of the paper in support of the statement, "There is some valley storage which does not decrease flood peaks." The upgrading of hydrographs does appear possible, however, and should be investigated. A closely analogous situation exists in electrical circuits, and the upgrading (in height, not discharge) of hydraulic waves in converging channels is a recognized phenomenon in tidal hydraulics.

Mr. Linsley clarifies several ideas in the paper by his discussion. The formula he presents for determining time of concentration in watersheds without records shows better correlation with data in the paper than the author's formula. To aid in further study by others, additional data are presented in the same manner in Table 6. The majority of values were developed from records available prior to 1938. The variation in the constants suggests judicious local application. Attention is again directed to the first two sentences under the heading, "Determination of the Instantaneous Hydrograph for Streams Without Flow Records," and to Mr. Snyder's closing two paragraphs.

Messrs. Linsley and Johnstone properly question the writer's use of uniform velocity in establishing time contours along the watercourses and suggest variations based on slope and efficiency of watercourses. These refinements were considered, but investigation showed their use to be unaccompanied by superior results. This may be a case of two wrongs making a right, as the likewise neglected principle of using greater storage values, K, on flows from more remote areas instead of a single average value is a logical refinement of opposite effect. In general, the use of a varied velocity of channel flow and a single value of K tends to increase the fault originally recognized by the writer in item 6(*b*) of the "Summary."

Messrs. Brater and Johnstone voice an interest in a more detailed description of the time-area concentration curve. As Mr. Snyder states, the curve is an old device[42,43] for calculating flow. It has fallen into disuse because its theory assumes the reductive influence of storage to be negligible. A storage correction factor was included in the paper to revitalize this useful tool. The time-area concentration curve is derived by marking time contours along the

[42] "Sewerage and Sewage Disposal," by L. Metcalf and H. P. Eddy, McGraw-Hill Book Co., Inc. New York, N. Y., 1922, p. 94.

[43] "Calculation of Flood Discharge by the Use of a Time-Contour Plan," by Cecil N. Ross, *Transactions*, Institution of Engrs. (Australia), Vol. II, 1921, p. 85.

watercourses, the time to the most remote point being equal to the "time of concentration." For the latter term, a standard definition such as that quoted by Professor Johnstone is intended. Fig. 4 illustrates why the terminus of this time is not at the end of the hydrograph (theoretically infinite in length) but well up on the recessional leg.

