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SYNTHESIS OF THE INLET HYDROGRAPH

by

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ABSTRACT

A method is presented for synthesizing the hydrograph of runoff from paved portions of a drainage area. The method involves an application of the equations of gradually varied unsteady flow in open channels to describe the mechanics of surface runoff. The pattern of rainfall must be specified, and the geometry of the area must be given. Also, values of friction coefficients in the Manning and Darcy-Weisbach equations must be selected, but the computed hydrograph is not sensitive to rather large variations of these coefficients. Data from overland flow experiments reported in the literature and measurements of rainfall and runoff collected by the Storm Drainage Research Project have been used to verify the synthesis procedure. Excellent agreement between the measured and computed hydrographs has been obtained.

The method of synthesizing the inlet hydrograph will provide a means for developing from rainfall data, records of runoff for inlet areas of sufficient length to have statistical significance. The method could be quite useful for studying the validity of assumptions implicit in the Rational Method or other existing design procedures. It has been used in this study to develop synthetic unit hydrographs for the gaged drainage areas. For difficult drainage problems, as where

water is to be pumped from a depressed highway underpass, this method should be very useful to the design engineer. A particular advantage of the method is that drainage area behavior can be simulated on a computer so the response of an area to arbitrary, spacially-varied rainfall can accurately be predicted.

ACKNOWLEDGEMENTS

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SYNTHESIS OF THE RUNOFF HYDROGRAPH

More than seventy-five years ago, Kuichling (1889) described the central problem of storm drainage design by stating: "The most important question which arises in the construction of a sewerage system whose function is also the removal of the surface drainage, is with regard to the amount of storm water that will find its way into the sewers..." The relationship between rainfall and runoff which he then described is the foundation of present storm drainage design practice in the United States, and his procedure for computing storm water runoff rates has come to be known as the "Rational Method" (Fair and Geyer, 1959).

During the past seventy-five years billions of dollars have been spent in the United States for storm sewerage facilities which were designed by using the Rational Method. In the next 30 to 40 years, the population of the United States may be expected to double, and the population of urban areas is increasing at a faster rate than the nation as a whole. During this period another United States must be superimposed on the United States. Estimates of expenditures before the year 2000 for storm drainage facilities to serve the growing population exceed 25 billion dollars. This does not include future highway drainage costs or the cost of enlarging or replacing existing facilities or of improving natural stream channels. Clearly, the designs for new facilities should be based on sound, rational principles. If improved procedures could save 5 per cent of the future estimated costs, this would amount to savings of more than $5 million annually. The present value of these future savings on new residential construction alone, until the year 2000, is more than $100 million. And the benefits of having future drainage capacity more adequately matched to their need would be worth many times the savings of construction costs.

When using the Rational Method, an estimate is made of the greatest rate of storm water runoff that will be equaled or exceeded on the average of once in T years. The value usually used for T is between 2 and 50 years. As larger values of T are selected, larger values of the design runoff rate are obtained. For T = 5 years, a recent study (Schaake, 1964) suggested that one out of every three runoff estimates by the Rational Method is presently in error by more than 20 per cent. If this error could be reduced to only 10 per cent, there would be significant savings.

Since 1949, the Hopkins Storm Drainage Research Project has been studying various aspects of the urban drainage problem.

Both the hydrologic and economic aspects of the problem are now being investigated. Economic studies (Knapp, 1965) have shown that economic decisions depend quite heavily on adequate rainfall-runoff relationships. The primary objective of the Storm Drainage Research Project is to develop a method for estimating the hydrograph of runoff at the storm water inlet from knowledge of rainfall and the physical features of the drainage area. By routing inlet hydrographs through the drainage system, the downstream runoff hydrograph can be estimated.

A method is presented here for estimating the hydrograph of runoff from paved portions of the drainage area. If this method could be extended to include runoff contributed from pervious areas, there could result a much improved method of urban drainage design.

The method to be presented involves an application of the equations of gradually varied unsteady flow in open channels to describe the mechanics of storm runoff. Overland flow of surface runoff and the flow of runoff in gutters and storm sewers are accounted for. If the pattern of rainfall as it occurred in time and space is known, the runoff hydrograph may be computed. Only paved areas have been studied thus far. To extend the method to unpaved areas, there must be proper accounting for the water that infiltrates into the soil or is retained on the irregular soil surface.

The equations for gradually varied unsteady flow in open channels have been used in recent years for flood prediction in rivers. Stoker (1957) has used these equations to study floods in the Ohio River and to study the behavior of a flood at the junction of the Ohio and Mississippi Rivers. More recently, Yevdjevich (1961) has discussed the use of these equations to study the flow of storm runoff in a storm drain.

The method presented is based on sound physical principles and probably will be of greatest value in synthesizing runoff records for use in statistical studies. Most practical problems would not justify direct use of this rather complex method for the design of storm drains. However, the method may be quite useful for developing unit hydrographs or for studying the validity of assumptions made when using the Rational Method or other design procedure. A particular advantage of the method is that drainage area behavior can be simulated on the computer so that the response of an area to any arbitrary rainfall input can be predicted. Also the sensitivity of the runoff hydrograph to various factors can easily be studied. This could be extremely helpful when attempting to describe drainage area behavior. This method also could be useful in formulating plans for collecting urban hydrologic data for specific studies.

Chapter 2

THE RUNOFF PROCESS IN URBAN AREAS

The runoff process begins when water falls as rain and ends when this water leaves the area as surface runoff, evaporation, transpiration, or underground flow of groundwater (Linsley, Kohler and Paulhaus, 1958; Horton, 1945). A simplified description of the various phases of the runoff process is illustrated in Figure 2-1. Urban drainage areas typically are quite different from natural drainage areas since much of an urban areas is impervious and drainage is provided by paved gutters and sewers as well as by natural stream channels. However, the basic runoff process is the same for both natural and urban areas.

The Phases of the Runoff Process

When rain falls on an urban area, some of the water lands on impervious surfaces and some on pervious surfaces that may or may not have some form of vegetal cover. Some or all of the rain reaching pervious surfaces may infiltrate into the soil profile. As rain continues, colloidal materials swell, reducing the rate of infiltration. Also, open spaces in the soil become filled and the rate of infiltration is

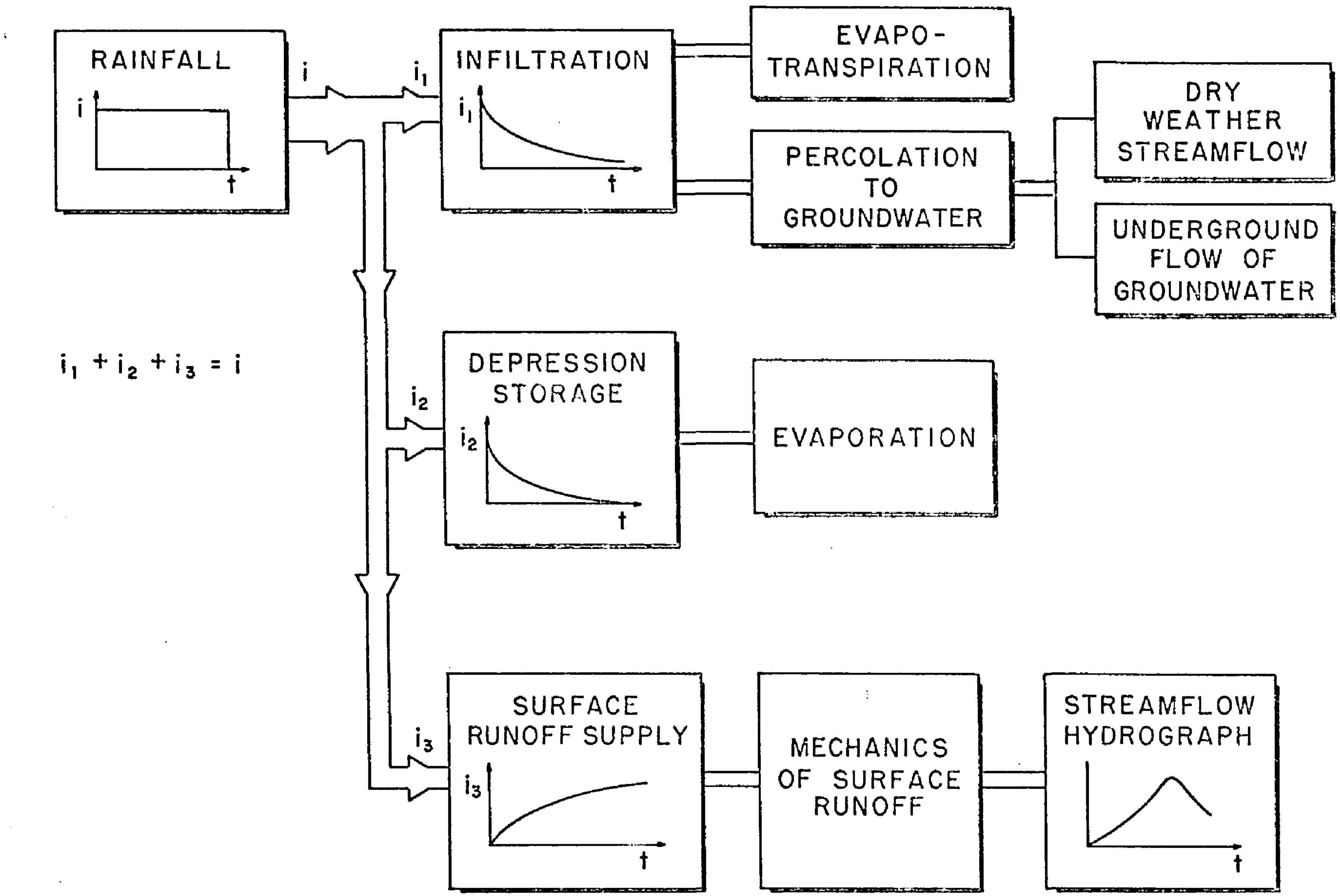


Figure 2-1 The Runoff Process

drastically reduced from an initial rate to the rate at which water can percolate downward by gravity and capillary forces.

After the rate of rainfall exceeds the infiltration capacity, flow of water over the surface can occur. As this flow begins, some of the water is trapped in surface depressions. These depressions may become filled and then overflow, but the water contained in them does not become surface runoff. This water either evaporates or infiltrates into the soil after rainfall ceases. Since there are many sizes and shapes of depressions throughout an area, it is possible only to consider their gross effect on the downstream runoff hydrograph.

Water that is not infiltrated or evaporated and is not stored in surface depressions eventually appears downstream as runoff. Runoff begins with overland flow, then there is flow in swales and gutters, and finally there is flow into a storm water inlet and into the sewer system.

The complete runoff process can be summarized briefly as shown in Figure 2-1. The rates which rainfall enters the various phases of the runoff process are indicated by: i_1, infiltration; i_2, depression storage; i_3, surface runoff supply rate. The greatest problem in synthesizing the runoff hydrograph in urban areas is one of accounting properly for the distribution of the water among these phases of the runoff process. For paved areas, the rate at which water enters depression storage can be neglected for practical purposes without introducing a substantial error in the estimated runoff hydrograph. During storms of importance to design engineers, the volume of depression storage on paved areas is a small part of the total rainfall volume. On the other hand, the loss of water by infiltration and depression storage in pervious areas is significant and the accuracy of runoff estimates will never be better than the accuracy with which these losses can be estimated.

The most accurate method in general use today for estimating runoff from rainfall involves: first, accounting for the amounts of water entering the various phases of the runoff process; and then, using a unit hydrograph to approximate the mechanics of surface runoff to obtain the downstream runoff hydrograph.

If records of rainfall and runoff for an area are available, a unit hydrograph may be derived from these records. But for areas where records of rainfall and runoff are not available, as is always the case where storm drains are to be provided, a synthetic unit hydrograph must be obtained from knowledge of the physical features of the area. There is no generally accepted procedure available for accurately obtaining a synthetic unit hydrograph for an ungaged area.

This study demonstrates how the mechanics of surface runoff can accurately be described by the equations for gradually varied unsteady flow in open channels. Using these equations, the runoff hydrographs from overland flow experiments by Izzard (1942-43) and runoff hydrographs observed for three paved areas gaged by the Storm Drainage Research Project have accurately been synthesized. For paved areas, such as parking lots, streets, highways, and airfield runways and parking aprons, infiltration is negligible and depression storage is small compared with the total rainfall during intense storms. For these areas, the equations of gradually varied unsteady flow in open channels describe, almost exactly, the complete runoff process.

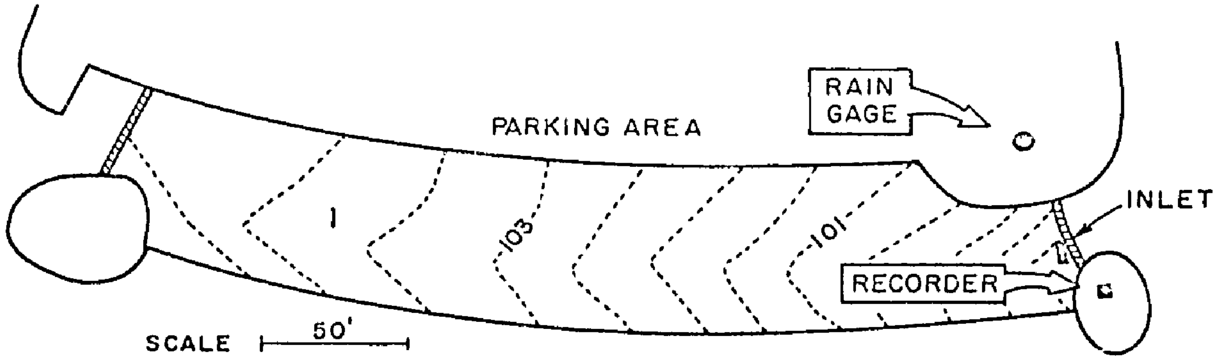
Hopefully, these equations can also be used to describe the mechanics of surface runoff for pervious parts of a drainage area. It would be necessary to account for the distribution of water in the various phases of the runoff process, but the accuracy of the result would be limited only by the accuracy of the accounting procedure.

A Representation of the Surface Runoff Phase

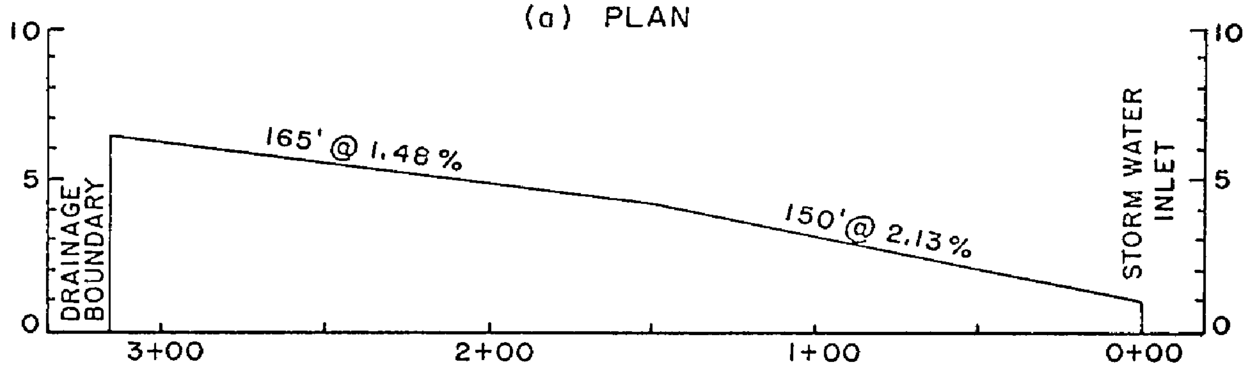
To describe the flow that occurs in the various parts of a drainage area during a storm it is necessary to divide the whole area into a number of component parts. The equations for gradually varied unsteady flow in open channels can then be used to describe the flow in each of the component parts of the larger areas.

Typically, a parking lot may drain to a swale, the swale draining to a storm water inlet. Such an area is illustrated in Figure 2-2a where a plan of the parking area SPL1 gaged by the Storm Drainage Research Project is shown. There is a swale running longitudinally through this area; a profile of the swale is shown in Figure 2-2b. How this area was partitioned into smaller components is shown in a schematic representation of the area, Figure 2-2c. For convenience, each of the component parts of the area are numbered and their physical characteristics are given in the table in Figure 2-2d. The outflow from component 6 flows into a storm water inlet where the runoff is measured.

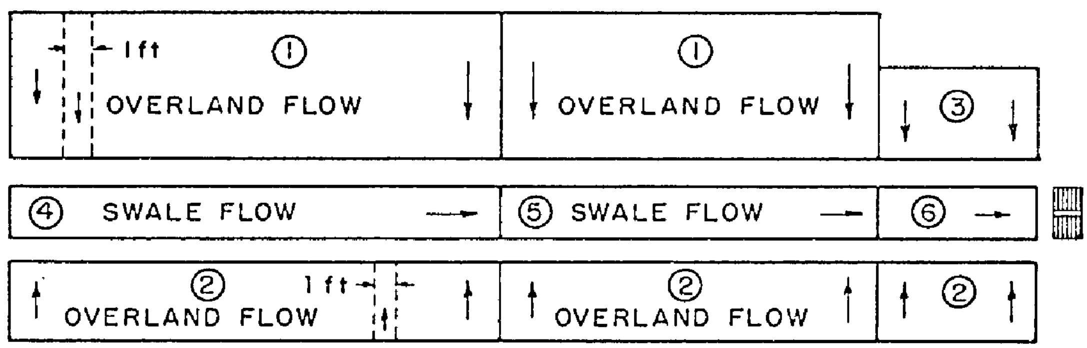
Beginning at the upstream edge of the area, components 1 and 2 refer to the parts of the area contributing overland runoff to component 4, the upstream part of the swale. Since the overland flow to the swale is measured in cfs per foot of swale length, only the outflow from a one foot wide strip of overland flow need be computed. The rectangular overland flow component is therefore composed of a sequence of one foot wide strips. The sum of the outflows from one foot wide strips from components 1 and 2 then represents uniform lateral inflow along the length of component 4. Outflow from component 4 is upstream inflow to component 5 which also receives uniform lateral inflow from components 1 and 2. Component 4 was terminated where there was a change in the slope of the swale as



(a) PLAN



(b) PROFILE OF SWALE



(c) SCHEMATIC REPRESENTATION OF DRAINAGE PLAN

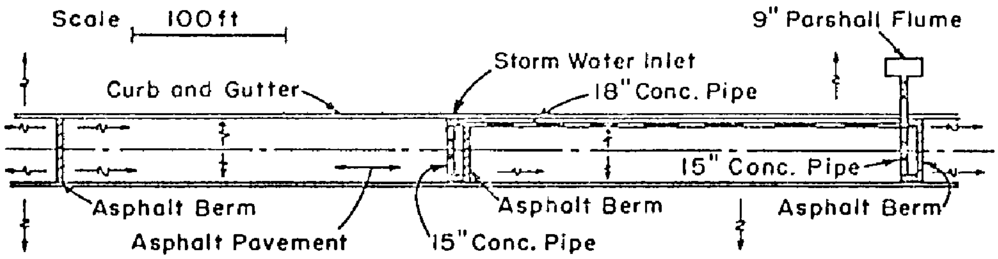
Component	Length (ft)	Width or Dia (ft)	Slope (ft/ft)	Side Slope	Inflow to Component	
					Lateral Inflow	Upstream Inflow
①	36	1	.019	—	Rainfall	—
②	20	1	.0167	—	Rainfall	—
③	25	1	.019	—	Rainfall	—
④	165	—	.0148	113:1	① and ②	—
⑤	100	—	.0213	113:1	① and ②	④
⑥	50	—	.0213	113:1	② and ③	⑤

(d) PHYSICAL CHARACTERISTICS OF COMPONENTS IN THE SCHEMATIC REPRESENTATION

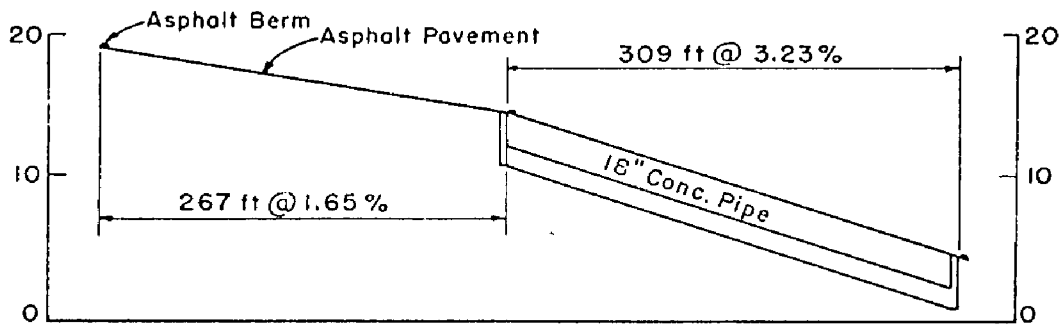
Figure 2-2 Physical Characteristics of Inlet Area SPL1 can be seen in the profile, Figure 2-2b. Outflow from component 5 is then upstream inflow to component 6 which also receives uniform lateral inflow from components 2 and 3. Component 5 was terminated at the point where the length of overland flow on one side is reduced from 36 feet to 25 feet.

The schematic representation of the area SPL1 illustrates how a small, paved inlet area can be divided into smaller components that together represent the essential features of the whole area. Each of the other paved areas gaged by the project also have been represented schematically as shown in Figure 2-3 and Figure 2-4.

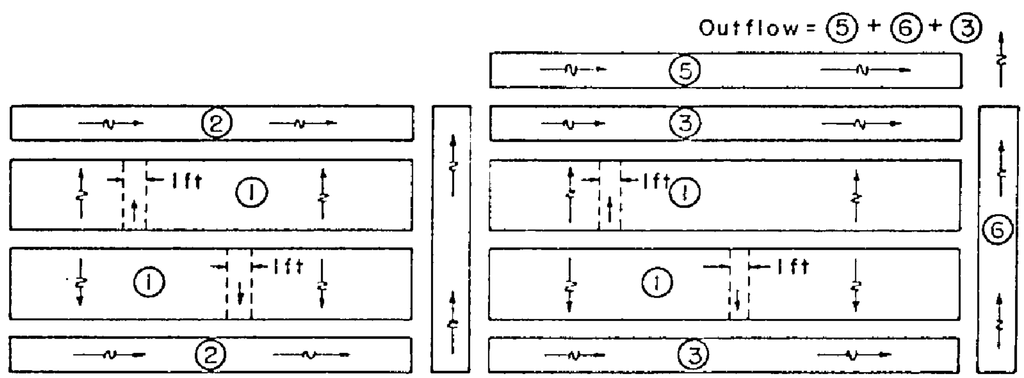
In describing schematically the overland flow contribution to swales and gutters in a drainage area, the direction of this flow has been assumed perpendicular to the gutter. Because the gutter has a longitudinal slope, the actual direction of overland flow is determined by the gutter slope as well as the cross-slope of the surface so that the actual direction of flow is not exactly the same as the assumed direction. However, it can be shown that the computed time of flow along the assumed direction is exactly the same as the time of flow along the actual direction if the flow is laminar. An error is, however, introduced because the overland flow actually enters the gutter at a point further downstream than is assumed in the representation used here. But this error is very small and can be neglected.