TABLE 6.—ADDITIONAL COMPARISON BETWEEN EQS. 18a AND 18b

Stream	s (ft per mile)	L (miles)	K (hours)[a]	A (sq miles)	C_1 (Eq. 18a)	C_2 (Eq. 18b)
North Branch, Potomac River, at Bloomington, Md.	45	49	10	287	1.4	0.081
Savage River at Bloomington, Md.	65	26	11	115	3.4	0.32
Georges Creek at Franklin, Md.	78	18	5	72	2.4	0.29
Wills Creek near Cumberland, Md.	40	35	8	247	1.4	0.092
Patterson Creek near Headsville, W. Va.	19	40	12	216	1.3	0.089
South Branch, Potomac River, near Petersburg, W. Va.	24	60	15	642	1.2	0.048
South Fork, South Branch, Potomac River, near Moorefield, W. Va.	25	54	13	283	1.2	0.071
Cacapon River at Yellow Spring, W. Va.	16	51	10	306	0.78	0.044
Back Creek near Jones Springs, W. Va.	11	40	10	243	0.81	0.052
Conococheague Creek at Fairview, Md.	9.8	62	30	494	1.5	0.068
North River near Burketown, Va.	37	18	9	375	3.0	0.16
Middle River near Grottoes, Va.	14	54	12	360	0.83	0.044
South River at Waynesboro, Va.	17	24	12	144	2.1	0.17
North Fork, Shenandoah River, near Strasburg, Va.	12	96	20	772	0.72	0.026
Passage Creek at Buckton, Va.	31	32	6	87	1.0	0.11
Cedar Creek near Winchester, Va.	30	26	6	101	1.3	0.12
Antietam Creek near Sharpsburg, Md.	11	49	24	281	1.6	0.097
Monocacy River near Frederick, Md.	5.9	62	28	817	1.1	0.038
Goose Creek near Leesburg, Va.	11	33	17	338	1.7	0.093
Seneca Creek at Dawsonville, Md.	17	17	6	101	1.5	0.15
Jackson River at Falling Spring, Va.	22	47	12	409	1.2	0.059
James River at Lick Run, Va.	17	78	28	1,369	1.5	0.040
Cowpasture River near Clifton Forge, Va.	13	69	24	456	1.3	0.059
Calfpasture River at Goshen, Va.	23	36	12	147	1.6	0.13
North River at Rockbridge Baths, Va.	27	45	18	329	2.1	0.12
North River near Lexington, Va.	21	58	20	487	1.6	0.071
North River near Buena Vista, Va.	16	70	24	649	1.3	0.053
Hardware River near Scottsville, Va.	17	22	18	116	3.4	0.32
Slate River near Arvonia, Va.	9.0	34	30	235	2.6	0.17
Rivanna River at Palmyra, Va.	8.0	50	82	675	1.8	0.070
Willis River at Flanagan Mills, Va.	4.2	55	72	247	2.7	0.17
Appomattox River at Farmville, Va.	8.7	34	32	306	2.8	0.16
Appomattox River at Mattoax, Va.	3.0	87	114	729	2.3	0.084
Appomattox River near Petersburg, Va.	2.9	122	168	1,335	2.3	0.064
Meherrin River near Lawrenceville, Va.	3.2	67	60	553	1.6	0.068
Meherrin River at Emporia, Va.	2.4	86	78	750	1.4	0.051
Roanoke River at Roanoke, Va.	17	48	12	388	1.0	0.052
Blackwater River near Union Hall, Va.	14	46	18	208	1.4	0.10
Pig River near Toshes, Va.	8.7	72	24	394	0.98	0.049
Snow Creek at Sago, Va.	18	18	14	60	3.2	0.41
Goose Creek near Huddleston, Va.	14	33	12	187	1.4	0.099
Otter River near Evington, Va.	18	34	22	325	2.7	0.15
Falling River near Brookneal, Va.	11	23	22	228	3.2	0.22
Dan River near Francisco, N. C.	35	43	4	124	0.55	0.049
Mayo River near Price, N. C.	20	38	14	260	1.6	0.10
Smith River at Bassetts, Va.	17	37	7	265	0.77	0.047
Smith River at Martinsville, Va.	13	48	13	374	0.98	0.051
Smith River at Spray, N. C.	9.4	71	23	538	0.99	0.043
Sandy River near Danville, Va.	21	21	5	113	1.1	0.10
Banister River at Halifax, Va.	7.2	46	60	552	3.5	0.15

[a] Time of concentration, not reductive storage factor.

Mr. Crawford presents interesting analyses of channel-storage correlation with stages at the discharge point and upstream from it, and shows the superiority of correlations which give weight to upstream values of inflow and stage. He calls attention to the potential possibilities of determining from discharge records other storage relations which would be too costly by cross-sectional surveys. The writer has seen area-capacity curves for small dam sites so developed which would require very detailed field surveys for equal accuracy.

Mr. Kennedy emphasizes the fallacy of constant base length in a unit hydrograph. Much of this fallacy may originate in the assumption of a finite base length. If storage theories and unit hydrograph theories are to be correlated, the concept of finite base length in a unit hydrograph must give way to a concept of an infinite length, a characteristic possessed by the writer's instantaneous hydrographs and their derivatives. Nevertheless, the practical differences, as shown by Fig. 15, are within the range of usually acceptable hydrograph computations. This is probably the reason that the practical simplicity of finite constant base length has so long outweighed the theoretical superiority of infinite base length.