(a) PLAN



(b) PROFILE

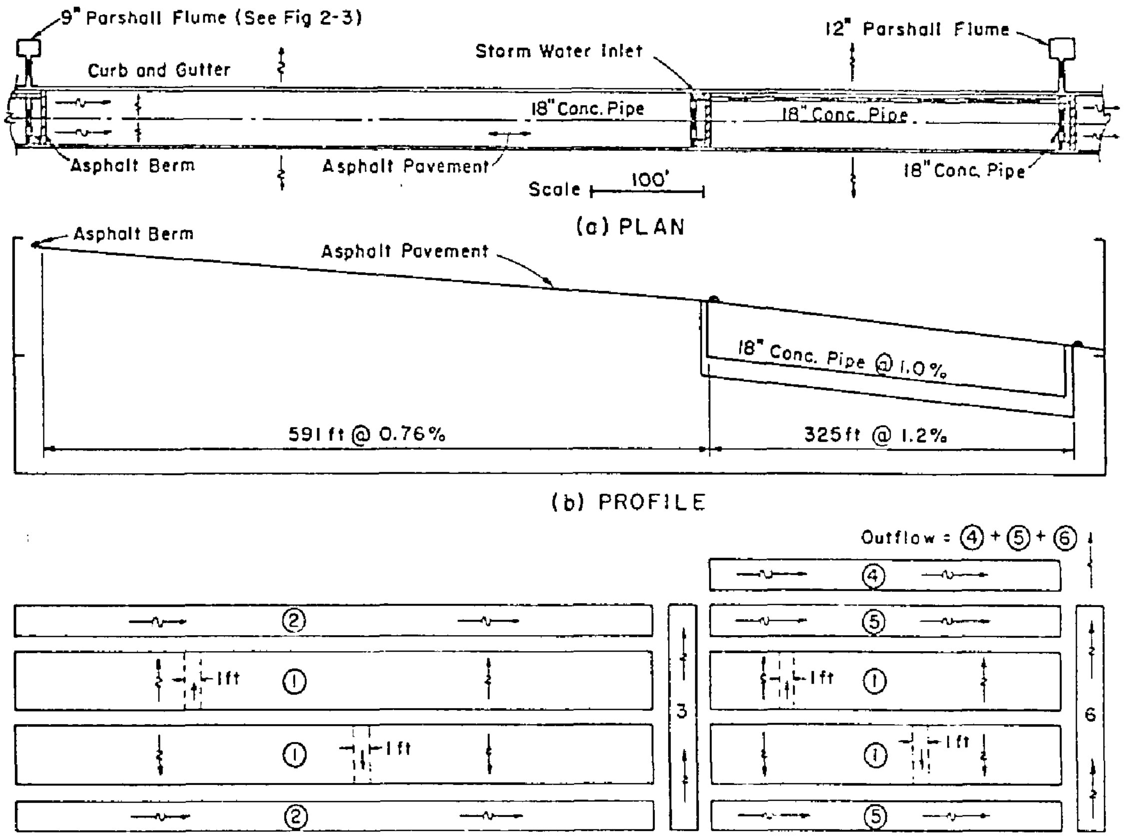


(c) SCHEMATIC REPRESENTATION OF DRAINAGE PLAN

Component	Length (ft)	Width or Dia (ft)	Slope (ft/ft)	Side Slope	Inflow to Component	
					Lateral Inflow	Upstream Inflow
①	22	1.0	.03	–	Rainfall	—
②	267	—	.0165	34:1	①	—
③	309	—	.0323	34:1	①	—
④	44	1.25	.0123	–	—	②
⑤	309	1.50	.0316	–	—	② and ④
⑥	44	1.25	.0041	–	—	③

(d) PHYSICAL CHARACTERISTICS OF COMPONENTS IN THE SCHEMATIC REPRESENTATION

Figure 2-3 Physical Characteristics of Inlet Area N9



(c) SCEMATIC REPRESENTATION OF DRAINAGE PLAN

Component	Length (ft)	Width or Dia (ft)	Slope (ft/ft)	Side Slope	Inflow to Component	
					Lateral Inflow	Upstream Inflow
(1)	22	1.0	.03	—	Rainfall	—
(2)	591	—	.0076	44:1	(1)	—
(3)	44	1.5	.0064	—	—	(2)
(4)	325	1.5	.010	—	—	(2) and (3)
(5)	325	—	.012	44:1	(1)	—
(6)	44	1.5	.034	—	—	(5)

(d) PHYSICAL CHARACTERISTICS OF COMPONENTS IN THE SCHEMATIC REPRESENTATION

Figure 2-4 Physical Characteristics of Inlet Area N12

The gaged areas considered in this study were basically rectangular in shape so that the overland flow components could be represented as rectangles. However, if the area had a more complicated shape, trapazoidal or triangular components also could have been used. There would then be a spacially variable lateral inflow to the gutter since overland flow lengths would vary along the gutter.

Because there is only one rain gage on each of the gaged areas, no data exist to describe the areal variability of the rainfall during the observed storms. It was assumed, therefore, that rainfall intensities were spacially uniform throughout the area. Although the spacial variability of the rainfall has been neglected, it could have been included if data were available. The basic approach in using the equations of flow is sufficiently general that areal variability of rainfall can and should be considered in future studies. Using the basic approach taken here, the sensitivity of the runoff hydrograph to the areal variability of rainfall observed on various sized drainage basins could accurately be studied. However, a somewhat more complicated computer program would be required than was used in this study. Also, the cost of the computations would be considerably greater than for spacially uniform rainfall.

Thus far, runoff from paved areas only has been studied. It would be possible, however, to consider runoff contributions from the pervious areas also. If the amount of water entering the other phases of the runoff process could be accounted for properly, the remaining water could be routed overland to a swale, a gutter, or to the upstream end of a paved overland flow segment. To describe exactly the location of all of the flow paths in pervious portions of an area would be an impossible task. Hopefully, the essence of the physical runoff process can be preserved by approximating the true flow conditions in the pervious areas with properly chosen planes of overland flow and segments of swale and gutter flow.

Chapter 3

SOLUTION OF THE EQUATIONS OF GRADUALLY VARIED UNSTEADY FLOW IN OPEN CHANNELS

The equations for gradually varied unsteady flow in open channels have been used in this study to describe the relationship between rainfall (more precisely, the surface runoff supply rate) and the area, depth, velocity and rate of flow of surface runoff in a drainage area. They account for the conservation of mass and momentum of the surface runoff at any point in space and time. Because of the complexity of the equations, analytic solutions cannot be obtained for the practical problem of synthesizing the hydrograph of runoff from rainfall. Therefore, numerical methods have been used to obtain solutions, and the computations were performed on a digital computer.

A very complete discussion of the application of the equations of flow to unsteady free surface flow in a storm drain was presented by V. M. Yevdjevich in 1961 in a report to the U. S. Bureau of Public Roads. This report was prepared as part of the initial phase of a broader program to develop a set of routing methods for flows in storm drains. A hydraulic model as well as the digital computer are being used in that study.

The equations of flow are presented below, the significance of each term is briefly described, and the assumptions implicit in the use of these equations are summarized. Since numerical techniques were used to solve the equations, various finite difference forms of these equations are discussed. Finally, the algorithm for solving the finite difference equations is presented.

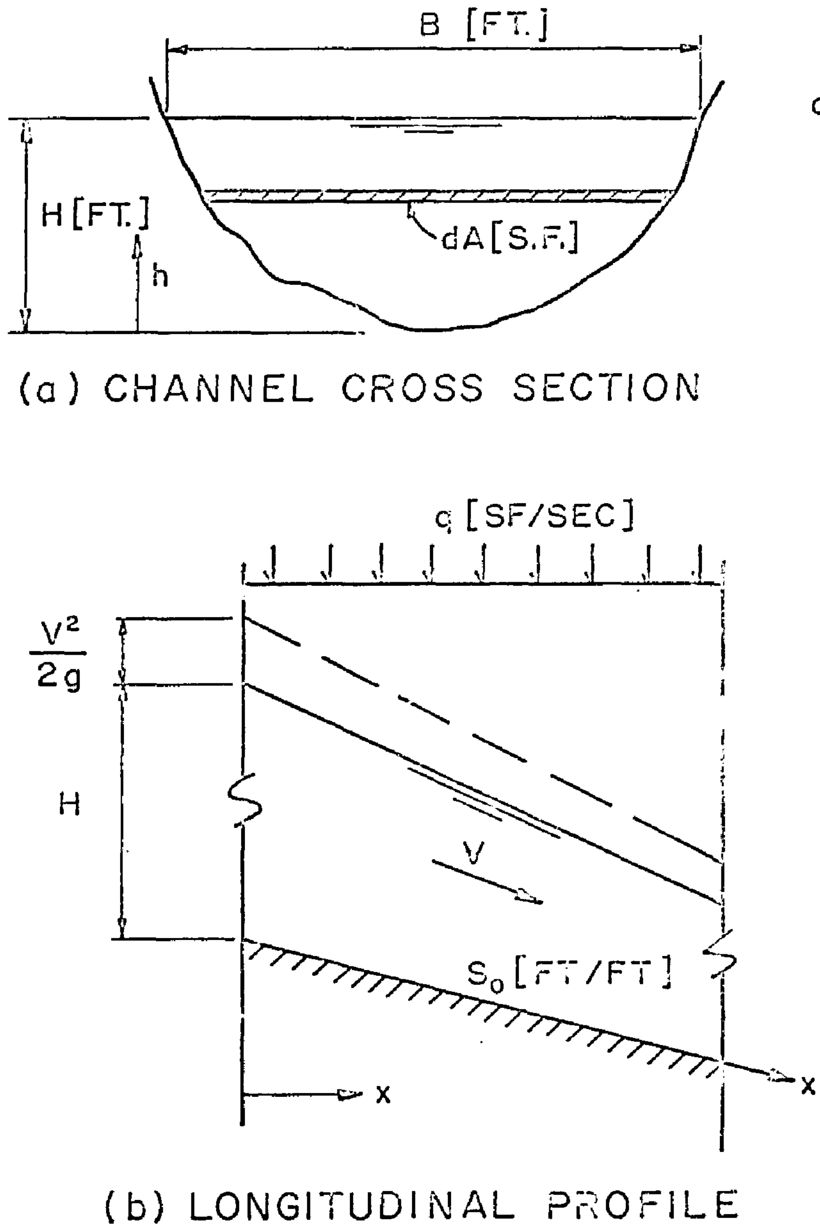
The symbols adopted for use are defined where they first appear and are listed in Appendix A. In Figure 3-1 the notation used in this chapter also is illustrated.

The Equations for Gradually Varied Unsteady Flow in Open Channels

The first equation is a continuity equation, usually written as:

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = q \qquad (3\text{-}1)$$

This equation is derived by considering the water entering and leaving an infinitesimal section of a channel (Yevdjevich, 1961). The term $\partial A/\partial t$ accounts for the change in storage with time in the infinitesimal section; the term $\partial Q/\partial x$ accounts for the difference between the outflow and the inflow to the infinitesimal section; and the term q is the lateral inflow in cfs/ft along the channel. This equation, in finite difference



$$dA = B\,dh$$

$$A = \int_0^H B\,dh \;[S.F.]$$

$$V = \frac{\int_A v\,dA}{A} \;[FT/SEC]$$

$$Q = AV \;[CFS]$$

Figure 3-1 Elements of Open Channel Flow form, is widely used for studying the effect of storage on flood peaks in reservoirs and open channels (Clark, 1945; Rouse, 1950).

The second equation, usually referred to as the momentum equation, refers to the dynamic behavior of the flow. It frequently is written as:

$$\frac{\partial V}{\partial t} + V\frac{\partial V}{\partial x} + g\frac{\partial H}{\partial x} + g(S_f - S_o) + \frac{V_q}{A} = 0 \qquad (3\text{-}2)$$

and is derived by considering all of the forces acting on a flüid element. Neglecting the first and last terms, the remaining terms are commonly used to compute backwater profiles for steady flow in reservoirs and stream channels. The first term, $\partial V/\partial t$, accounts for the local acceleration of the fluid. The convective terms, $V\frac{\partial V}{\partial x}$ and $g\frac{\partial H}{\partial x}$, relate to changes in kinetic and potential energy respectively. The terms gS_f and gS_o account for friction along the channel and the component of gravitational force in the direction of flow respectively. The last term, qV/A, accounts for the momentum that must be imparted to the lateral inflow by the water flowing in the channel.

During this study, the range of values obtained for each term in the momentum equation are presented in Table 3-1 for both overland flow and gutter and pipe flow conditions. Since the term $\partial V/\partial t$ was found to be much smaller than the other terms, it probably can be neglected without significantly affecting the computed hydrograph. Neglecting this term would reduce by a factor of 2 or more the computer time required since it would be possible to use larger values of Δt in the finite difference equations. The relation between the term $\partial V/\partial t$, the magnitude of Δt, and required computer time is discussed in this chapter.

Table 3-1. Range of Values of Each Term in the Momentum Equation

Term	Overland Flow (ft/sec2)	Pipe, Gutter Flow (ft/sec2)
$\frac{\partial V}{\partial t}$	0 - .01	0 - .03
$V\frac{\partial V}{\partial x}$	0 - .01	0 - 3
$g\frac{\partial H}{\partial x}$	0 - .10	0 - 6
gS_f	0 - 1.3	0 - 1.1
gS_o	0 - 1.3	0 - 1.1
$\frac{Vq}{A}$	0 - .001	0 - .05

Evaluation of the Friction Term, S_f

The most important term in the dynamic equation is the friction term S_f. Since the magnitude of this term is usually larger than any of the other terms, except S_o, the method of evaluating S_f would be expected to have greater effect on the computed hydrograph than any other term in the dynamic equation.

To determine S_f, a uniform flow formula has been used. Where there was laminar flow (as apparently occurs during overland flow), the Darcy-Weisbach formula [1]

$$S_f = \frac{f}{H} \frac{V^2}{2g} \tag{3-3}$$

was used. The coefficient, f, is found from

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[1] Some investigators when studying flow in rectangular channels prefer to use the equation $S_f = \frac{f'}{H} \frac{V^2}{8g}$. Therefore, f' = 4f and f'R = 4fR. The factor of 4 has the effect of giving values of f' more nearly the same value as are obtained for circular conduits flowing full. The factor 4 results from the fact that the hydraulic radius of a circular conduit is D/4 and D is used where H appears in the Darcy-Weisbach formula. For wide rectangular channels, the hydraulic radius is equal to the depth of flow. Since the theoretical value of the product fR for rectangular channels is not the same as for circular pipes flowing full, there is little to be gained by including an arbitrary factor of 4 in the Darcy-Weisbach equation. This factor has been omitted in this study.

$$f = C/R \tag{3-4}$$

where C is a constant and

$$R = VH/\nu \tag{3-5}$$

is the Reynolds number. Where there was turbulent flow, Manning's equation

$$S_f = \left(\frac{nV}{1.486\ r^{2/3}} \right)^2 \tag{3-6}$$

was used, where n is a coefficient related to the channel roughness and r is the hydraulic radius.

It has been assumed that the value of S_f occurring for gradually varied unsteady flow at any point in a drainage basin is the same as the value of S_f at the same velocity and hydraulic radius during uniform steady flow at that same point. The error introduced by this assumption probably is less than the error involved in selecting proper values for the coefficients C and n.

The sensitivity of the computed hydrograph to different values for C and n was investigated. From a practical point of view, the runoff hydrograph was found to be insensitive to these coefficients. This was rather surprising since S_f is the most important term other than S_o in the dynamic equation.

The results of this sensitivity analysis are presented in Chapter 5.

Assumptions Used in Deriving the Equations of Flow

The assumptions used in deriving the equations of flow can be summarized as follows (Yevdjevich, 1961; Chow, 1959):

1. Accelerations normal to the direction of flow have been neglected.
2. Velocities normal to the direction of flow have been neglected.
3. Velocities are assumed to be uniform throughout a section normal to the direction of flow. This assumption has been made more restrictive than necessary since coefficients can be introduced into the dynamic equation to account for a variable velocity profile. However, if these coefficients are introduced, it is then assumed that the coefficients are the same as for steady flow conditions at the same discharge and depth of flow.
4. Friction resistance is assumed to be the same as for steady uniform flow at the same velocity and depth of flow.

The importance of these assumptions in this study can be investigated by considering their possible effect on the runoff hydrograph. Since the flow is gradually varied, accelerations and velocities normal to the direction of flow are small compared to those in the direction of flow. Furthermore, the most significant terms in the dynamic equation are the terms gS_o and gS_f. Certainly, if the runoff hydrograph is insensitive to small perturbations of the term gS_f, the hydrograph would be at least as insensitive to variations in the other terms. The sensitivity analysis presented in Chapter 5 provides sufficient justification for the above assumptions, where the objective is synthesis of the inlet hydrograph.

A Finite Difference Approximation to the Equations of Flow

The two equations of flow can be written in finite difference form as

$$\frac{\Delta A}{\Delta t} + \frac{\Delta Q}{\Delta x} = q \tag{3-7}$$

$$\frac{\Delta V}{\Delta t} + \frac{\Delta V}{\Delta x} + g\,\frac{\Delta H}{\Delta x} + g(S_f - S_o) + \frac{qV}{A} = 0 \tag{3-8}$$

To be computationally useful, the solution of the finite difference equations should approach the solution of the partial differential equations as $\Delta x \longrightarrow 0$ and $\Delta t \longrightarrow 0$. The strategy followed in this study has been to divide the channel into a number of intervals of length, Δx, and then the above difference equations are solved at successive intervals of time, t.

To assure that the solution of the finite difference equations approximate the solution of the differential equations, a restriction must be placed on the selection of Δx and Δt. This restriction is that the ratio $\Delta x/\Delta t$ must satisfy

$$\frac{\Delta x}{\Delta t} \geq \lambda = V + \sqrt{gA/B} \qquad (3\text{-}9)$$

at all grid points (Forsythe and Wasow, 1964; Dronkers, 1964). This restriction is related to the existence, in the x - t plane, of characteristic curves described by the equations

$$\frac{dx}{dt} = V + \sqrt{gA/B} \qquad (3\text{-}10)$$

$$\frac{dx}{dt} = V - \sqrt{gA/B} \qquad (3\text{-}11)$$

Small disturbances in the flow at any point in the x - t plane are propogated along curves defined by these equations. These curves also define the regions of influence and dependence of the solution at points in the x - t plane on the solutions at other points. Thus, it is necessary that the inequality (3-9) be satisfied.

To appreciate the significance of this restriction, some typical numerical values may be helpful. The value of Δx in this study varied from 3 ft to more than 50 ft. Values of the velocity V varied from 0.3 ft/sec for overland flow to 5-10 ft/sec for gutter and pipe flow. The term $\sqrt{gA/B}$ is about as large as V. Therefore, the value of Δt typically was limited to from 1 to 3 seconds. A considerable number of computations must therefore be made to obtain runoff hydrographs for storms that last for 30 to 60 minutes or more. Neglecting the term $\partial V/\partial t$ would eliminate one of the characteristic curves, and the remaining curve would be defined by $dx/dt = V - \frac{gA/B}{V}$ allowing a larger interval Δt and reducing the number of computations required by a factor of greater than 2.

The manner in which the terms $\Delta Q/\Delta x$, $\Delta V/\Delta x$, and $\Delta H/\Delta x$ must be evaluated also depends on the characteristic equations. If the term $V - \sqrt{gA/B}$ is positive, the flow is supercritical; if it is negative, the flow is subcritical. For supercritical flow, all disturbances propogate downstream; but for subcritical flow, disturbances propogate both upstream and downstream. This phenomena must be preserved in the finite difference representation of the partial differential equations. Therefore, the terms $\partial Q/\partial x$, $\partial V/\partial x$, and $\partial H/\partial x$ were evaluated using different finite difference schemes for supercritical flow.

Referring to Figure 3-2, the following equations were used to approximate derivatives with respect to x for subcritical flow.

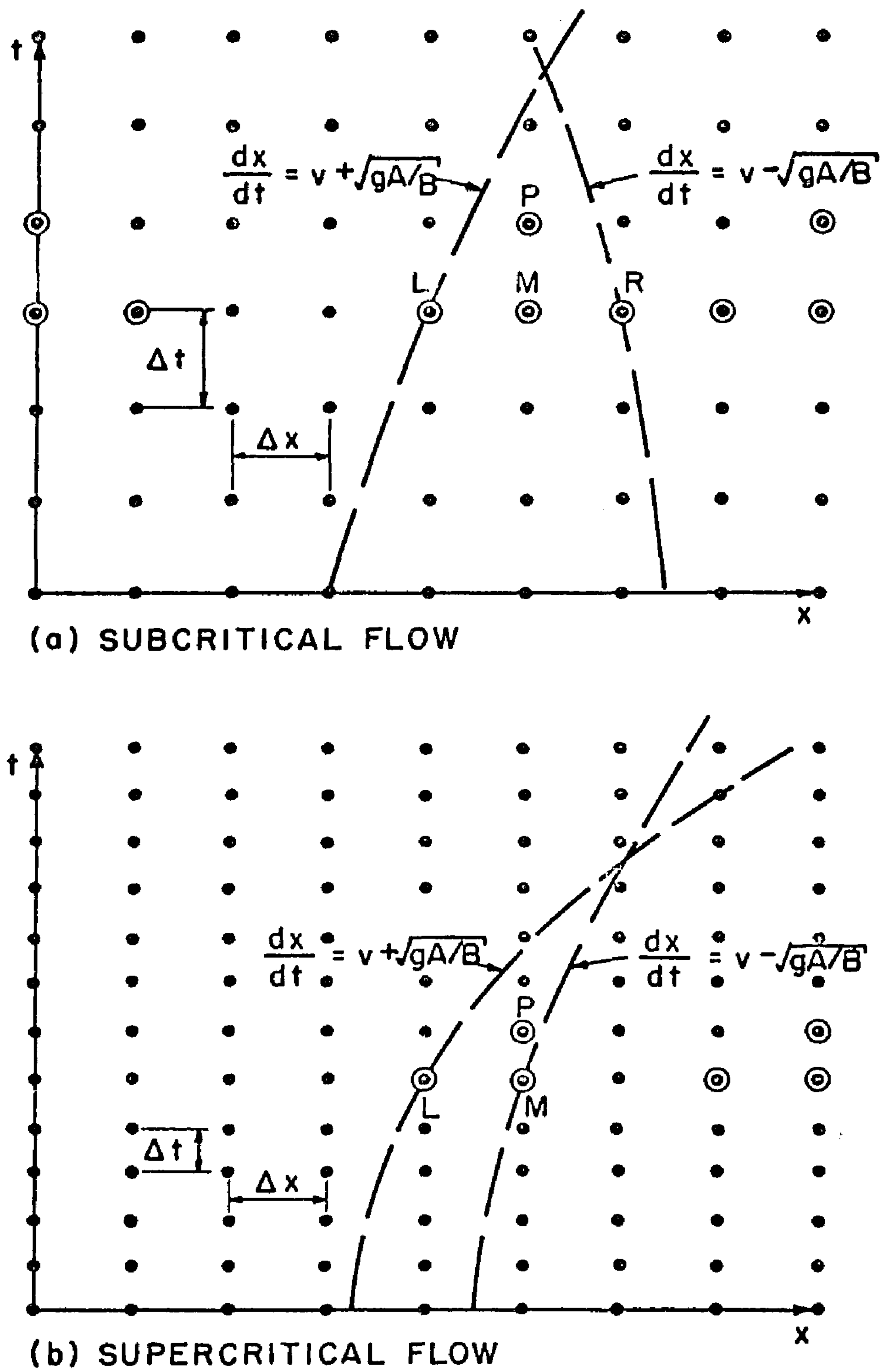

Figure 3-2 Pattern of Grid Points for the Finite Difference Equations

$$\frac{\Delta V}{\Delta x} = \frac{V(x+\Delta x) - V(x-\Delta x)}{2\Delta x} = \frac{V_R - V_L}{2\Delta x} \tag{3-12}$$

$$\frac{\Delta H}{\Delta x} = \frac{H(x+\Delta x) - H(x-\Delta x)}{2\Delta x} = \frac{H_R - H_L}{2\Delta x}$$

$$\frac{\Delta Q}{\Delta x} = \frac{Q(x+\Delta x) - Q(x-\Delta x)}{2\Delta x} = \frac{Q_R - Q_L}{2\Delta x}$$

These are centered difference approximations to the derivatives, evaluated at the point M in Figure 3-2. Points upstream as well as downstream are used to approximate the derivatives.