Mr. Cochrane has very thoroughly examined the procedure for deriving unitgraphs utilizing storage concepts, and presents some explanations which should be helpful to those who were mystified by the writer's presentation. In a simple manner, he clarifies the twofold effect of storage: First, the greater effect of storage in creating time lags between flows from different zones, or the time-area diagram; and, second, the further modification of hydrograph shape accomplished by storage.

The order in which these steps have been utilized is the reverse of that in Fig. 8, but the result is the same regardless of order. The order in Fig. 16 gives a clear picture of the mechanical effects, and would be necessary if one wished to use variable values of K for different zones. At the sacrifice of this possibility, the order of Fig. 8 involves less work. However, a unit hydrograph for an entire area may produce very disappointing results if applied to a storm characterized by great nonuniformity of runoff distribution. Mr. Cochrane presents a very practical application of storage principles which considers both nonuniformity of runoff distribution and some variation in storage effect.

Another method of accounting for nonuniformity, suggested by Mr. Turner, is to extend the time-area concentration curve to express not only shape but also different runoff distribution—that is, multiply the zone areas by the depth of runoff in each zone before embarking on the routing procedure of Table 1. This procedure takes care of distribution with respect to area and is a fine tool for studying the magnitude of the effect of areal nonuniformity of runoff distribution.

Both procedures abandon the unit hydrograph and require complete recalculation of each flood with attendant increased labor. Other less logical, but extremely useful techniques to account rapidly for nonuniformity include adjustments of chronological sequences of runoff used with the unit hydrograph. Thus, concentration of volume on more remote parts of a simple watershed can be simulated in some degree by using an effective runoff volume for a later time than actually was the case. Extreme intensity of rainfall within a unit period can be simulated by increasing the computational amount in that period and decreasing the values on either side of the period. The most flexible technique yet utilized by the writer is the development of unit hydrographs for very small zones, which are routed downstream to gaging stations and tabulated in the manner of Table 2. This procedure is followed for as small areas as desired to express nonuniformity to the necessary degree. These subunitgraphs can always be added together for use on areas as large as justified by the uniformity of any storm and can be used separately if necessary. Although this procedure was discussed under the heading, "Unit Hydrographs for Large Drainage Areas," any watershed is large from the viewpoint of detailed, exacting hydrograph computation.

Refined techniques are necessary and desirable to establish more suitably correct procedures and to confirm academic viewpoints as to the effect of certain qualitative variables. However, a computation is only a guide to judgment. After a hydrograph is computed with a unitgraph or other appropriate tool, judgment demands some modification of the answer if the premises of the methods and the facts of the occurrence are not in accord. At times a wide line, a small scale, or a little freehand "artistic license" may have as much merit as hours of computing. In their proper place, these too are proper tools.

Mr. Snyder presents several of the practical aspects of hydrograph calculation and the mathematical basis for a very simple and useful method of routing over long reaches of river where storage-discharge relationships are such as to produce essentially constant time elements. He calls attention to elements of logical and mathematical fallacy which may appear small to some investigators, but which must be faced squarely and solved before the hydrograph computations can be considered accurate enough to prove or disprove some of the questions raised in this paper.

As Mr. Snyder states, the Muskingum method of flood routing loses its usefulness when applied to too long reaches. However, this may also occur because the storage-discharge relationship for a long reach of river is much less closely approximated by a straight line than is the relationship for short reaches. The storage item in flood-routing formulas such as Eq. 5 deals with the volume stored in a reach at any instant of time, whereas the storage capacity of channels having nearly constant elements of flow time ($n = 1.0$ in Eq. 3) refers more closely to the volume under the high-water profile. Although this latter volume is the total storage volume utilized in the reach of river during flood passage, it is more than is used at any instant during that time. The difference becomes a material one in long reaches. Nevertheless, if storage solutions are confined to the length of reach for which Eq. 5 would be applicable, Eq. 6*b* is a correct solution. The Muskingum method is a correct solution of Eq. 6*b* only to the extent that a finite value of T is a satisfactory approximation of dt. Mr. Snyder expresses the opinion that values of T used in the preparation of Figs. 2 and 3 were too short.