For supercritical flow, a backward difference scheme was used:

$$\frac{\Delta V}{\Delta x} = \frac{V(x,t) - V(x-\Delta x, t)}{\Delta x} = \frac{V_M - V_L}{\Delta x} \tag{3-13}$$

$$\frac{\Delta H}{\Delta x} = \frac{H(x,t) - H(x-\Delta x, t)}{\Delta x} = \frac{H_M - H_L}{\Delta x}$$

$$\frac{\Delta Q}{\Delta x} = \frac{Q(x,t) - Q(x-\Delta x, t)}{\Delta x} = \frac{Q_M - Q_L}{\Delta x}$$

Only points at M, or upstream from M are used to estimate the derivatives at point M where the flow is supercritical.

The remaining terms in Equations 3-7 and 3-8 were evaluated for both subcritical and supercritical flow as follows:

$$\frac{\Delta A}{\Delta t} = \frac{A(x,\ t+\Delta t) - A(x,t)}{\Delta t} = \frac{A_P - A_M}{\Delta t} \qquad (3\text{-}14)$$

$$\frac{\Delta V}{\Delta t} = \frac{V(x,\ t+\Delta t) - V(x,t)}{\Delta t} = \frac{V_P - V_M}{\Delta t}$$

$$A = \frac{A(x,\ t+\Delta t) + A(x,t)}{2} \quad \frac{A_P + A_M}{2}$$

$$S_f = \frac{S_f(x,\ t+\Delta t) + S_f(x,t)}{2} \quad \frac{(S_f)_P + (S_f)_M}{2}$$

When Equations 3-14 are substituted into Equations 3-7 and 3-8, there result two equations for two unknowns, $A(x,\ t+\Delta t)$ and $V(x,\ t+\Delta t)$. These are the only unknown quantities since values of A, Q, H, and V are known at time t. These values are known either from the initial conditions that must be specified for the first iteration or from the solution obtained from the previous iteration. One iteration consists of solving for A and V (thus also Q and H) at time $(t+\Delta t)$ at all grid points along the length of the channel. The grid points at the boundaries require special consideration. Equation 3-7 is used to obtain $A(x,\ t+\Delta t)$ and Equation 3-8, $V(x,\ t+\Delta t)$. The algorithm used to determine $A(x,\ t+\Delta t)$ and $V(x,\ t+\Delta t)$ is obtained after substituting 3-3 to 3-6 into 3-7 and 3-9. Then referring to Figure 3-2 and to Equations 3-3 and 3-14

$$A(x,\ t+\Delta t) = A_P = A_M - (\Delta Q/\Delta x)\Delta t + q\Delta t \qquad (3\text{-}15)$$

$$V(x,\ t+\Delta t) = V_P = (-K_1 + \sqrt{K_1^2 - 4K_2K_3})/2K_2$$

where

$$K_1 = 1 + \frac{g\Delta t}{2(A_M+A_P)} + \frac{\Delta V}{\Delta x}\frac{\Delta t}{2} \qquad \text{(Turbulent Flow)}$$

$$K_2 = \frac{g\Delta t}{2}\left[\frac{n}{1.486(\frac{H_M+H_P}{2})^{2/3}}\right]^2$$

$$K_3 = \frac{V_M}{2}\frac{\Delta V}{\Delta x}\Delta t + g\Delta t(\Delta H/\Delta x - S_o + 2K_3V_M^2/g\Delta t)$$

(3-16a)

$$V(x,\ t+\Delta t) = \frac{V_M\left[1 - \frac{CV\Delta t}{(H_M+H_P)^2} - \frac{\Delta V}{\Delta x}\frac{\Delta t}{2}\right] - g\Delta t\left[\frac{\Delta H}{\Delta x} - S_o\right]}{1 + \frac{\Delta V}{\Delta x}\frac{\Delta t}{2} + \frac{CV\Delta t}{(H_M+H_P)^2}} \qquad (3\text{-}16b)$$

(Laminar Flow)

The decision whether to use Equations 3-12 or 3-13 to evaluate the derivatives with respect to x depends whether the flow is subcritical or supercritical at the points L, M, and R.

This decision involves the directions of the characteristic curves leaving these points. Where the flow was supercritical at some points, but not at others, the specific force of the flow at the points L, M, and R was checked to see if a hydraulic jump would occur. This procedure assumes the flow is steady, but the acceleration forces in the direction of flow are small compared to the pressure and momentum forces so the results are not significantly affected by this assumption. If the flow is supercritical or subcritical at all points, there is no question which equations to use.

Initial Conditions

The initial conditions used in this study were V(x,0), A(x,0), Q(x,0) and H(x,0) = 0 throughout the drainage area. These conditions satisfy the partial differential equations as well as the finite difference equations since all but 2 terms in the momentum equation are initially put equal to zero. The two remaining terms in the momentum equation are

$$\frac{\partial V}{\partial t} = gS_o$$

at the instant t = 0. At some small increment of time ϵ after the instant t = 0, the velocity assumes a non-zero value and the momentum equation becomes, for overland flow

$$\frac{\partial V}{\partial t} + g(S_f - S_o) = 0 \qquad (3\text{-}17)$$

After time ϵ, the term $\partial V/\partial t$ is much smaller than either of the terms gS_f or gS_o.

Boundary Conditions

The boundary conditions that have been used depend whether the flow is subcritical or supercritical at the boundary. For supercritical flow at the upstream end of the channel there are two characteristic curves originating at the upstream boundary since $dx/dt = V \pm \sqrt{gA/B}$ is positive for both curves. Therefore, two boundary conditions must be given at the upstream boundary if the flow is supercritical there. If the flow is subcritical at the upstream boundary, only one boundary condition is given there. At the downstream boundary, a characteristic curve originates only when dx/dt is negative as occurs when the flow is subcritical there. Therefore, not more than one boundary condition can be imposed at the downstream boundary, and this only when there is subcritical flow there.

The possible situations for required boundary conditions are summarized in Table 3-2. Here, depending on whether the flow is subcritical or supercritical, and whether the boundary is upstream or downstream; the required number of boundary conditions at that boundary are indicated.

It is readily seen from this table that at least one boundary condition is always required at the upstream boundary.

Table 3-2.

Boundary	Number of Boundary Conditions Required Supercritical Flow	Subcritical Flow
Upstream	2	1
Downstream	0	1

This condition therefore is always given by the inflow hydrograph at the upper end of the channel.

If the flow is supercritical at the upstream boundary, the depth of flow at this boundary may also be given as a boundary condition. Assuming the depth of flow from the channel contributing from upstream of the channel in question were known, some decision rule could be formulated based on the energies of the flow in the transition between the two channels. However, since the objective here relates only to the runoff hydrograph, small inaccuracies in flow depths can be tolerated. Therefore, the flow is assumed to be at critical depth at the upstream boundary whenever the flow immediately downstream from this boundary is supercritical.

The final situation is where the flow is subcritical at the downstream boundary. Here, upstream depths of flow are influenced by the depth of flow at the boundary. The best procedure to follow at this point depends to a great extent on the geometry of the drainage area. If there are very flat slopes (i.e., less than about .001 ft/ft) the depth of flow in the next channel downstream should be considered. Where slopes are steeper than about .001 ft/ft, a much simpler procedure, from the programming point of view, can be used. In this study, equations 3-13 were always used to compute the velocity and area of flow at the downstream boundary, even when the flow was subcritical. The possibility of a backwater effect at the downstream boundary was therefore eliminated. Depths of flow obtained in this way at the downstream boundary were found to be very close to the depths that would obtain for steady uniform flow at the same rate of flow and channel slope.

One other alternative for establishing the downstream boundary condition for subcritical flow arises when the channel discharges freely. Here, the depth of flow at the boundary is known to be at the critical depth for the given rate of flow. However, for this case in which the slope of the channel is flatter than the critical slope for uniform steady flow at a given discharge rate, very small intervals of Δx must be used near the boundary to describe the drawdown curve of the water surface. Since the ratio $\Delta x/\Delta t$ must be larger than $\lambda = V + \sqrt{gA/B}$, the interval Δt must be reduced in proportion to the reduction in Δx required to obtain the drawdown curve.

Because the object here is to obtain runoff hydrographs, the minor improvement that could be expected for a considerable increase in computer time needed to define the drawdown curve, does not justify the expense.

Chapter 4

RESULT OF COMPUTER SIMULATION OF THE RUNOFF PROCESS - OVERLAND FLOW

Before attempting to simulate the runoff from an entire drainage area, a number of overland flow tests conducted by Izzard in 1942-43 were simulated. These overland flow tests were made for two types of paved surface and a blue grass turf surface. One of the impervious surfaces was a commercial slate roofing-felt impregnated with asphalt, and tests simulated were run on this surface.

The overland flow tests were conducted using a flume 72 feet long by 6 feet wide (Izzard, 1944, 1946). Lengths of overland flow for various tests were between 12 feet and 72 feet. The slope of the flume was varied between .001 ft/ft and .04 ft/ft. The rainfall intensity was approximately 2 or 4 inches per hour. Rainfall was sprinkled uniformly over the area at a steady rate until the runoff at the end of the flume reached steady state. Only the downstream discharge was measured and the rainfall intensity was computed from the steady state runoff rate. Runoff measurements were made at time intervals as small as 5 seconds depending on the rate at which the runoff rate changed with time.

The tests selected for simulation were chosen on the basis of a factorial design where each of the factors: length, slope and rainfall intensity were at high and low levels. The complete factorial was used so that all combinations of high and low levels of the three factors were considered in the eight tests selected. The levels of the factors and the run numbers corresponding to each test are indicated in Table 4-1.

Table 4-1. Factorial Design of Selected Overland Flow Tests for Computer Simulation

Simulation Run Number	Izzard's Run Number	Length (ft)	Slope (ft/ft)	Rainfall Intensity (in/hr)
7	121	12	.005	3.58
8	133	72	.005	3.67
9	173	72	.04	3.68
12	134	72	.005	1.87
13	122	12	.005	1.73
14	174	72	.04	1.90
15	167	12	.04	3.66
16	168	12	.04	1.73

Evaluation of the Friction Term, S_f

Before the overland flow tests could be simulated, coefficients in the equation for S_f had to be selected. For the small Reynolds numbers that occur during overland flow, the flow is essentially laminar. However, the raindrop impact causes disturbance of the flow and provides additional energy that must be dissipated. This has the effect of increasing the resistance to flow, and larger coefficients in the laminar friction equation result than are determined theoretically.

The Darcy-Weisbach equation (3-3) was used to compute the friction slope, S_f, for the overland flow tests. For convenience, this equation is presented again.

$$S_f = \frac{f}{H}\frac{V^2}{2g} \qquad (4\text{-}1)$$

where

$$f = \frac{C}{R} \qquad (4\text{-}2)$$

and C is a constant. The Reynolds number is found from

$$R = \frac{VH}{\nu} \qquad (4\text{-}3)$$

For laminar flow in a rectangular channel, the value C = 6 is obtained by balancing the forces acting on an element of the fluid and assuming the shear stress is related to viscosity by the equation

$$\tau = \mu\frac{dv}{dH} \qquad (4\text{-}4)$$

The roughness of the boundary has no effect on the flow in this theoretical description.

To obtain a value for C to be used in the simulation, it is recalled that initially the terms A, V, Q, and H are equal to zero. Therefore, in the equations of flow, all partial derivatives with respect to x are initially equal to zero. If a uniform rainfall is then applied, the values of the terms in the continuity equation evaluated at the downstream end of the channel are

$$\frac{\partial A}{\partial t} = q, \quad \frac{\partial Q}{\partial x} = 0 \qquad (4\text{-}5)$$

for an interval of time after t = 0. If the channel were infinitely long and if there were no drawdown effects at the end of the channel, Equation 4-5 would hold at the downstream end as long as there were a steady uniform rainfall intensity. In the momentum equation, the terms $\partial H/\partial x$ and $\partial V/\partial x$ similarly, are initially equal to zero at the downstream end. Then, neglecting the terms $\partial V/\partial t$ and Vq/A, which are small compared to gS_o and gS_f, the momentum equation is reduced to

$$S_f \approx S_o \qquad (4\text{-}6)$$

for an initial increment of time after t = 0.

After substituting Equations 4-6, 4-5, 4-3, and 4-2 into 4-1, the following equation is obtained

$$Q = \frac{2gS_o H^3}{C \nu} \qquad (4\text{-}7)$$

where Q is the runoff rate in cfs at the downstream end, and

$$H = \frac{i\ t}{43200}$$

where i is the rainfall intensity in in/hr and t is time in seconds. The runoff rate, in in/hr is then given by

$$Q = \frac{2gS_o\ i^3 t^3}{C \nu L (43200)^2} \qquad (4\text{-}8)$$

Equation 4-8 can be used as a similarity relationship so that all data from the beginning of all tests can be used to obtain a single estimate for C. If Equation 4-8 is rewritten as

$$Q^* = \frac{Q}{K} = \frac{1}{C}\, t^3 \qquad (4\text{-}9)$$

values of Q^* can be computed from the data for each test and plotted as a function of t. Data from each of the eight tests listed in Table 4-1 have been plotted in this way, as illustrated in Figure 4-1. A straight line was fitted to the data by eye. If the flow were laminar during the tests, the slope of this line on log-log paper would be 3:1. A line with this slope fits the data very well, and the value obtained for C was C = 10.

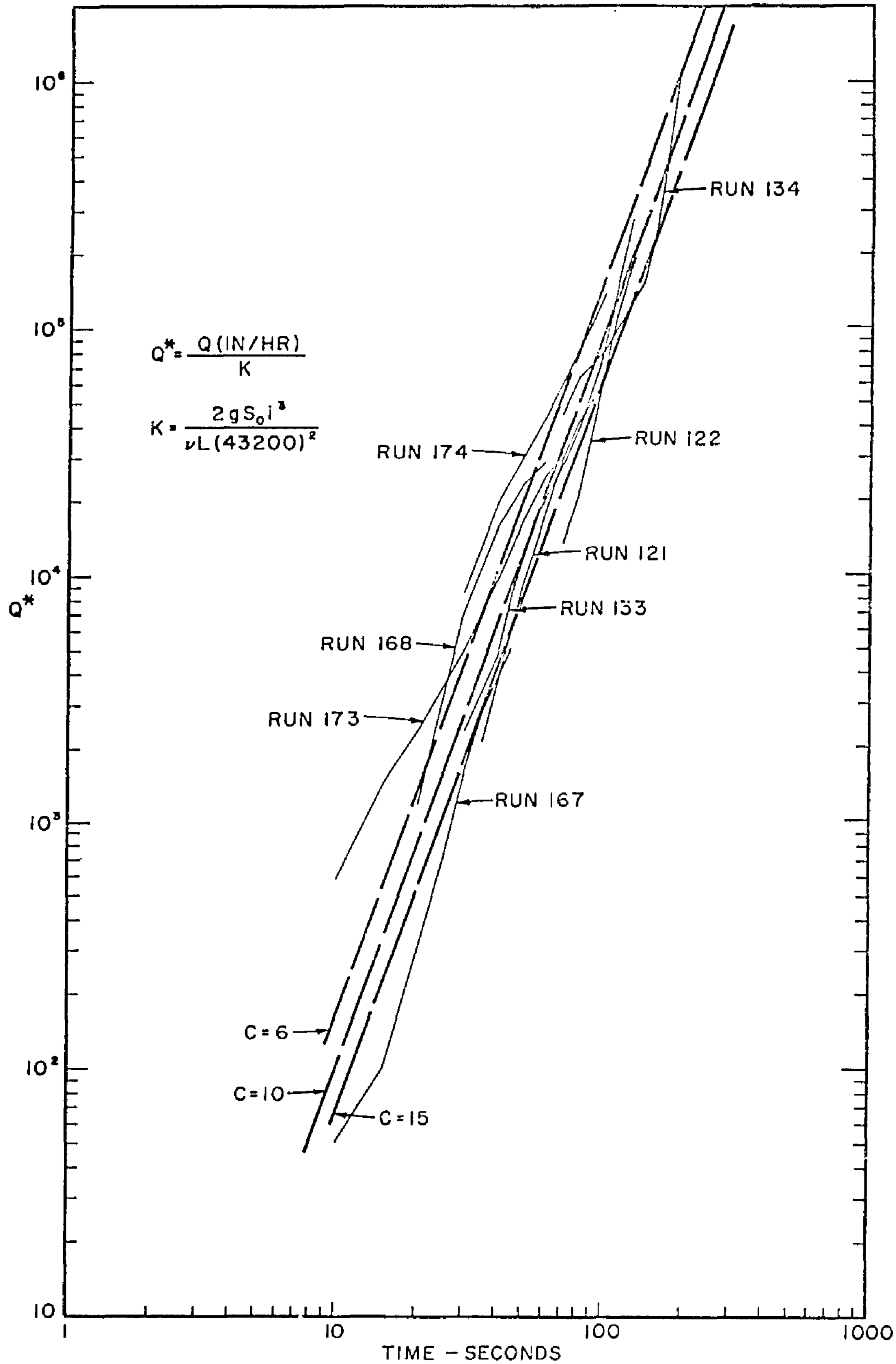

Figure 4-1 Rising Limb of the Overland Flow Hydrograph

This value is higher than the theoretical value of C = 6, but is lower than a similar value estimated for C by Izzard using the same data. In his study, Izzard computed the surface profile of the overland flow when the flow reached steady state, and compared the volume of water under this profile to the volume detained on the surface as computed from the measured hydrograph. Following this procedure for a number of runs, Izzard obtained the value C[1] = 58/4 = 14.5 when the rainfall intensity was 3.6 in/hr (Izzard, 1946). The difference between these values of C is not significant from a practical point of view since the ultimate interest is in the runoff hydrograph. This point will be developed more fully in terms of a sensitivity analysis in the next chapter. However, these differences, as well as the occurrence of a sharp increase in discharge at the end of rainfall during many of the tests suggests there is a much more complicated relationship between the Reynolds number, rainfall intensity, and the friction coefficient than is accounted for by the laminar flow equation.

Woo and Brater (1962) have reported the results of experiments conducted by them where steady uniform rainfall was applied to a flume approximately 30 ft long by 6 in wide. The rainfall applicator was carefully designed to faithfully simulate raindrops as they naturally occur. The rainfall intensity was

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[1] See footnote attached to Equation 3-3.

set at each of three levels and two conditions of surface roughness were studied. The slope of the flume was varied from .001 ft/ft to .06 ft/ft. When the flow reached steady conditions, the water surface profile and discharge were measured. The rainfall intensity was computed from the discharge. Values of C obtained by Woo and Brater were sensitive to both the slope of the flume and to the intensity of the rainfall. For a surface of sand, screened between sieve sizes 16 and 20 and glued to masonite, values of C increased from 10 to 25 as the slope increased from .003 to .06 ft/ft. Masonite, with the rough side up, produced slightly lower values. The investigators present numerous curves relating the coefficient f and the Reynolds number for various combinations of slope, rainfall intensity, and surface roughness.

Results of Computer Simulation

Each of the overland flow tests listed in Table 4-1 were simulated on the computer using the finite difference form of the equation of flow. For overland flow lengths of 72 ft, the value used for Δx was 12 ft. For overland flow lengths of 12 ft, the value used for Δx was 3 ft. The value used for Δt was 3 seconds. An attempt was made to use larger values of Δt, but the computations became unstable even though the ratio $\Delta x/\Delta t$ satisfied the convergence restriction given by Equation 3-9.

The results of the computer simulation are presented in Figures 4-2 to 4-5. The observed runoff hydrograph is indicated by the dashed curve, and the computed hydrograph, by the solid curve. There is excellent agreement between the observed and computed hydrographs. Where observed and computed values were too close to show both curves, only the solid curve for the computed hydrograph has been shown.

In every test, the computed hydrograph reached steady state before the observed hydrograph. This phenomena could be explained by a variable coefficient C. In the computations, C was always equal to the constant value of 10, but if C were increased as a function of the Reynolds number and the rainfall intensity, better agreement between the two curves could be obtained. By changing the value of C when rainfall ceases, the sharp increase in runoff at the end of rainfall could also be reproduced. However, the amount by which C should change at the end of the rain is a function of the Reynolds number as well as the rainfall intensity.

Yu and McKnown (1964) studied overland flow data from tests by the Los Angeles District of the Corps of Engineers from 1948 to 1954. These tests extended over a higher range of Reynolds numbers than the tests by Izzard. Yu and McKnown report that above R = 2000 the sharp increase in flow after secession of rain was not observed to occur on a concrete surface. The relative increase in discharge, expressed as

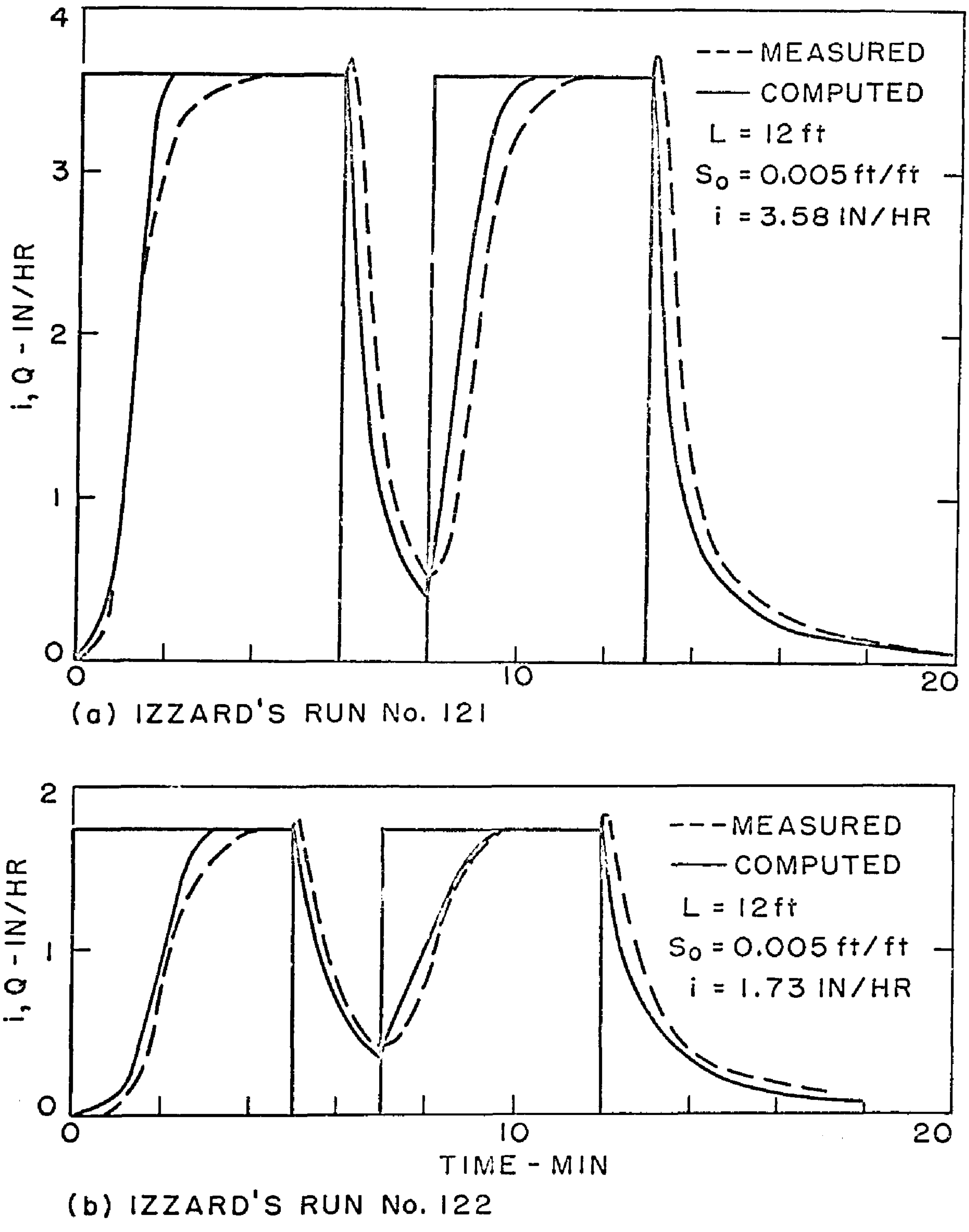

Figure 4-2 Comparison of Measured and Computed Overland Flow Hydrographs - Izzard's Run Nos. 121 and 122