The inadequacies of methods of mathematical solution receive too little recognition. Most engineers find it necessary to adopt arithmetic approximations, such as Eqs. 1 and 6*c*, for the solution of calculus equations, such as Eqs. 2*a* and 6*b*.[44] The practises are so common that they escape much comment. The simplicity of the Muskingum method encourages these procedures, since it depends first on the substitution of a finite value of time T for the infinitesimal time dt. Then, too, many convenient, quick solutions evolve from various lengths of time T which make one or more of the constants either 1.0 or 0. Values of T as large as K or larger conveniently obscure some of the

[44] "Graphical Integration of the Flood-Wave Equations," by Harold A. Thomas, *Transactions*, Am. Geophysical Union, Vol. 21, 1940, Pt. I, p. 597.

effects of the inadequacy of the theory recognized by the writer in his discussion of Fig. 2, and have been accepted by some as a basic requirement. The differences are as given by Mr. Snyder. In an effort to prove whether such a basic requirement added anything to the over-all accuracy of hydrograph solutions, the writer has for several years assumed that the shortest practical time element, T, was the most appropriate. In general, practical applications have been no more faulty, and in some cases apparently better, whereas the practical benefits of greater mathematical consistency and rapid checking procedures accrue to (or result from) closer adherence to the mathematical premises.

Mr. Snyder's presentation of the mathematics of a simple method of lag routing should increase the practical appeal and use of this simple procedure. To be completely general, the solution may need some provision (1) to cover the flow conditions in which the lag of the crest discharge is greater than the lag between the centers of mass of the inflow and outflow, and (2) to cover the conditions in which the recession rate of outflow exceeds the recession rate of inflow. Flow conditions requiring this modification are evident in Fig. 18, in which the hydrographs of the Appomattox River at Mattoax are shown to have a materially smaller rate of recession than does that portion of the hydrograph at Petersburg which encompasses the flow volume from above Mattoax.

Inspection of Fig. 17 and Eq. 25 suggests the applicability of a negative value of T_s. The writer finds the mathematics of a solution evasive, however, and doubts that this is a practical solution. Nevertheless, it is explanatory of a relationship between K and time of concentration (T_c), which still baffles him. However, it is assumed for the purpose of the next paragraph that negative values of T_s are applicable.

The failure of the writer to solve this possible adaptation of lag routing is the source of his confusion in statements under the heading, "The Hydrograph as an Index of Storage," about which Mr. Snyder rightfully comments. The values of K applicable to smaller watersheds appear to be as large as 0.5 of the time of concentration (T_c); yet those for larger watersheds have a much smaller ratio of T_c and occasionally a smaller dimensional quantity. Obviously this is possible if T_s could have a negative value. For the three gages on the Appomattox River—Farmville, Mattoax, and Petersburg, each site being downstream from the preceding site—the values of K which the writer regards as most appropriate are, respectively, 9 hours, 12 hours, and 9 hours, whereas the most acceptable times of concentration, T_c, are 32 hours, 114 hours, and 168 hours. (These values for Petersburg differ from values in Table 1, which were the very first computations made for that stream. The difference between the values of T_c equal to 144 hours and 168 hours is dimensionally large, but the same percentage difference in the more usual range of the writer's application, 24 hours or less, would represent but 4 hours, which is close to the limiting error fixed by rainfall data. Furthermore, that percentage difference is quite comparable with the expected difference due to seasonal variation in flow conditions, a factor usually ignored, but by no means absent.)

The Appomattox River presents the most perplexing flow conditions to which the writer has yet applied the unit hydrograph theory and the longest time of concentration for comparable drainage area that he has ever encountered, as well as a unique shape. Although the illustrated hydrographs for the Smith River at Bassetts and the James River at Lick Run are certainly the more common type, exceptions, like the illustrated hydrographs of Appomattox River and Meherrin River, are necessary to prove the rule. Judging from records of rounded and tree-shaped watersheds typical of the glaciated and alluvial streams of the Midwest, it would appear illogical that a stream could rise for a longer period than it fell, or that streams could have unit hydrographs like those presented for the Appomattox and Meherrin rivers. Mr. Sherman's doubt that any unit hydrographs were presented in the paper is therefore very understandable.