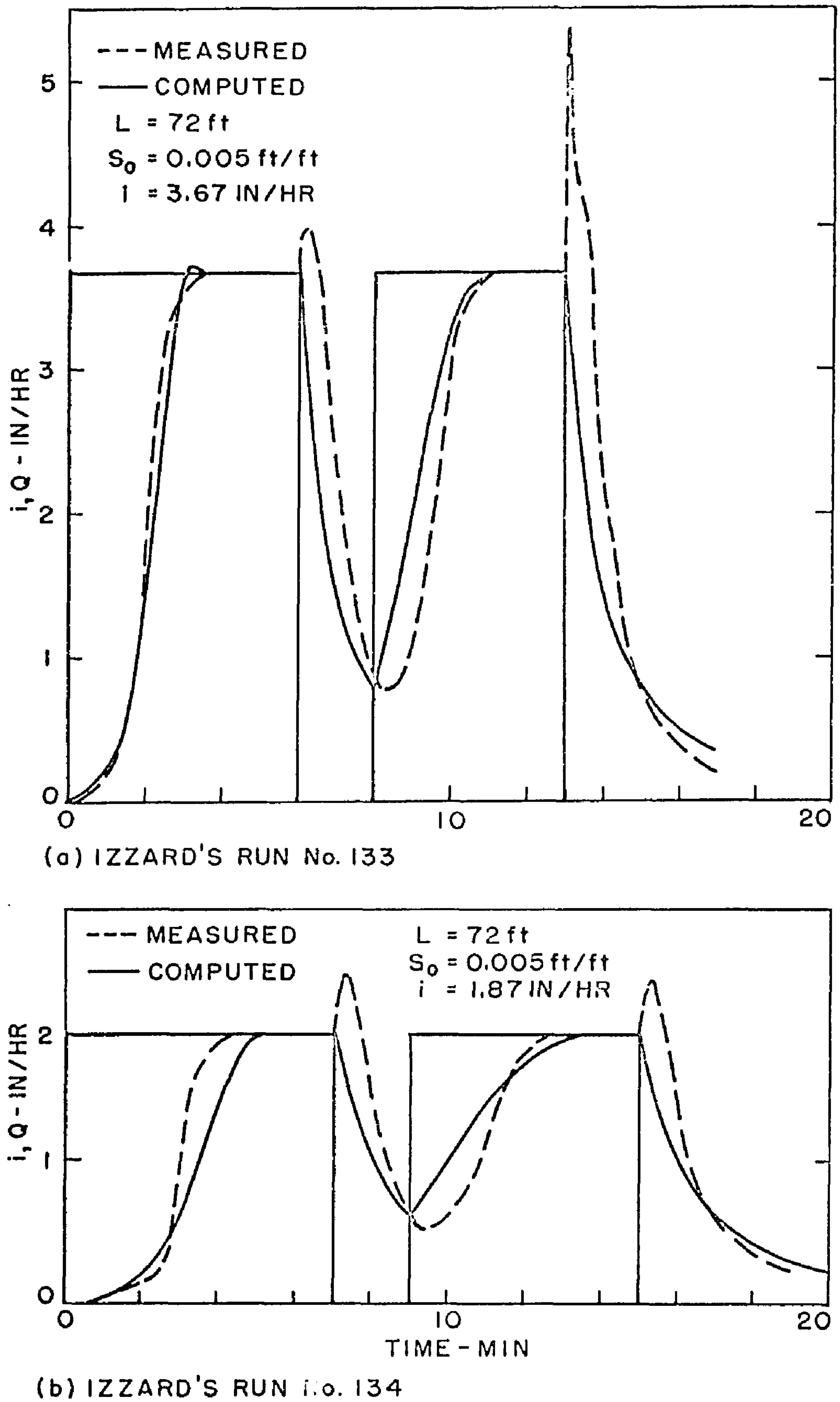

Figure 4-3 Comparison of Measured and Computed Overland Flow Hydrographs - Izzard's Run Nos. 133 and 134

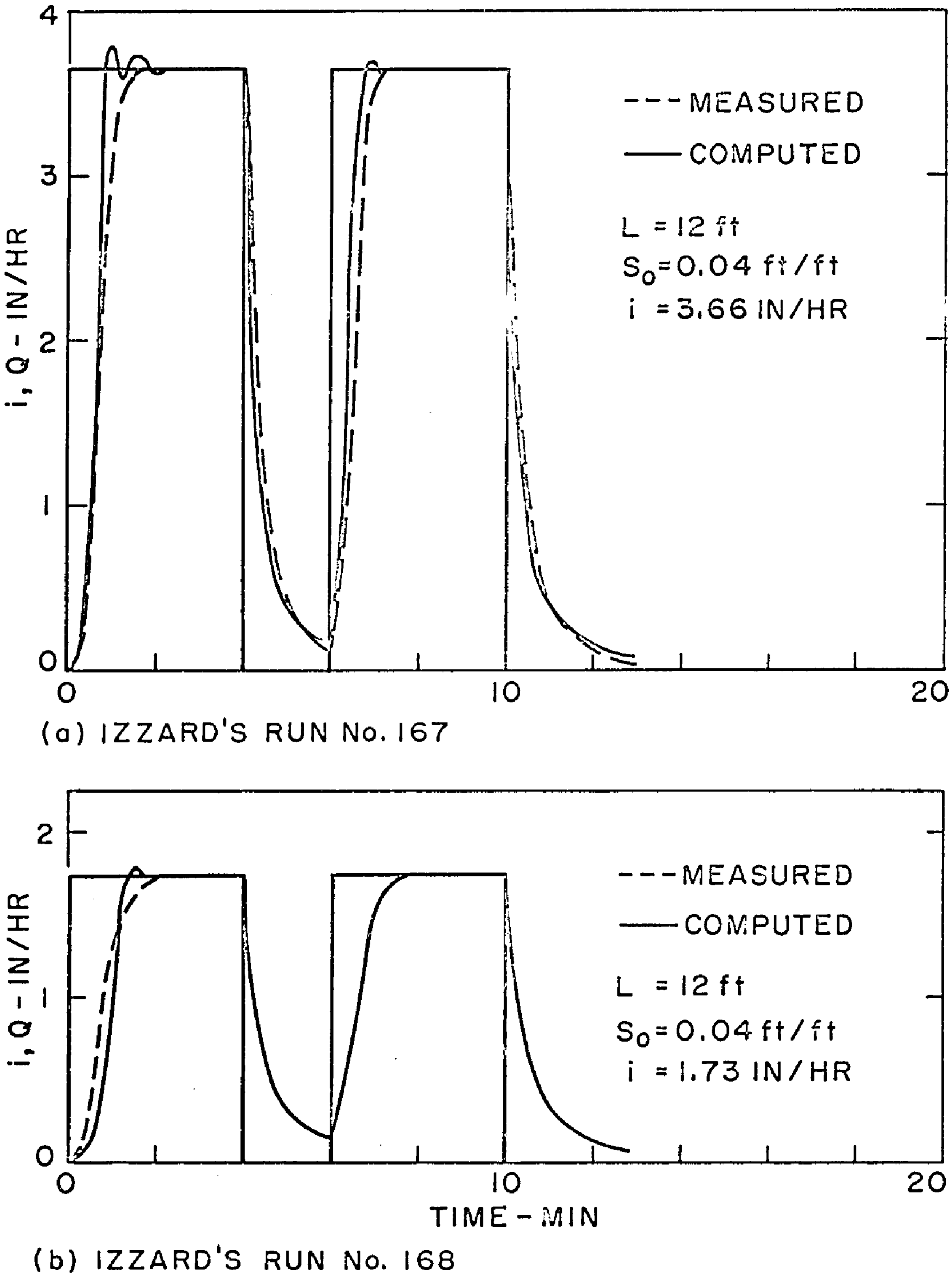

Figure 4-4 Comparison of Measured and Computed Overland Flow Hydrographs - Izzard's Run Nos. 167 and 168

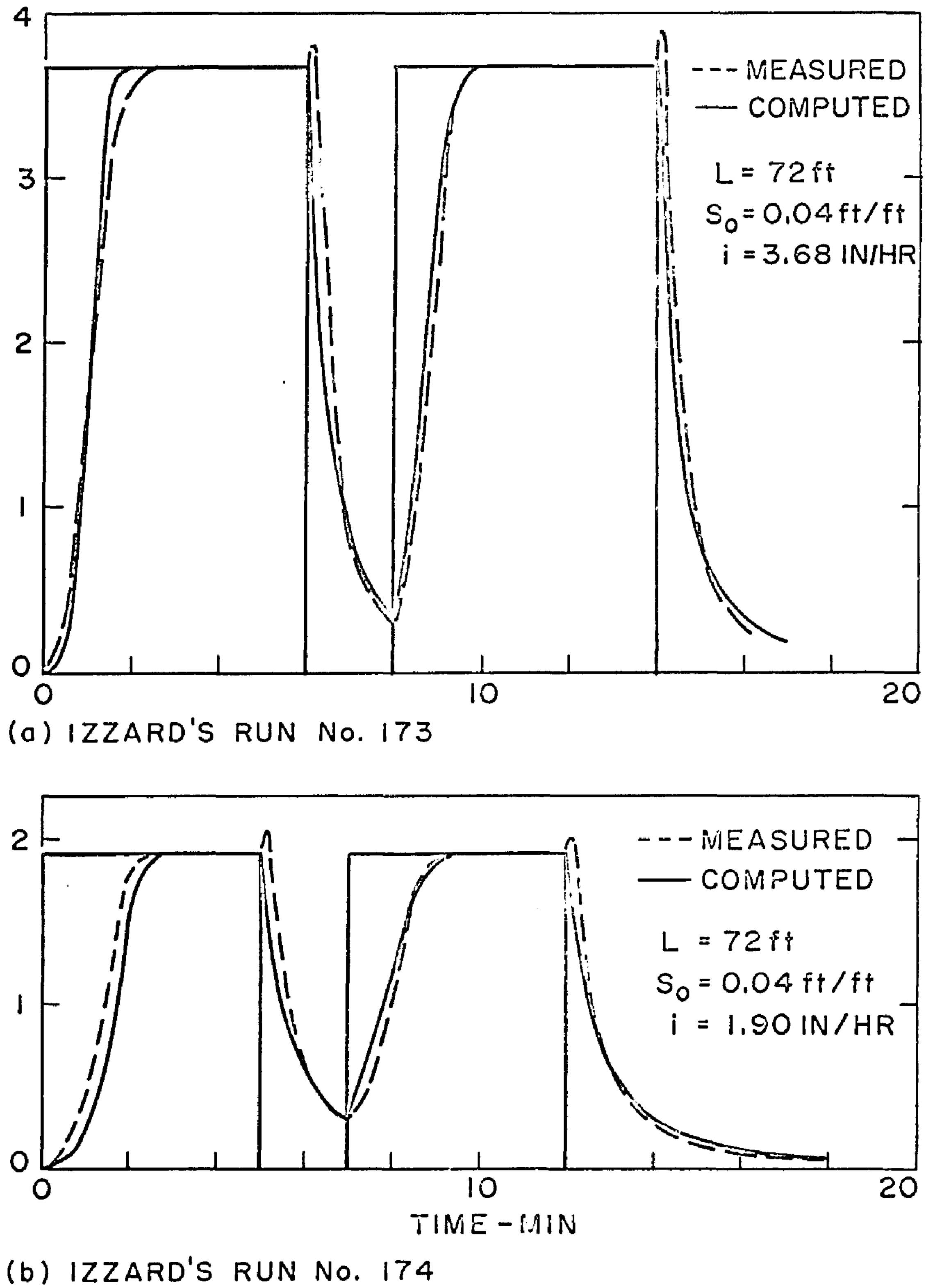

Figure 4-5 Comparison of Measured and Computed Overland Flow Hydrographs - Izzard's Run Nos. 173 and 174

$\Delta Q/Q$, was found to be greatest at a Reynolds number of about 600. During Izzard's tests, the Reynolds numbers were less than $R = 500$.

Considering the variability of the values of C associated with the scatter of points in Figure 4-1 and the variability of values of C reported by Woo and Brater as well as Yu and McKnown, it was quite surprising that the computed hydrographs in Figures 4-2 to 4-5 agreed well with the observed hydrographs. A careful study of these hydrographs reveals that at any instant of time the computed runoff rates may differ by a rather wide margin from the observed runoff rates. These differences primarily reflect the variable nature of the coefficient C.

However, in most cases, the time required to reach a given runoff rate is not much different between the measured and computed hydrographs. This might suggest that attention should be given to large variations in the cube root of the coefficient C rather than to variations in the value of C itself (cf Equation 4-8 since $t \sim \sqrt[3]{C}$ in this equation).

Chapter 5

RESULTS OF COMPUTER SIMULATION OF THE RUNOFF PROCESS - THE INLET HYDROGRAPH

Runoff hydrographs for three paved areas have been synthesized using a finite difference form of the equation of flow to simulate the dynamic behavior of a drainage area. Measured rainfall was used as input to the mathematical model and the synthesized runoff hydrograph was then compared with the measured hydrograph from the gaged area. Excellent agreement was found between the synthesized hydrograph and the measured hydrograph for every storm that was simulated.

A total of 13 storms were simulated for the three areas. The synthesized and measured hydrographs are presented in Figures 5-1 to 5-12.

Description of the Gaged Areas

Two of the gaged areas, N9 and N12, are adjacent sections of East Cleveland Avenue in Newark, Delaware. The third area, SPL1, is a parking lot on the Johns Hopkins University campus. The physical features of the areas are illustrated in Figures 2-2 to 2-4. For convenience, some of the principal features of these areas also are summarized in Table 5-1.

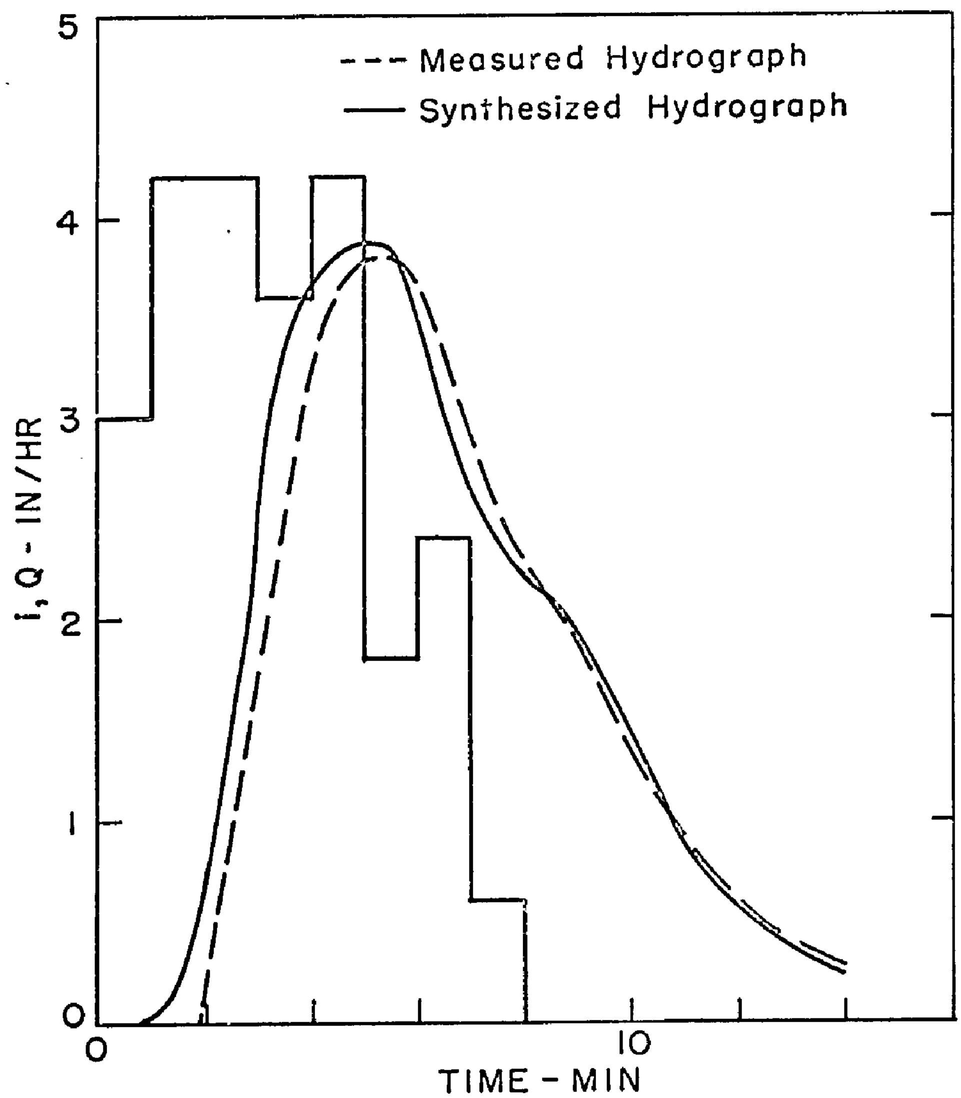

Figure 5-1 Comparison of Measured and Synthesized Runoff Hydrographs - Storm 15N9

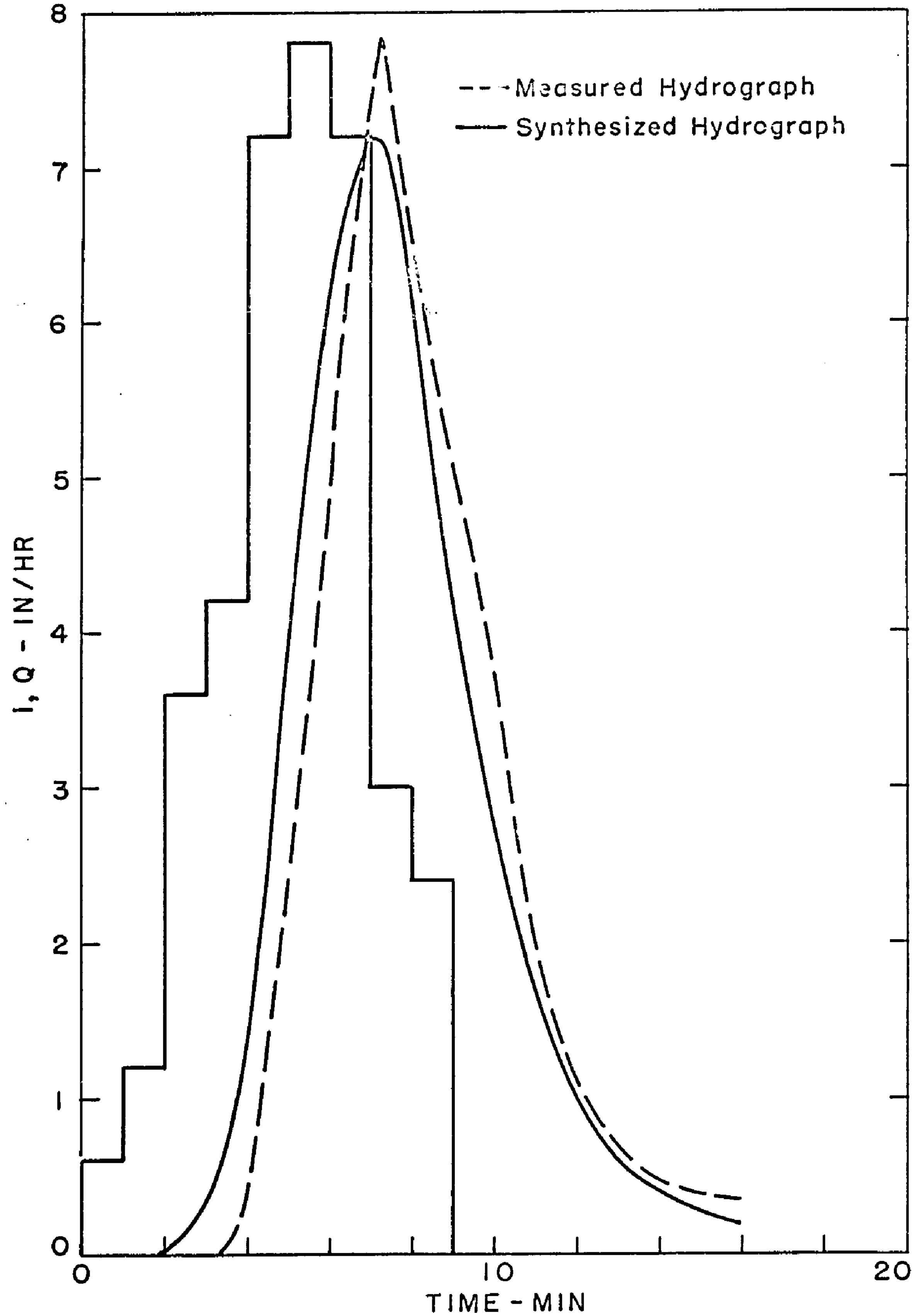

Figure 5-2 Comparison of Measured and Synthesized Runoff Hydrographs - Storm 19N9

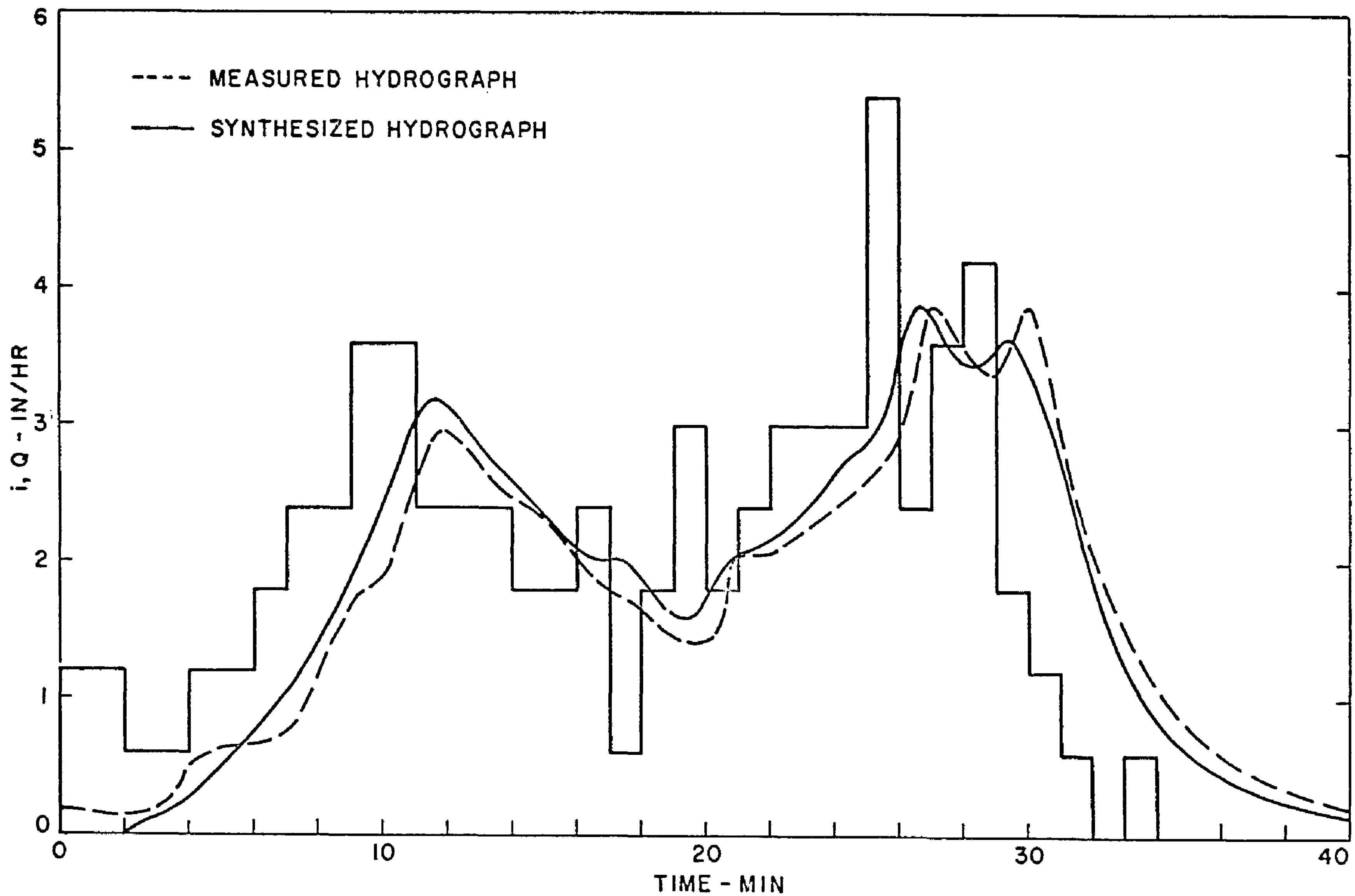

Figure 5-3 Comparison of Measured and Synthesized Runoff Hydrographs - Storm 23N9

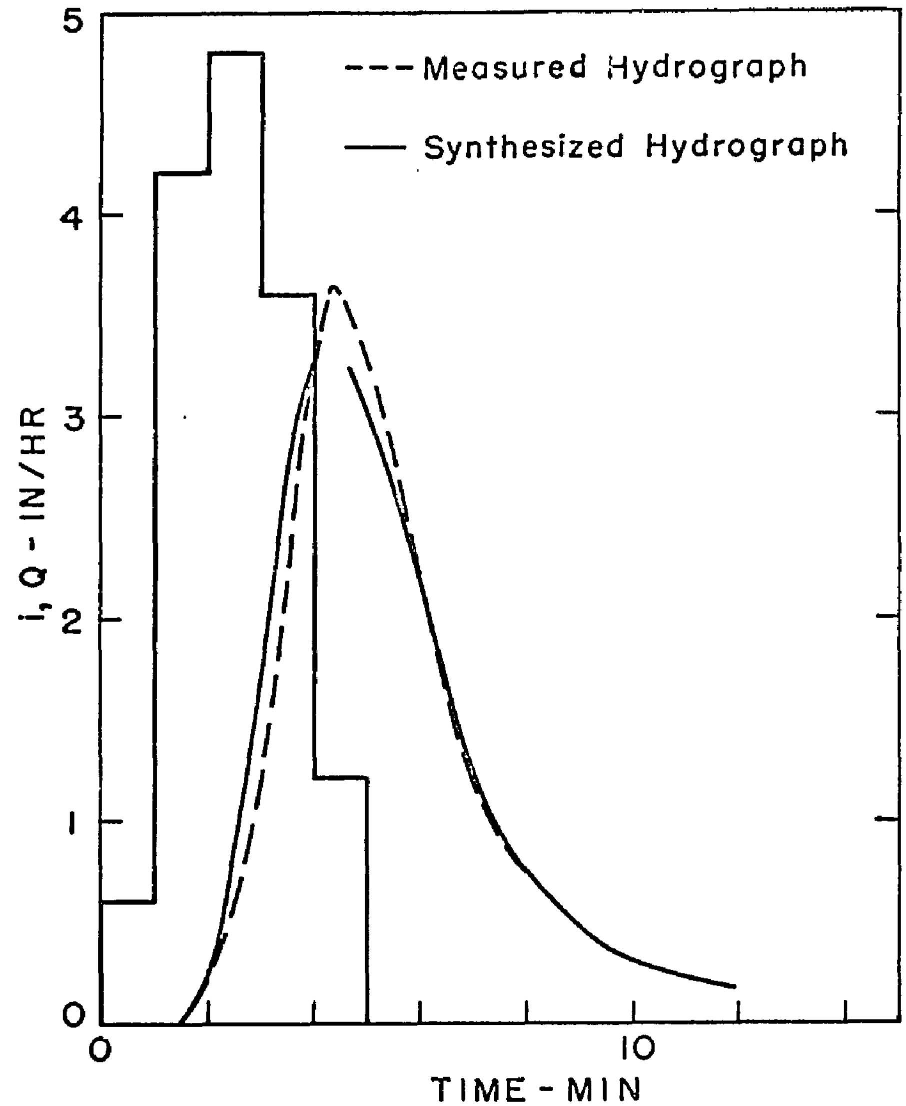

Figure 5-4 Comparison of Measured and Synthesized Runoff Hydrographs - Storm 25N9

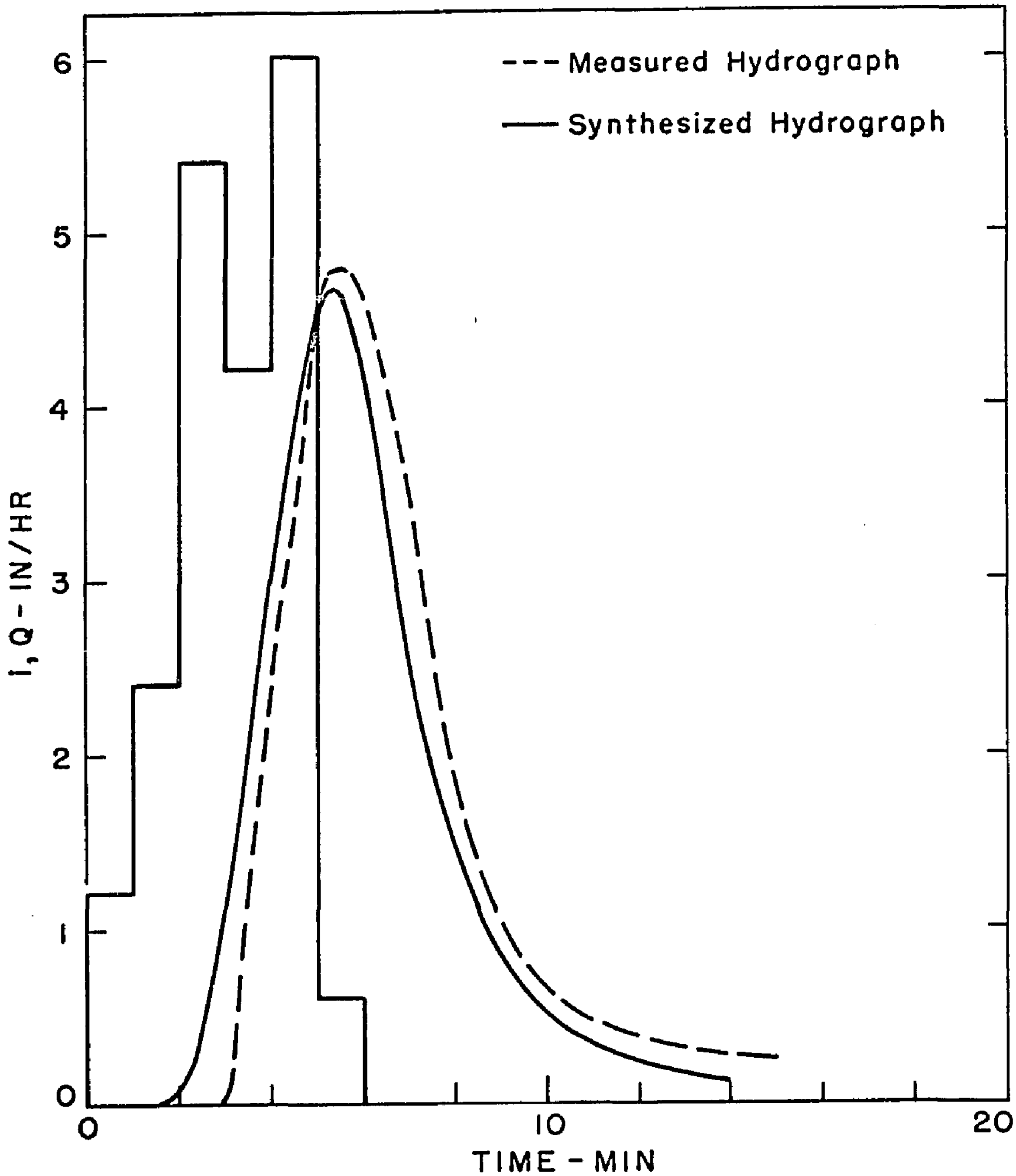

Figure 5-5 Comparison of Measured and Synthesized Runoff Hydrographs - Storm 47N9

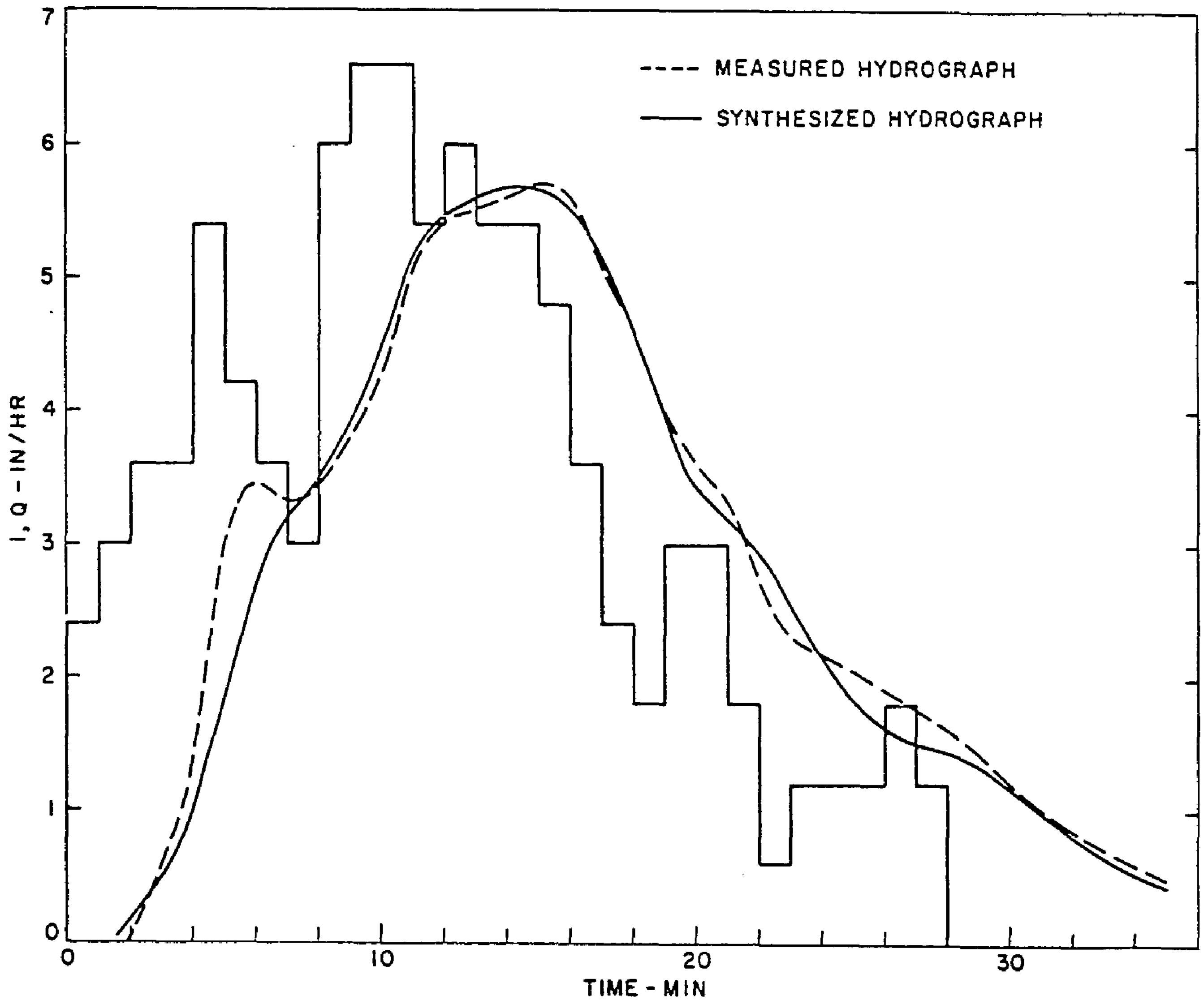

Figure 5-6 Comparison of Measured and Synthesized Runoff Hydrographs - Storm 8N12

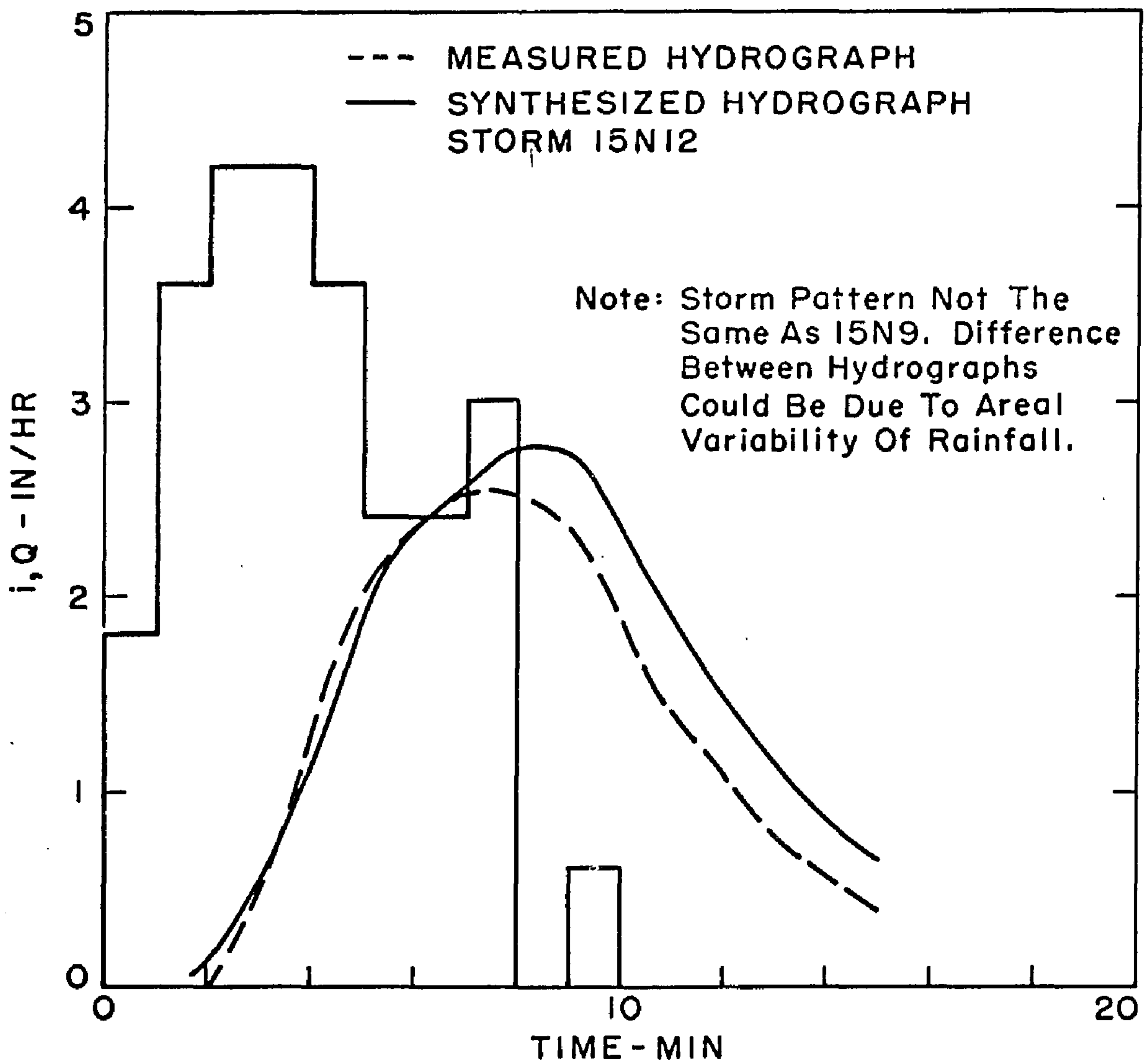

Figure 5-7 Comparison of Measured and Synthesized Runoff Hydrographs - Storm 15N12

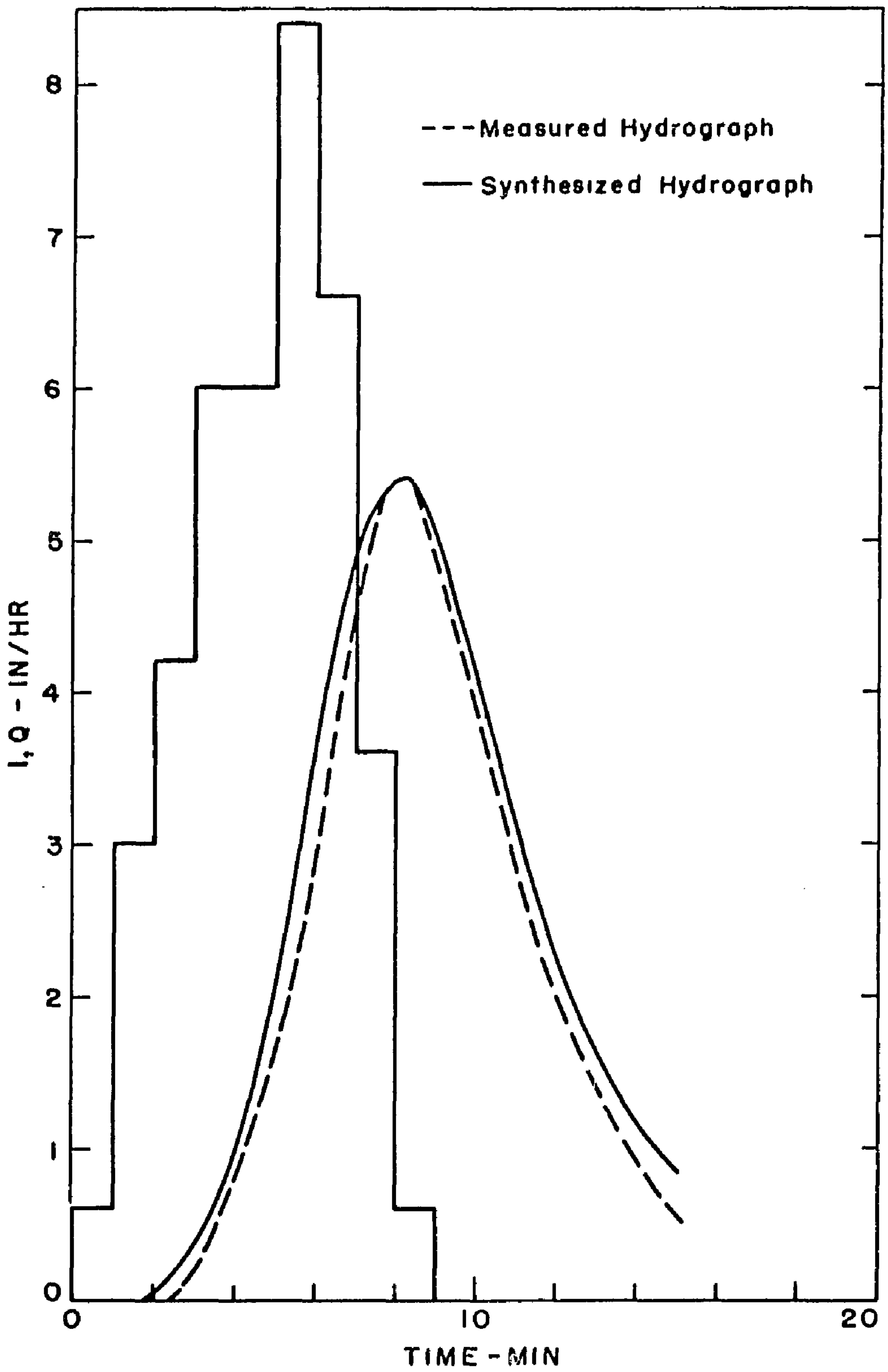

Figure 5-8 Comparison of Measured and Synthesized Runoff Hydrographs - Storm 19N12

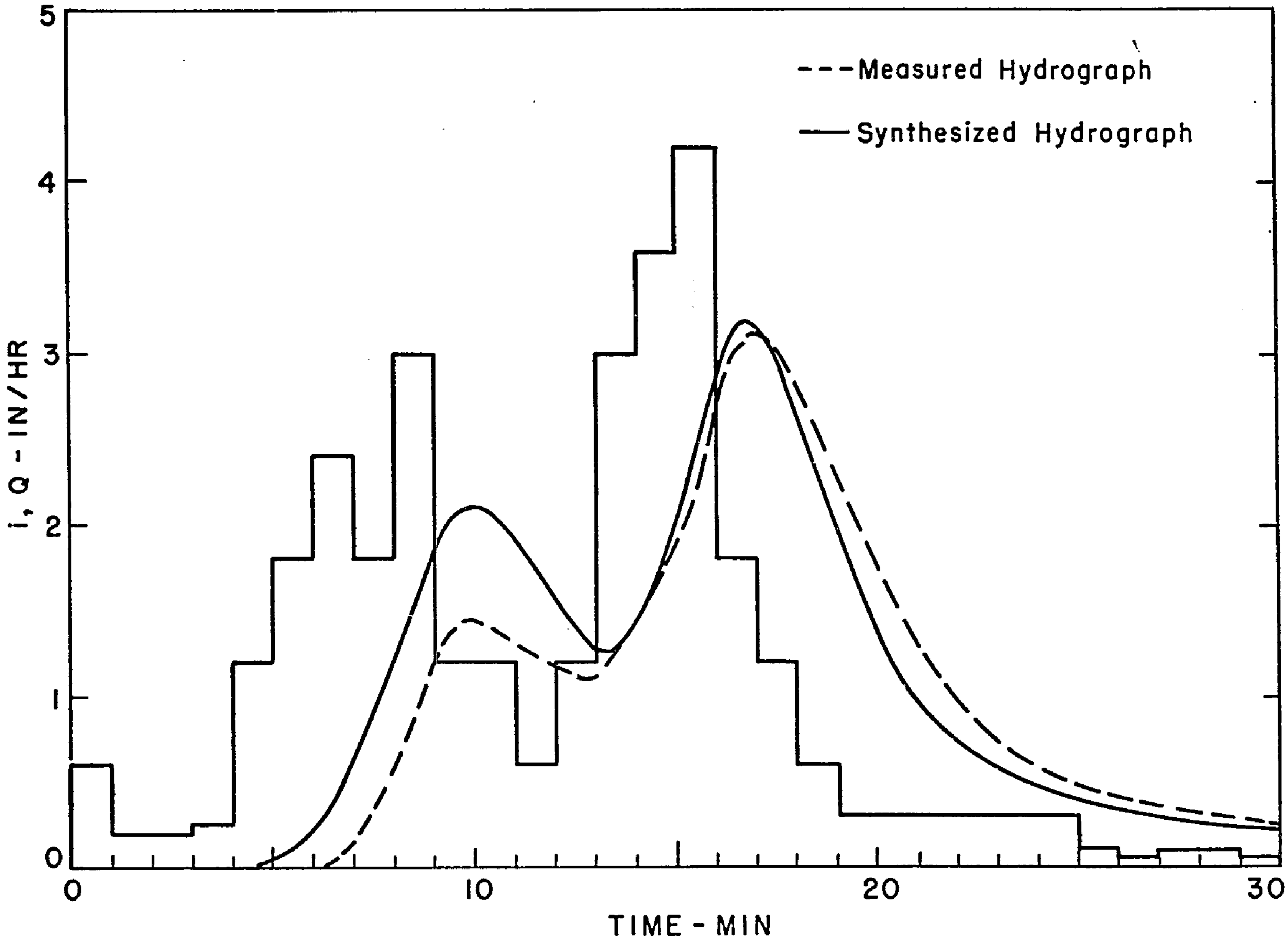

Figure 5-9 Comparison of Measured and Synthesized Runoff Hydrographs - Storm 3SPL1