Mr. Turner provides a well-balanced discussion of the paper, confirming statements and expressing appropriate reserve and doubt where his experience and understanding of conventional concepts and procedures do not justify presented conclusions. His comparison of results obtained from the outlined unitgraph procedure and from his similar procedure[31] encourages confidence in the principles applied. His extension of the method to include varying rates of rainfall on different parts of the area is quite practical.

Professor Johnstone outlines some logical elements of greater and lesser refinement whose merit would depend on the extent of the use for them. He expresses several views in sharp conflict with those of the paper. The propriety of defining a unitgraph to include nonsurface runoff is basicly criticized Granting that some investigators have presented hydrographs attributed to surface runoff, the writer questions whether such a claim can be substantiated or disproved, since there is no feasible method of distinction, no acceptable specific definition of either kind of runoff, or any impervious boundary between the flow channels utilized by each. Practise in the application of the terms seems to justify distinctions satisfactory to the user and determinable only to the extent of their compatibility with a preferred theory. The writer indicated that separate unitgraphs could be used for surface and subsurface flow, if warranted, and that there was nothing fixed about the 70:30 ratio. However, in computations of subsurface flow there seems little reason to reject that definable quality which the use of the unit hydrograph theory has provided in surface flow analysis.

In his demonstration that there "* * * should * * * be no possibility of multiple solutions" of conventional unit hydrograph derivations from rainfall and stream flow data only, Professor Johnstone adequately presents the reasons why an investigator who uses all the data available can secure: (*a*) A distressingly large number of different solutions if he assumes he is able to determine runoff from rainfall; or (*b*) no solution if he admits inability to determine runoff from rainfall. In the storage concepts presented in the paper, there are additional considerations which eliminate the necessity for dependence on rainfall and runoff determinations. Until runoff determinations become more reliable, however, the real accuracy of the method must remain partly shrouded in unproved hope.

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TRANSACTIONS

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ANALYSIS OF RIGID FRAMES BY SUPERPOSITION

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WITH DISCUSSION BY MESSRS. F. S. MERRITT, RALPH W. STEWART, JOHN E. GOLDBERG, A. W. FISCHER, LEON BLOG, ARTHUR B. MCGEE, JAROSLAV POLIVKA, ODD ALBERT, W. C. SPIKER, DAVID B. HALL, A. A. EREMIN, VICTOR R. BERGMAN, AND DAVID M. WILSON.

SYNOPSIS

The purpose of this paper is to demonstrate the usefulness of the principle of superposition in the analysis of rigid frames by the slope-deflection and moment distribution methods. A number of examples are presented to show how the principle may be used to advantage in attacking a variety of problems. The principle of superposition is usually stated in textbooks on structural analysis; but, in the writer's opinion, its application is not emphasized to the extent that its importance warrants. In the study of continuous structures, in particular, a thorough working knowledge of the principle is indispensable.

It will be noted that no new method of analysis is proposed. Instead, a method of procedure is outlined which coordinates the principle of superposition with existing methods and, at the same time, places emphasis where it properly belongs.

PRELIMINARY CONSIDERATIONS

The scope of the paper is limited to the analysis of frames in which applied loads cause linear displacements of the joints, in addition to angular displacements (rotations), and consequent angular displacements of the members. It is assumed that each member of any frame considered has constant section throughout its length. However, the method of procedure which is presented is general, since it is based upon fundamental principles, and can be applied with little additional labor to the analysis of frames having members of varying section after fixed-end moments, carry-over factors, and stiffness values have been evaluated.

Notation.—The following letter symbols, used in this paper, conform essentially to American Standard Letter Symbols for Mechanics, Structural Engineering and Testing Materials,[2] prepared by a Committee of the American

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