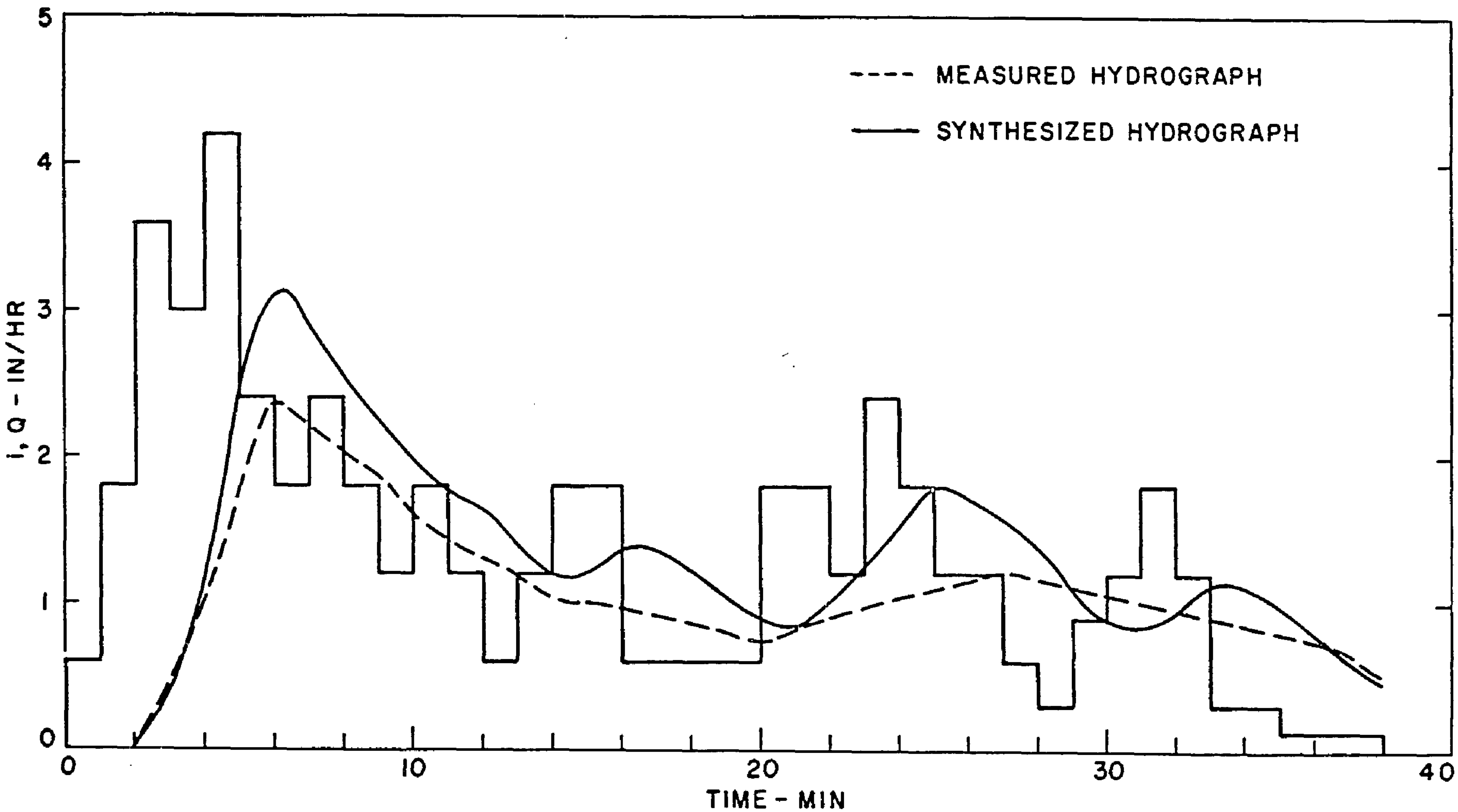

Figure 5-10 Comparison of Measured and Synthesized Runoff Hydrographs - Storm 6SPL1

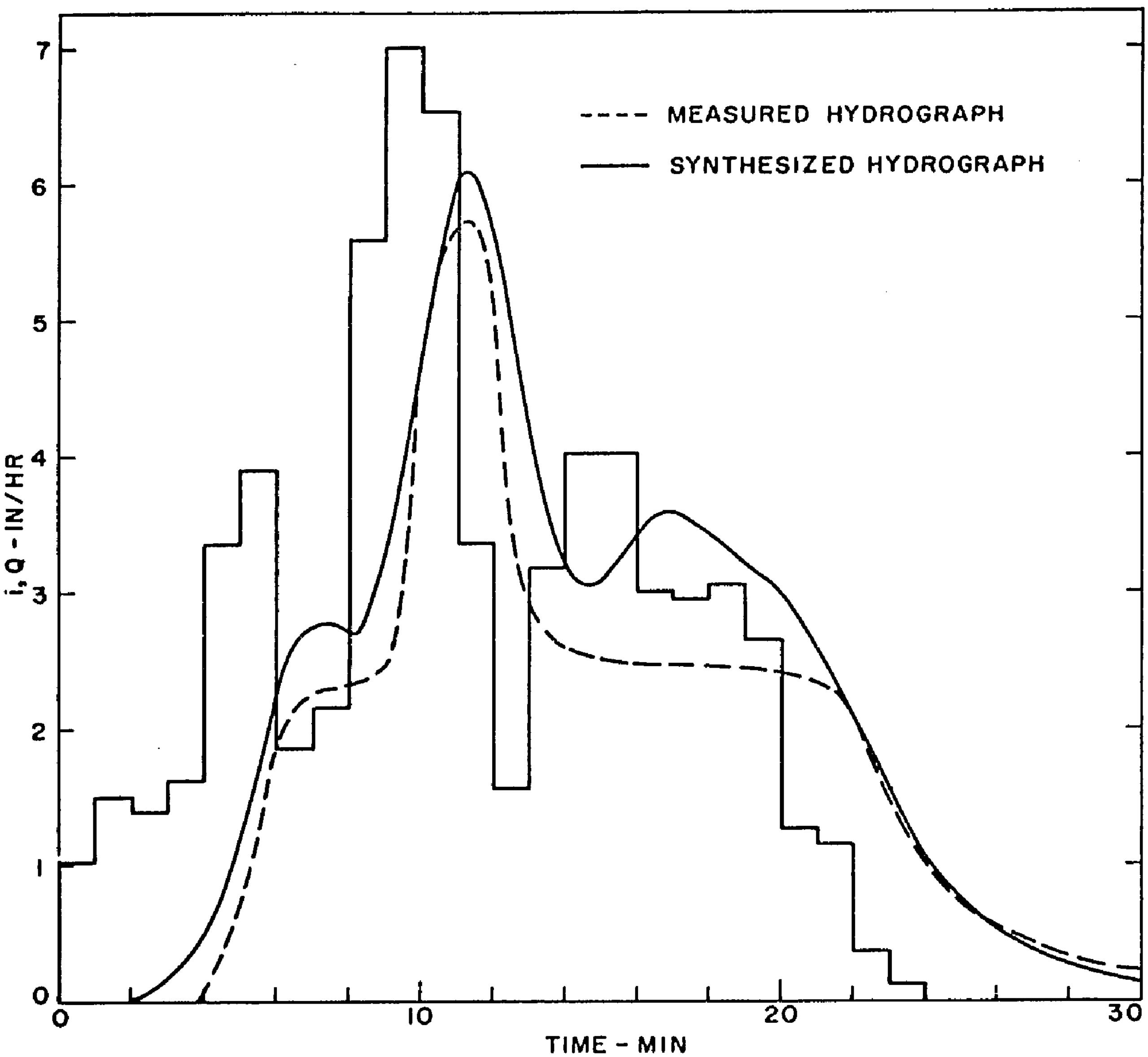

Figure 5-11 Comparison of Measured and Synthesized Runoff Hydrographs - Storm 13SPL1

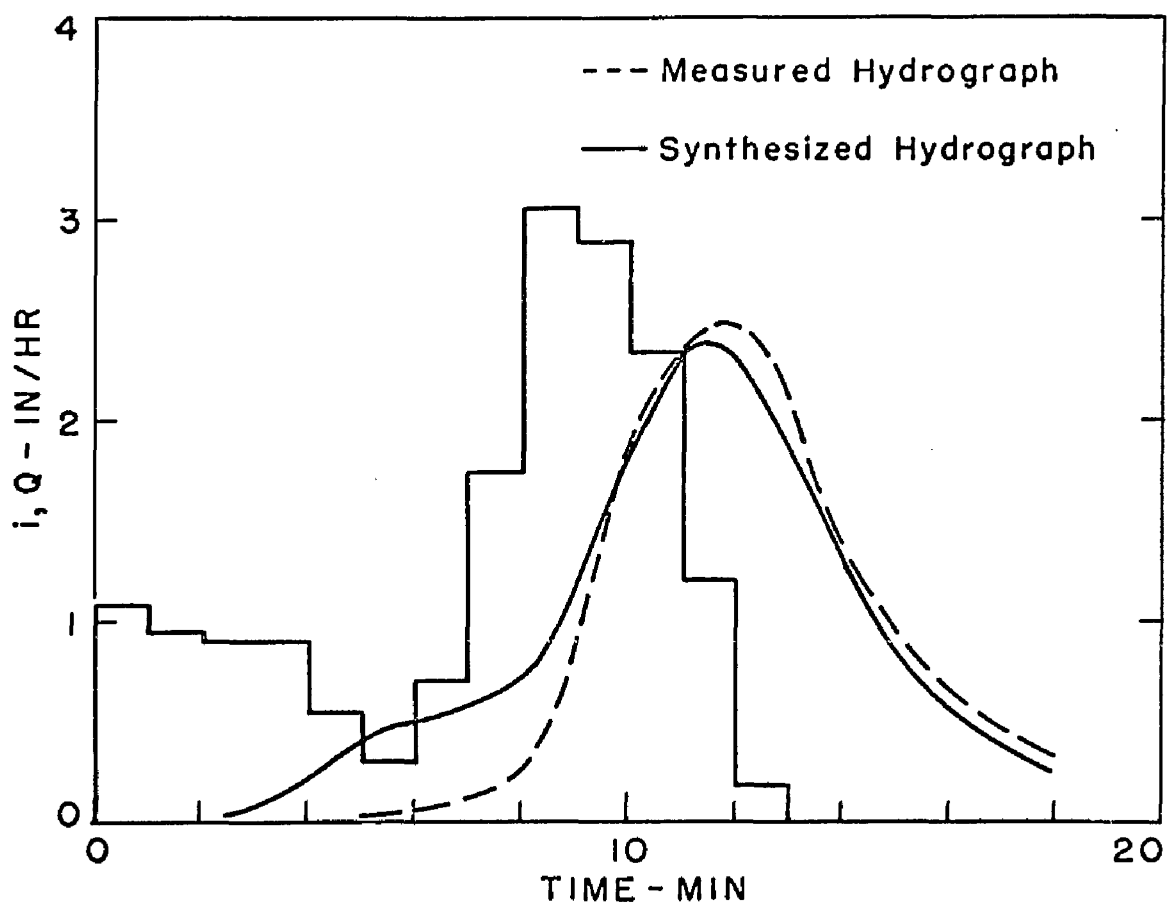

Figure 5-12 Comparison of Measured and Synthesized Runoff Hydrographs - Storm 18SPL1

Table 5-1. Physical Features of the Paved Inlet Areas

Station	Area - Acres	Imperviousness - %	Average Channel Slope	Average Surface Slope
N9	0.636	100	.0250	.0300
N12	0.955	100	.0092	.0300
SPL1	0.395	100	.0179	.0182

Runoff from area N9 is measured with a 9-inch Parshall flume; runoff from area N12, with a 12-inch Parshall flume. Rainfall on these areas is measured in increments of .01 inch by a tipping bucket rain gage. There is a separate rain gage immediately adjacent to each of the areas. The rainfall and runoff events at each gage are recorded on a single chart, there being separate recorders for the two areas. The chart speed of both recorders is 1/10 inch per minute, which allows the data to be read from the charts in one minute time intervals. The data obtained from the Newark areas are the most reliable record of rainfall and runoff that have ever been obtained for small paved areas. The gages were installed in 1959 by the University of Delaware in connection with a research project sponsored by the Delaware State Highway Department. Since 1962 the Department of Civil Engineering at the University of Delaware has very graciously cooperated with the Hopkins Storm Drainage Research Project in continuing the operation of the gages.

Runoff from area SPL1 is measured with a weir located in the storm water inlet to which this area drains. A digital record of depth of flow over the weir in increments of .02 ft is obtained. The record of rainfall and runoff is obtained on a single chart moving at a speed of $1\frac{1}{2}$ inches per minute. A more complete description of the instrumentation for the area is given in the 1962 Progress Report of The Storm Drainage Research Project.

The method used to divide each of the inlet areas into component parts was described in Chapter 2. The physical characteristics of each component are given in Chapter 2 in Figures 2-2 to 2-4.

Friction Coefficients for the Paved Inlet Areas

In addition to the physical characteristics of each component part of a drainage area, a friction coefficient must also be selected before the computations can be made. From the study described in Chapter 4, a laminar friction coefficient for overland flow components was obtained. For flow in gutters and pipes, friction coefficients are also needed.

Flow in gutters, swales and pipes is usually turbulent during storms, so the Manning equation has been used to compute the friction slope S_f. Coefficients in the Manning equation were selected on the basis of judgement; and the runoff hydrograph was computed and tested for sensitivity to changes of the Manning coefficient and the laminar flow C value as well. The coefficients selected for both laminar and turbulent flow are found in Table 5-2.

Table 5-2. Friction Coefficients Used in the Synthesis of the Runoff Hydrograph

Surface	Coefficient
Overland Flow (Impervious Surface)	C = 10
Concrete Pipes	n = .013
Concrete Curb and Gutter, Adjacent Asphalt Pavement	n = .020
Tar and Chips, Triangular Swale	n = .025

Sensitivity of the Computed Hydrograph to Changes in the Friction Coefficients

One of the most significant results obtained during this study resulted from a test of the sensitivity of the computed hydrograph to changes in the laminar and turbulent flow friction coefficients. It has been found that differences in n and C values, that would be expected in estimates of these coefficients by engineers exercising reasonably sound judgement, have very little effect on the computed runoff hydrographs for the drainage areas considered in this study. It appears that the geometry of the area and the continuity restrictions on the flow have a far more dominant effect on the shape of the hydrograph and the timing and magnitude of the peak rate of runoff than have the variations in friction coefficients estimated by engineering judgement.

The sensitivity analysis was made for the most significant storm observed on each of the areas N9 and SPL1. A factorial experiment was conducted where the Manning coefficient for gutter flow was assigned an upper and lower value, and the coefficient C also was assigned an upper and lower value. There are 4 possible combinations of upper and lower values of these coefficients, so, for each area, 4 hydrographs were computed using the various combinations of friction coefficients. In Table 5-3, these friction coefficients are given.

Table 5-3. Friction Coefficients for the Sensitivity Analysis

Coefficient	Storm 19 N9		Storm 13 SPL1	
	Upper Value	Lower Value	Upper Value	Lower Value
n, Gutter Flow	.017	.013	.025	.020
C, Overland Flow	20	15	20	15

The hydrographs computed using the friction coefficients in this table are shown in Figures 5-13 and 5-14 for storms 19 N9 and 13 SPL1 respectively. For convenience the computed hydrographs obtained using the friction coefficients given in Table 5-2 are shown in these figures by the solid curves. Only where there was sufficient difference to draw additional curves are portions of the hydrographs obtained in the sensitivity analysis also shown. For all computations, the value of n for pipe flow was assumed to be .013.

It appears there is also another result of the sensitivity analysis that was not anticipated but is nevertheless quite logical and extremely important. For the areas studied, the runoff hydrograph seems to be more sensitive to the value of n used for the gutter flow than to the value of C used for overland flow. For larger areas, where the lengths of gutter and pipe flow from the upper end of the area become very long relative to the lengths of overland flow, it would be expected that the runoff hydrograph would be more sensitive to n values over the entire length of flow than to C values. But, even for paved areas smaller than one acre, it appears that the nature of the gutter flow has a greater effect on the runoff hydrograph than the friction associated with overland flow.

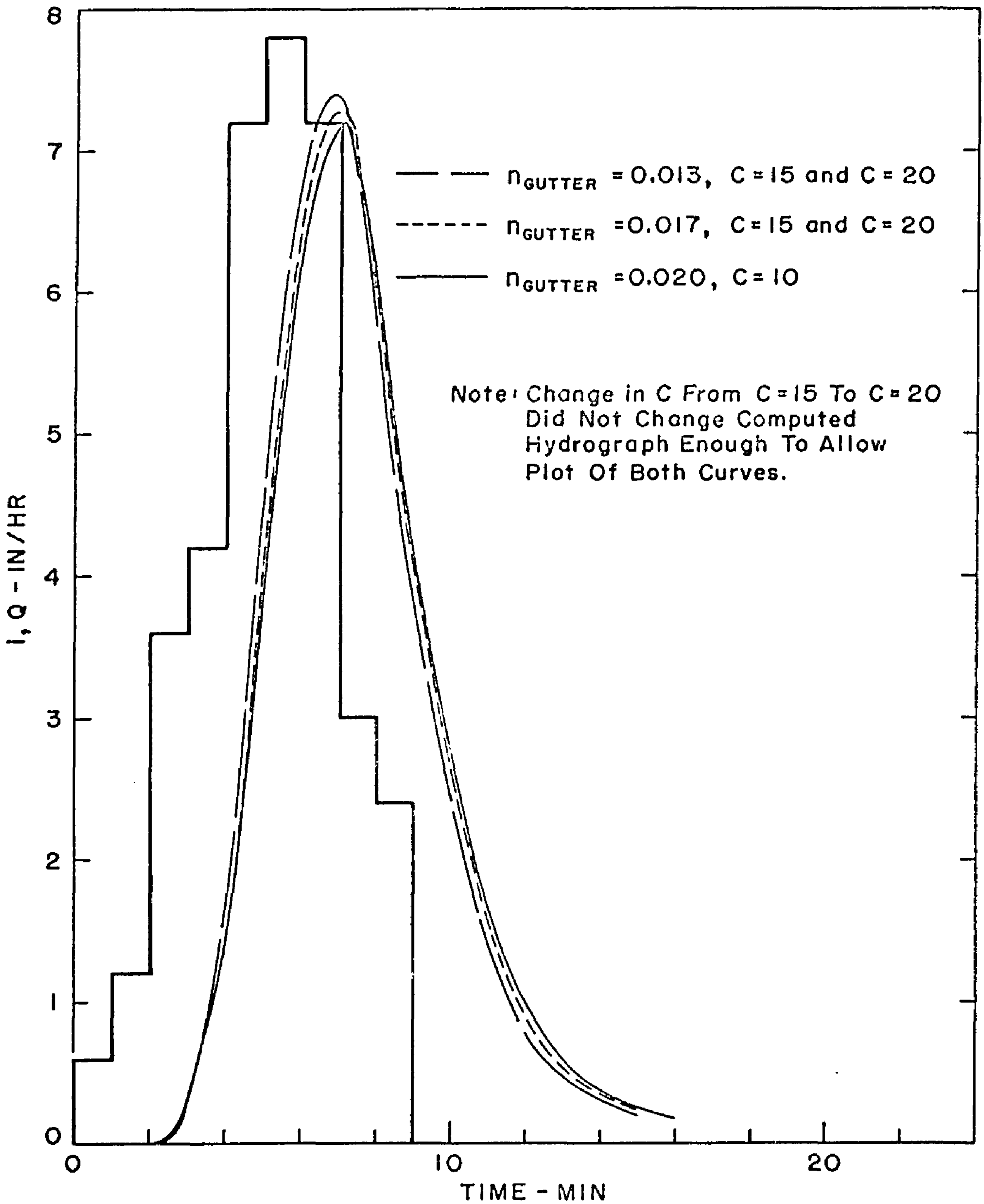

Figure 5-13 Sensitivity of Computed Hydrographs to Friction Coefficients - Storm 19N9

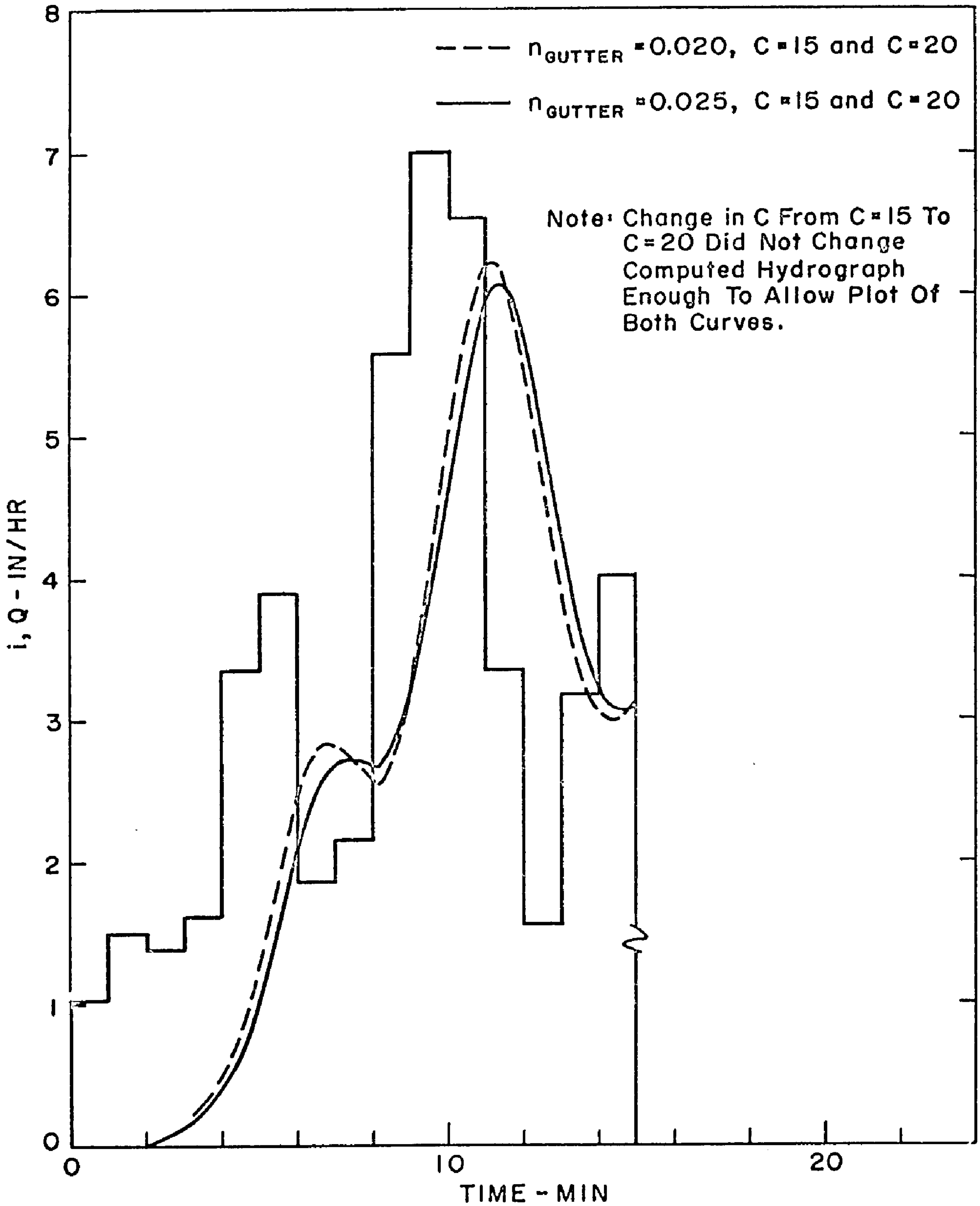

Figure 5-14 Sensitivity of Computed Hydrographs to Friction Coefficients - Storm 13SPL1

Chapter 6

A COMPARISON OF SEVERAL METHODS OF ESTIMATING RUNOFF RATES FROM SMALL URBAN AREAS

Because large amounts of computer time are required to solve the equations of flow, less complex methods of describing drainage area behavior are needed for many practical problems. This chapter has been prepared to illustrate how well the results obtained by solving the equations of flow compare with results from other, less complex methods.

Initially, a unit hydrograph method was used to estimate the complete runoff hydrograph for several of the storms studied in the previous chapter. The hydrographs computed with the unit hydrograph method are illustrated in Figures 6-7 to 6-12. Also shown in these figures are the measured hydrographs and the hydrographs synthesized using the equations of flow.

Other methods have also been used to compute the peak runoff rate for each of the 12 storms. These results are given in Table 6-1, and the statistical distributions of differences between the computed and measured peak rates are shown in Figure 6-14.

Before proceeding to present the results obtained with the unit hydrograph method, the unit hydrograph method and the procedure used to obtain the unit hydrograph are first described.

<u>The Unit Hydrograph</u>

The unit hydrograph method of estimating the runoff hydrograph approximates the behavior of a drainage area as a linear system. The unit hydrograph is the hydrograph of a unit volume of surface runoff from a storm producing surface runoff at a constant rate and lasting for a unit period of time. If the duration of the storm is one minute, and a unit volume of surface runoff occurs, the resulting hydrograph is called the 1-minute unit hydrograph. The hydrograph of runoff for any one minute storm of constant intensity can be computed from the 1-minute unit hydrograph by multiplying the ordinates of the 1-minute unit hydrograph by the proper coefficient. For example, let a unit volume of runoff be equal to one inch. Then a storm of 1 in/hr and lasting one minute produces 1/60 of an inch of surface runoff. The runoff hydrograph for this one minute storm is obtained by multiplying the ordinates of the 1-minute unit hydrograph by the coefficient 1/60.

Any storm lasting for many minutes may be described as a sequence of one minute storms (see Figure 6-1). The runoff hydrograph from each one minute storm in this sequence can be obtained as in the preceeding example. By superimposing the runoff hydrograph from each of the one minute storms, the runoff hydrograph for the complete storm may be obtained.

From the unit hydrograph for any duration of uniform rain, the unit hydrograph for any other duration may be obtained.

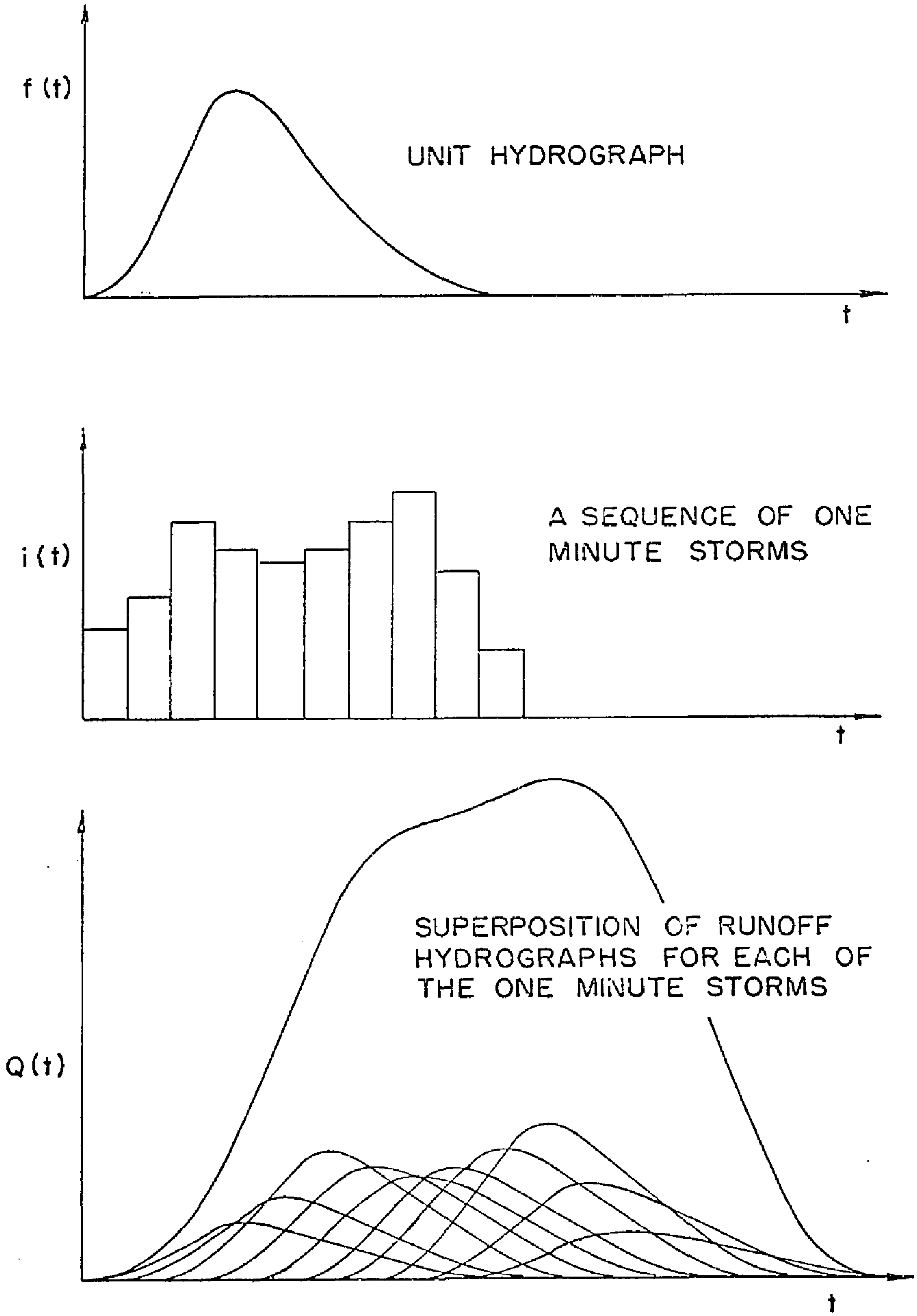

Figure 6-1 The Unit Hydrograph Description of the Runoff Process

As the duration becomes shorter, the resulting unit hydrograph approaches the instantaneous unit hydrograph. The instantaneous unit hydrograph (IUH) is the hydrograph of runoff that would result if an inch of water were spread uniformly over an area and then allowed to run off.

The IUH also has another, very significant meaning from a mathematical point of view. The ordinates of the IUH represent the relative effect of antecedent rainfall intensities on the runoff rate at any instant of time. By plotting the IUH with time increasing to the left rather than to the right (see Figure 6-2), and then superimposing this plot over the rainfall hyetograph (plotted with time increasing to the right as in Figure 6-2); the relative weight given to antecedent rainfall intensities (as a function of time into the past) is easily observed. In other words, the runoff rate at any time is computed as a weighted average of the previous rainfall intensities. Therefore, the computed runoff hydrograph is the weighted, moving average of the rainfall pattern, and the weighting function is the time-reversed image of the unit hydrograph.

Stated mathematically, the runoff rate at any time is given by

$$Q(t) = \int_0^t f(\tau) \quad i(t-\tau)d\tau \qquad (6\text{-}1)$$

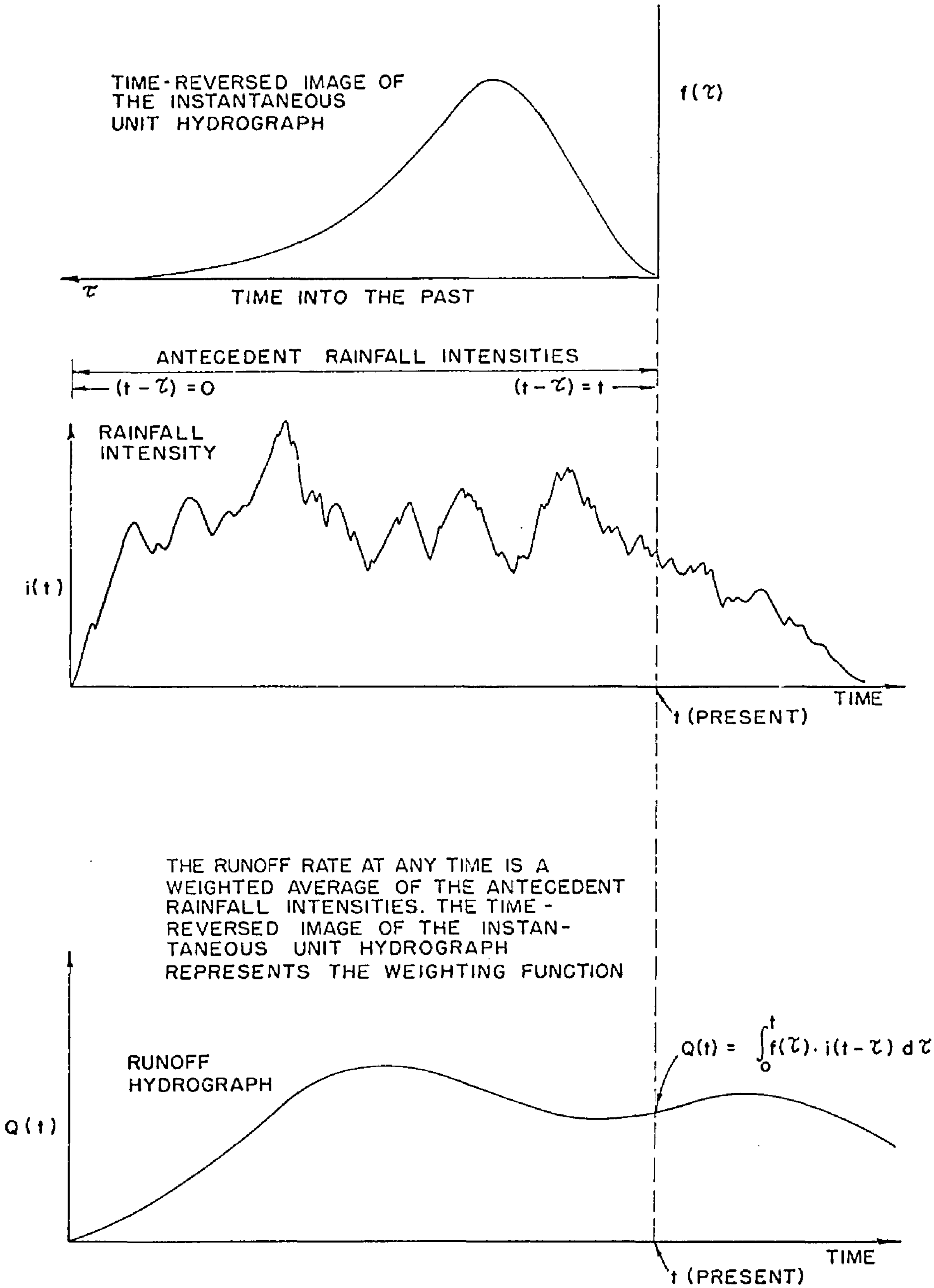

Figure 6-2 Calculation of Runoff Rates with the Instantaneous Unit Hydrograph

where $Q(t)$ is the surface runoff rate at time t, $f(\tau)$ is the ordinate of the IUH at time τ, and $i(t-\tau)$ is the rainfall intensity (after abstraction of the appropriate infiltration losses, etc.) at time $t-\tau$. The variable τ represents time into the past so that time $t-\tau$ occurs before time t. The limits on the integral allow τ to vary between the present time (i.e., $\tau = 0$, $t-\tau = t$), and the time rainfall began (i.e., $\tau = t$, $t-\tau = 0$). The integral gives a continuous weighting of previous rainfall intensities by the ordinates of the IUH.

Synthesis of the Unit Hydrograph

For gaged areas, there are numerous methods for obtaining a unit hydrograph from measurements of rainfall and runoff. For small ungaged urban areas, however, accurate methods for obtaining a unit hydrograph have not been developed. To distinguish between the unit hydrograph obtained from a record of rainfall and runoff and the unit hydrograph obtained for an ungaged area, the unit hydrograph for an ungaged area is known as a synthetic unit hydrograph. Studies by Dooge (1959), Nash (1959) and others have suggested equations to describe the shapes of synthetic unit hydrographs, but methods for evaluating unknown parameters in these equations remain quite subjective for urban areas.

To illustrate how well the unit hydrograph method can be expected to estimate the runoff hydrograph for an ungaged urban area, the equations of flow have been used to obtain synthetic unit hydrographs. The shapes of the synthetic unit hydrographs obtained in this way are similar to the shape of the Gamma or Peason Type III statistical distribution. The equation of this statistical distribution has been suggested by both Dooge (1959) and Nash (1959) to describe the instantaneous unit hydrograph.

The equations of flow have been used to simulate a storm of constant rainfall intensity lasting until the runoff reached steady state. Then, the rainfall was ended and the runoff allowed to recede. From the resulting synthetic runoff hydrograph, two estimates of the IUH were obtained. An IUH was derived from the rising part of this runoff hydrograph, and another estimate of the IUH was derived from the recession part.

If the theory of the unit hydrograph exactly represented the behavior of a drainage area (that is, if a drainage area behaved as a linear system), these two estimates of the IUH would be identical.

The ordinates of the IUH have been obtained from the response to a constant rainfall intensity, I, by using the equation

$$\frac{dQ}{dt} = f(t) \cdot I \qquad (6\text{-}2)$$

which is derived by differentiating Equation 6-1. That separate estimates of the IUH can be obtained from the rising and recession parts of this runoff hydrograph requires a mathematical description of the pulse of rain which lasted for a total duration of time, T.

Let the function U(t) be a unit step function, i.e.,

$$U(t) = 0, \ t < 0$$
$$= 1, \ t \geq 0 \qquad (6\text{-}3)$$

Then a pulse of constant rainfall intensity, I, and lasting for a duration, T, can be described as

$$i(t) = I \cdot U(t) - U(t-T) \qquad (6\text{-}4)$$

Substituting 6-4 into 6-1 and differentiating 6-1 gives

$$\frac{dQ(t)}{dt} = I \cdot f(t), \quad t < T \qquad (6\text{-}5)$$

$$\frac{dQ(t)}{dt} = I \cdot f(t) - f(t-T), \quad t > T \qquad (6\text{-}6)$$

After runoff reaches steady state, and before time T, $\frac{dQ}{dt} = 0$. Therefore, f(t) = 0 after steady state is reached. Thus, in Equation 6-6, the term f(t) = 0 since this equation applies for t > T and f(t) = 0 if t is larger than T. Introducing subscripts to distinguish between the two estimates of the IUH, Equations 6-5 and 6-6 are rewritten and simplified.

$$f_1(t) = \frac{\frac{dQ}{dt}}{I} , \quad t < T \tag{6-7}$$

$$f_2(t-T) = - \frac{\frac{dQ}{dt}}{I} , \quad t > T \tag{6-8}$$

In Figures 6-3 to 6-5 the results of simulating the pulse of rainfall for each of three drainage areas are shown. The IUHs derived from Equations 6-7 and 6-8 are shown below the corresponding rising and recession portions of the runoff hydrograph.

A rainfall intensity of 3 inches per hour was arbitrarily chosen, and different unit hydrographs would have resulted if a different intensity were selected. To illustrate the effect of rainfall intensity on the unit hydrograph, intensities of 1 in/hr and 6 in/hr were also simulated for area SPL1. The resulting instantaneous unit hydrographs are shown in Figure 6-6. As would be expected, the unit hydrographs indicate a quicker response of the area to larger rainfall intensities.

Runoff Hydrographs Computed with the Synthetic Unit Hydrograph

Runoff hydrographs for several of the storms studied in the previous chapter were computed using 1-minute unit hydrographs

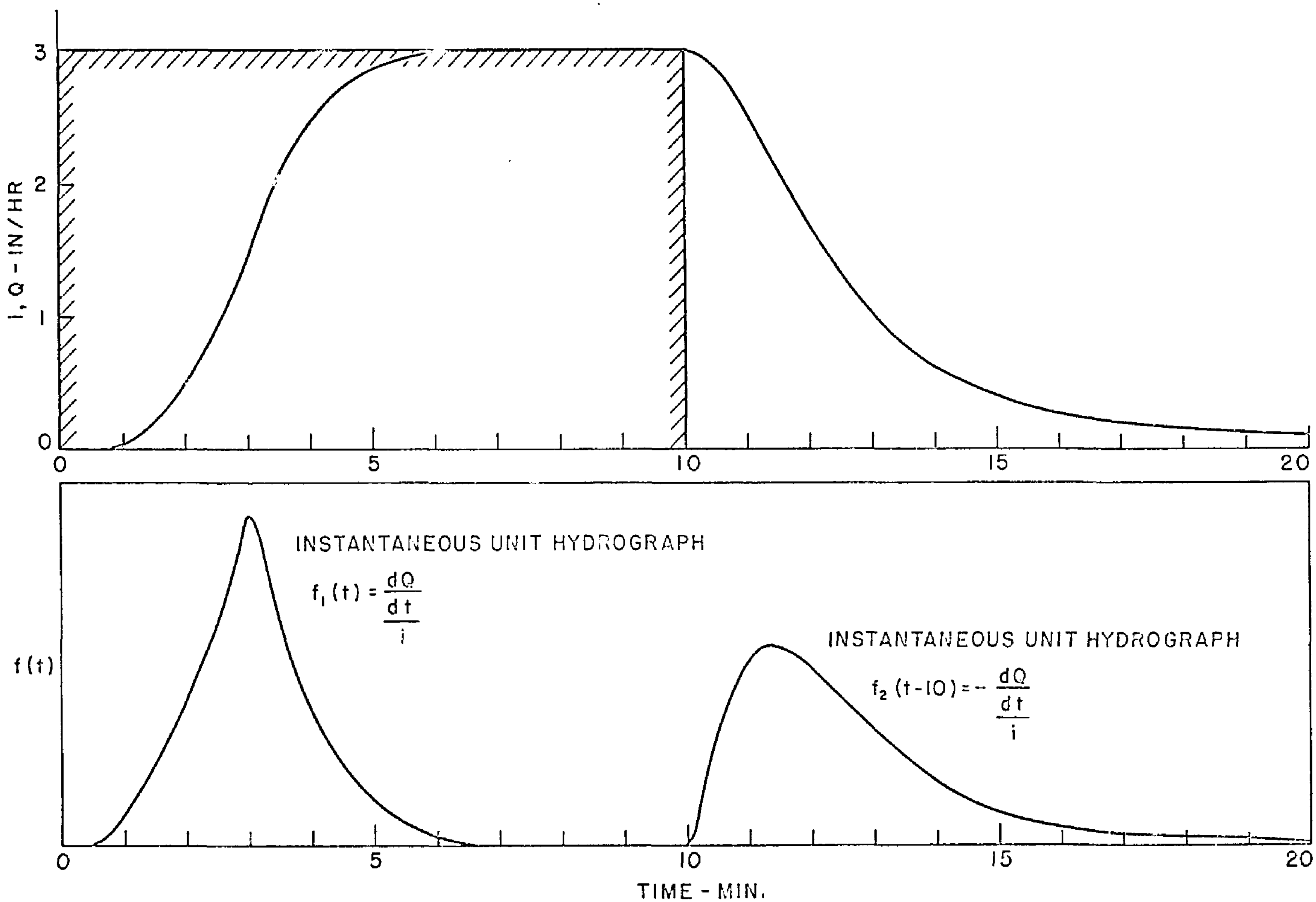

Figure 6-3 Instantaneous Unit Hydrograph - Inlet Area N9

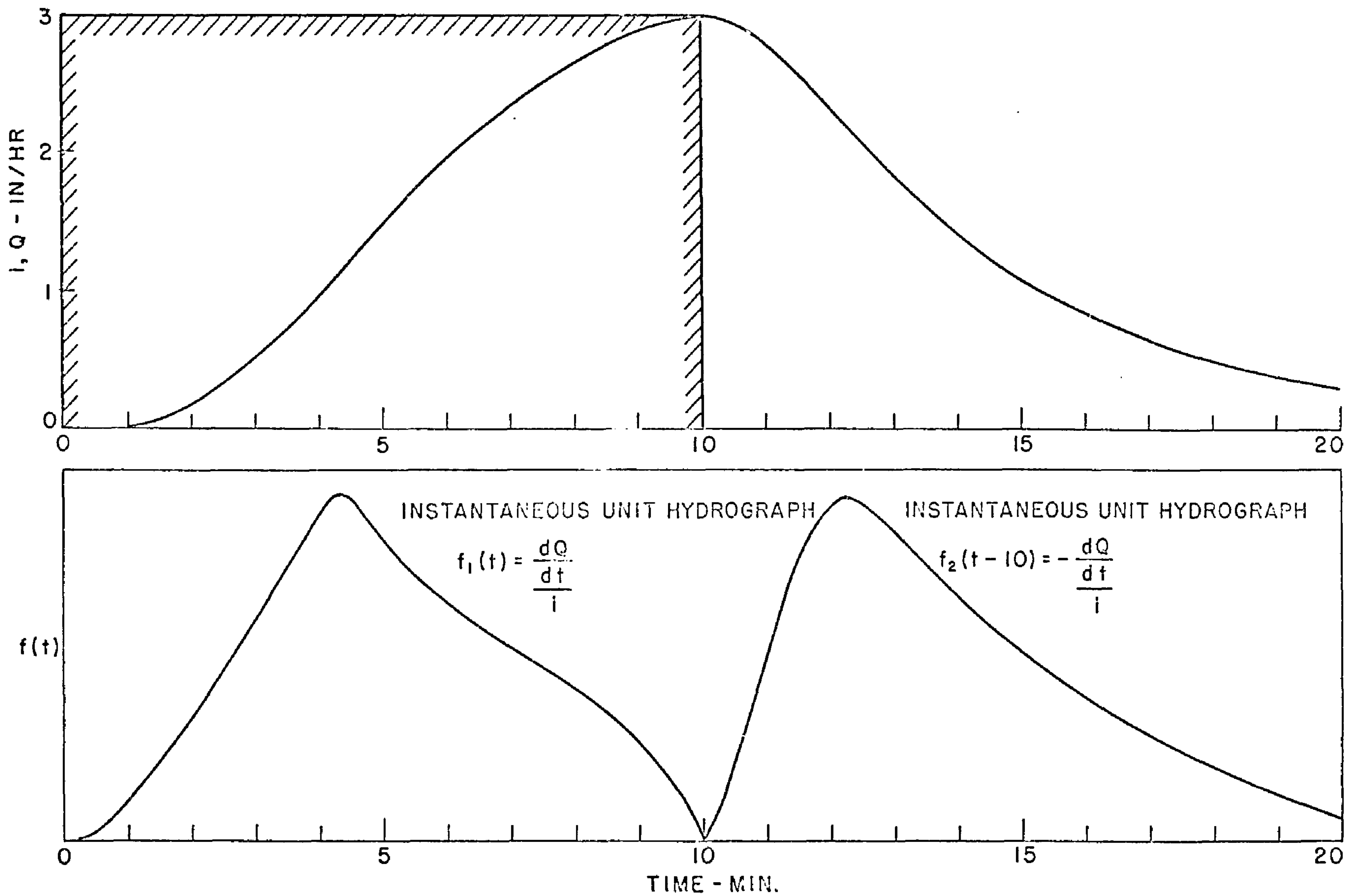

Figure 6-4 Instantaneous Unit Hydrograph - Inlet Area N12

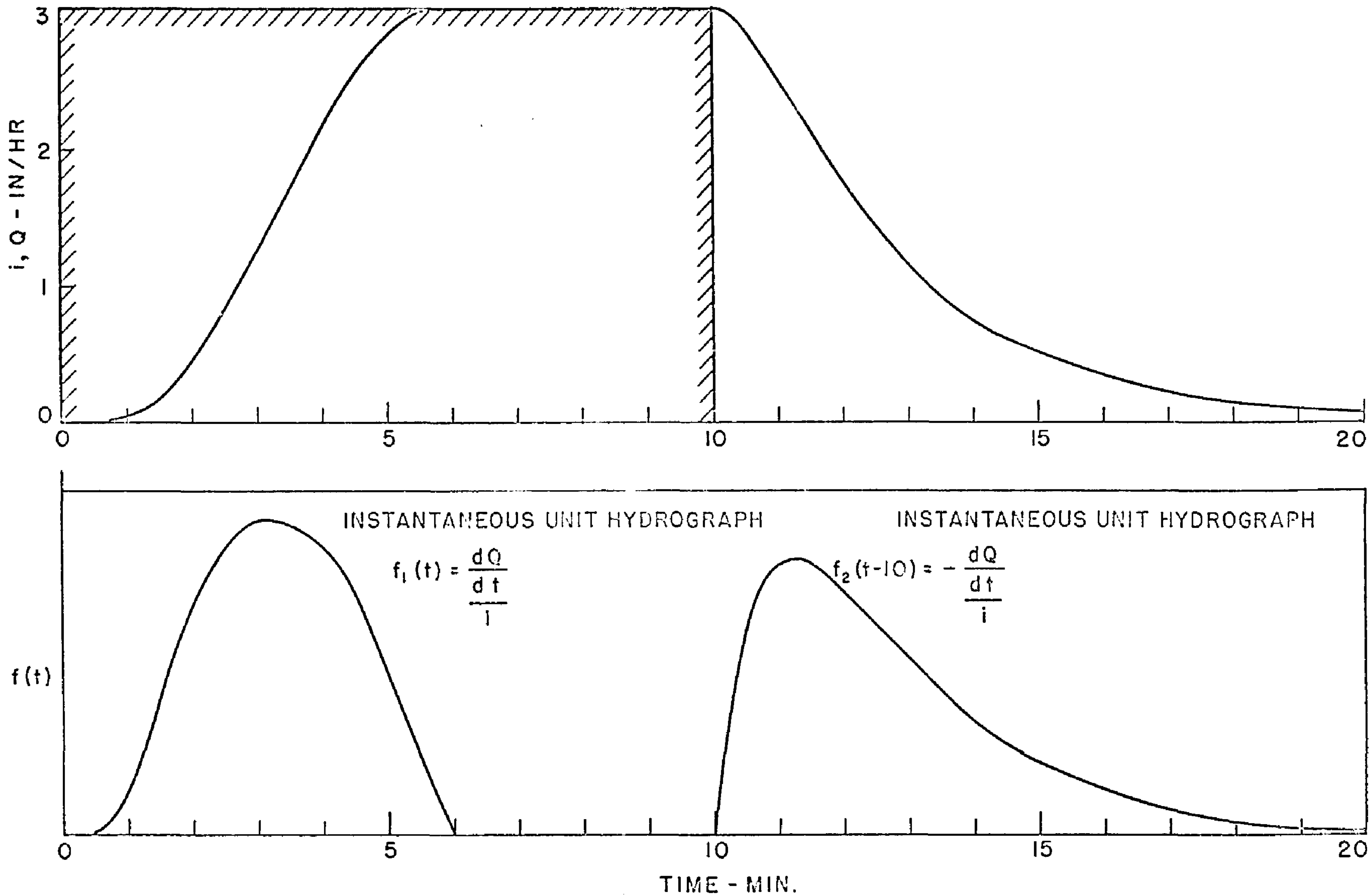

Figure 6-5 Instantaneous Unit Hydrograph - Inlet Area SPL1

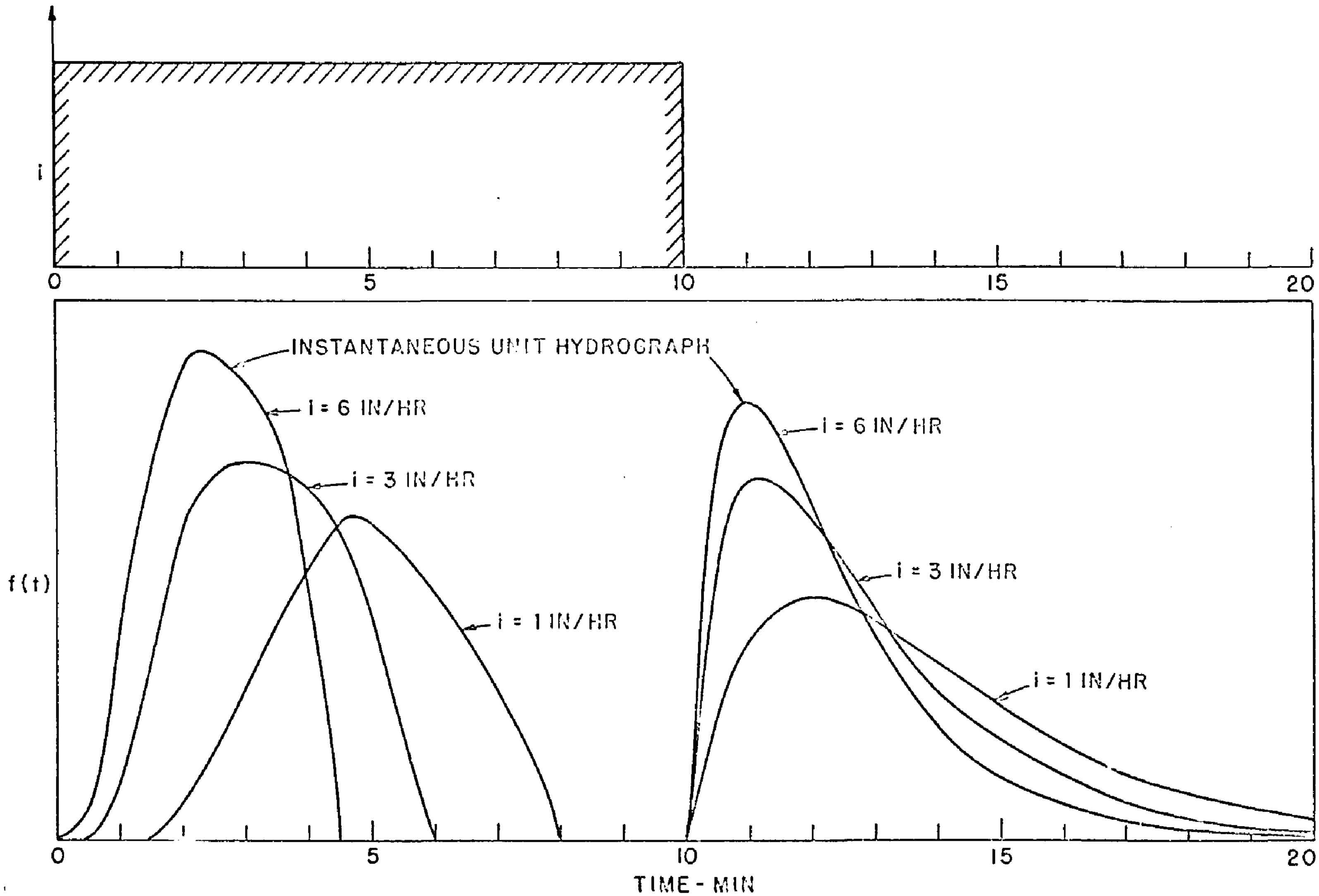

Figure 6-6 Sensitivity of Instantaneous Unit Hydrographs to Rainfall Intensity - Inlet Area SPL1

derived from the instantaneous unit hydrographs. For each storm, hydrographs were computed using both the rising and the recession unit hydrographs. The computed hydrographs are presented in Figures 6-7 to 6-12. Shown also on these figures are measured hydrographs and the synthetic hydrographs computed by the equations of flow. Since the equations of flow were used to derive the unit hydrograph, the runoff hydrographs represent an approximation to the solution of the equations of flow.

In all cases the curves from the recession unit hydrograph give the best approximation to the measured hydrograph.

It is interesting how well the unit hydrograph approximates the solutions to the equations of flow. It is significant to recall that in Figure 6-6 the two most similar instantaneous unit hydrographs are those obtained from the recession to the square pulse, particularly the two labeled 3 in/hr and 6 in/hr. This suggests that the unit hydrograph approximation obtains its best accuracy during intense rain when there already is water stored on the area. To show that the difference between the 3 in/hr and 6 in/hr recession instantaneous unit hydrographs is small, both of these unit hydrographs have been used to compute the runoff hydrograph from Storm 13 SPL1. As illustrated in Figure 6-13 there is little difference between the two hydrographs.

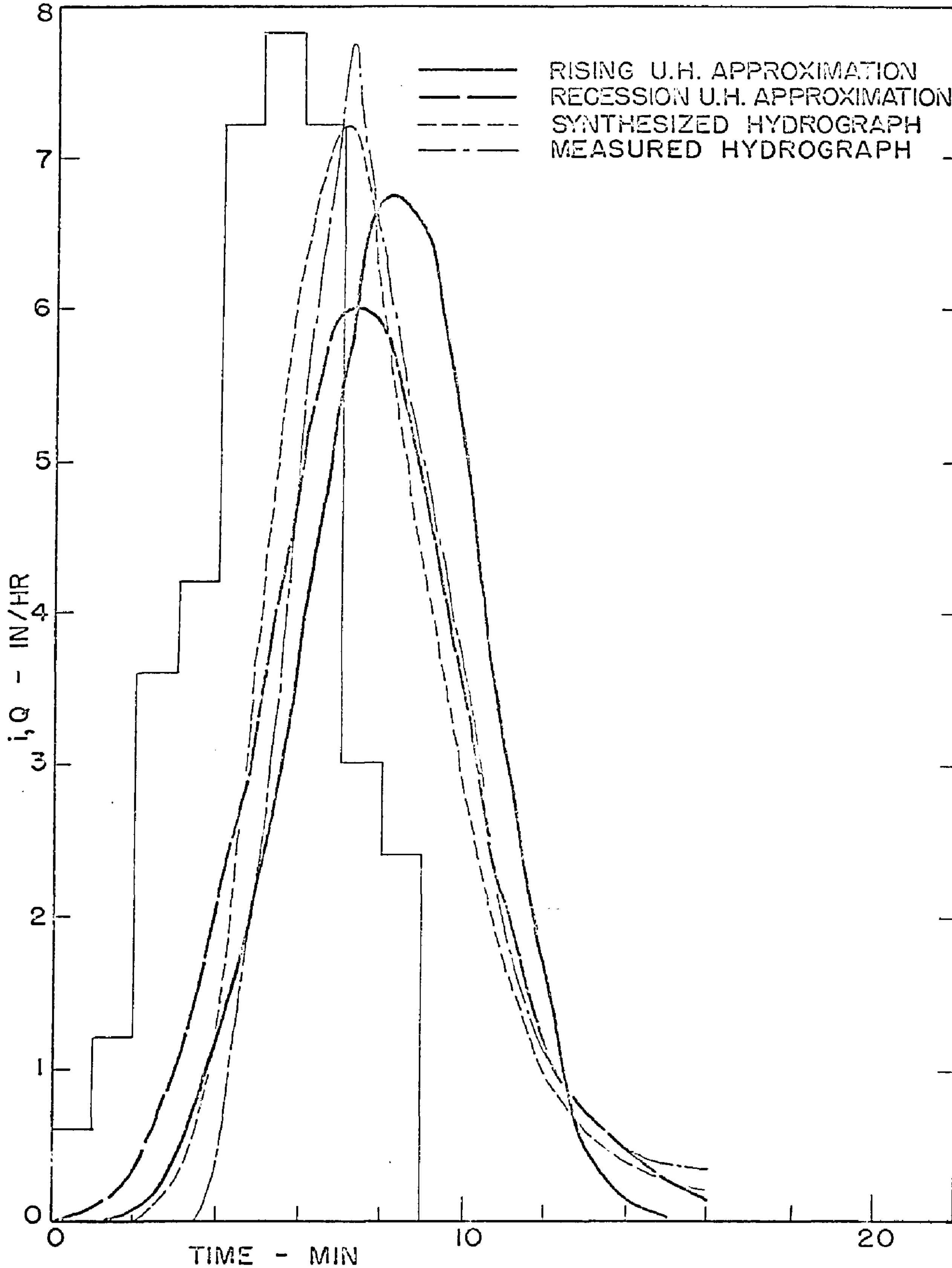

Figure 6-7 Unit Hydrograph Approximation to the Solution of the Equations of Flow - Storm 19N9

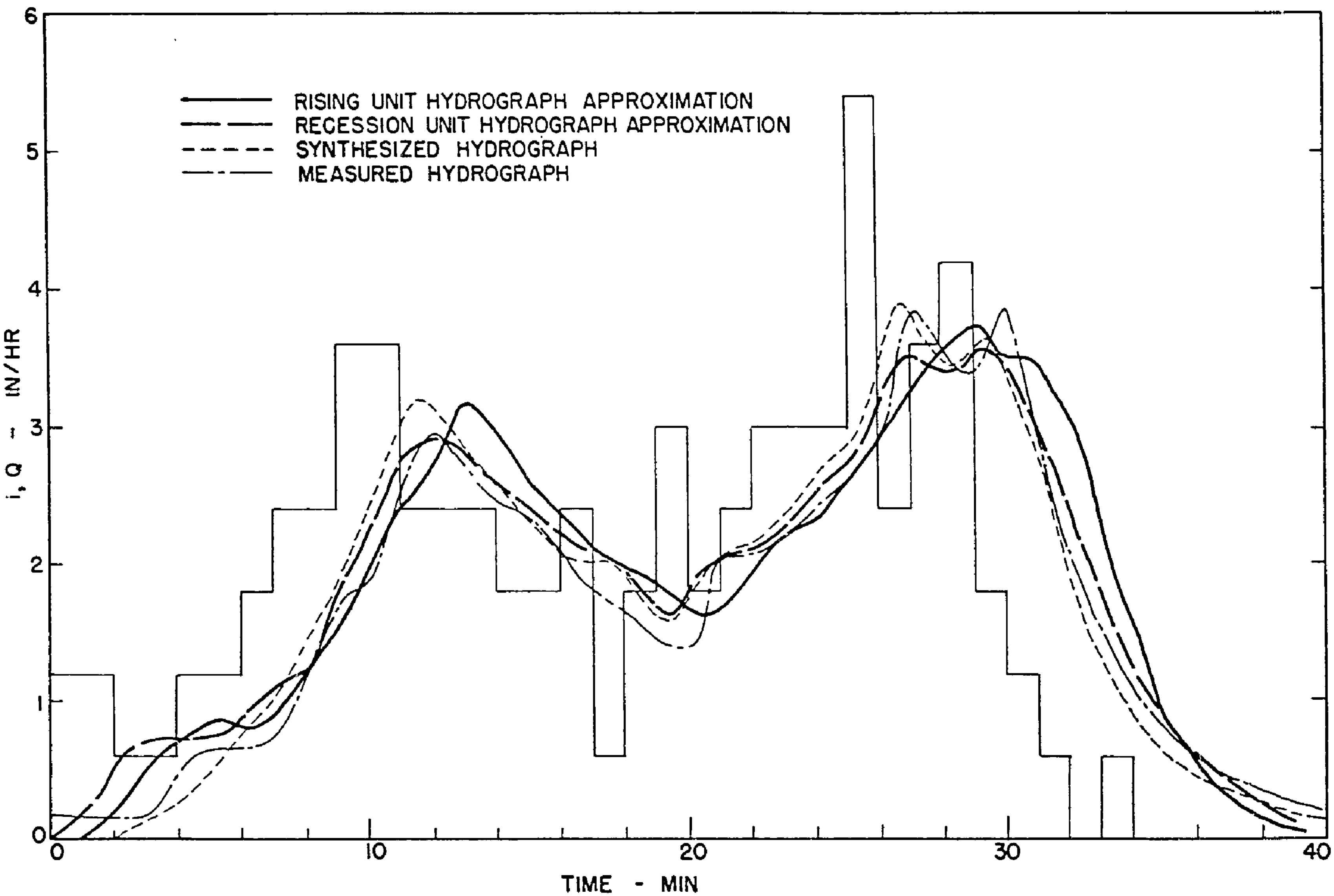

Figure 6-8 Unit Hydrograph Approximation to the Solution of the Equations of Flow - Storm 23N9

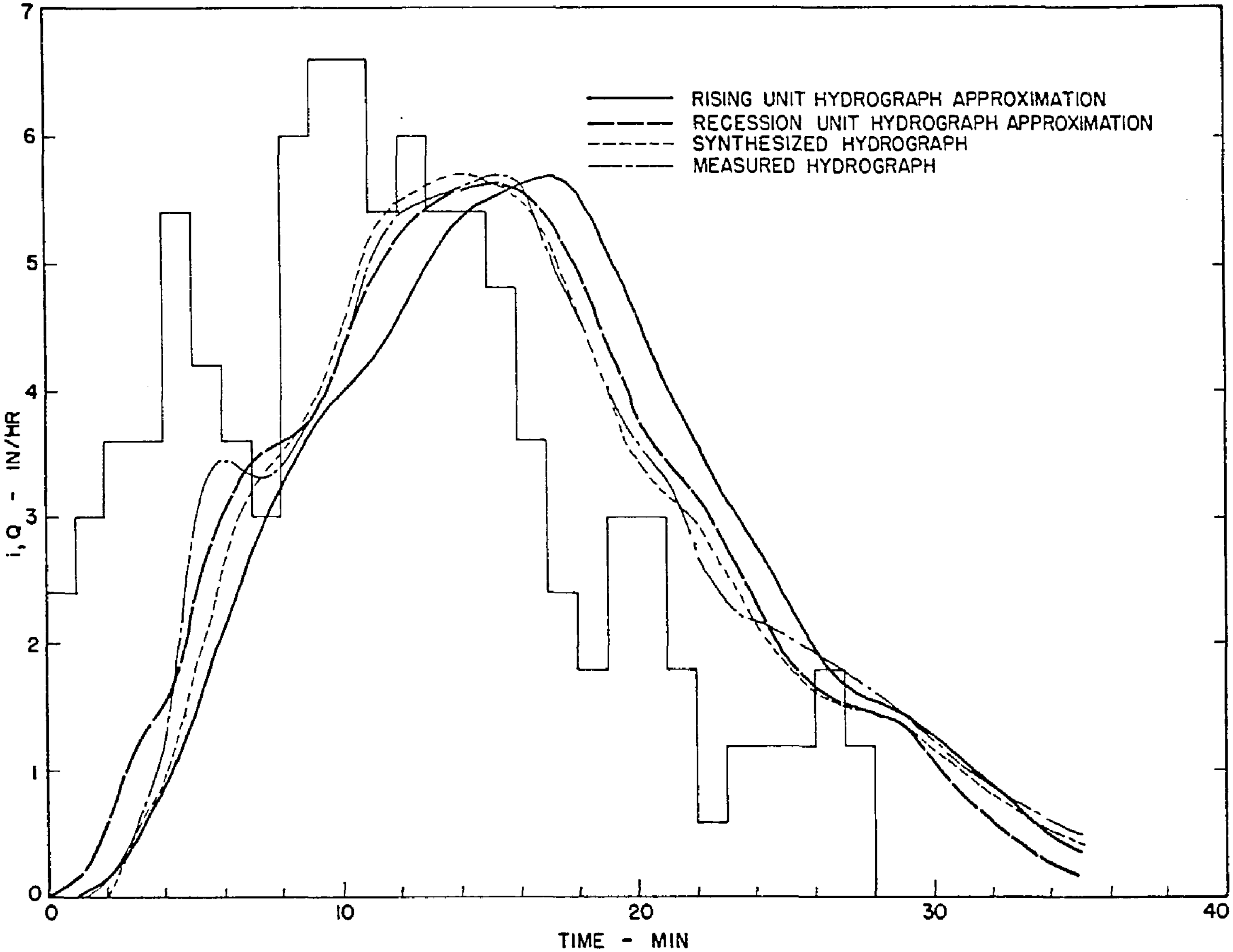

Figure 6-9 Unit Hydrograph Approximation to the Solution of the Equations of Flow - Storm 8N12

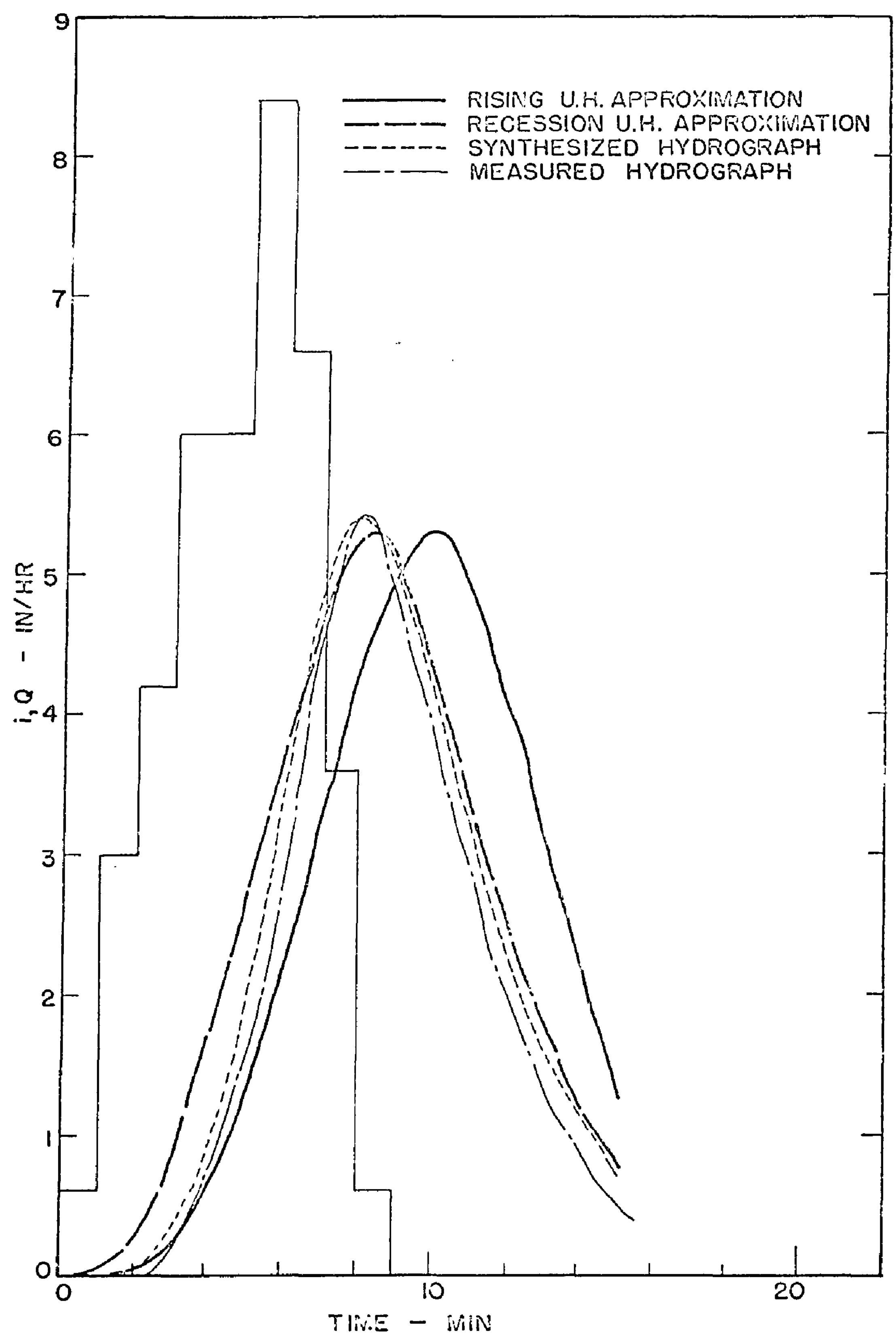

Figure 6-10 Unit Hydrograph Approximation to the Solution of the Equations of Flow - Storm 19N12

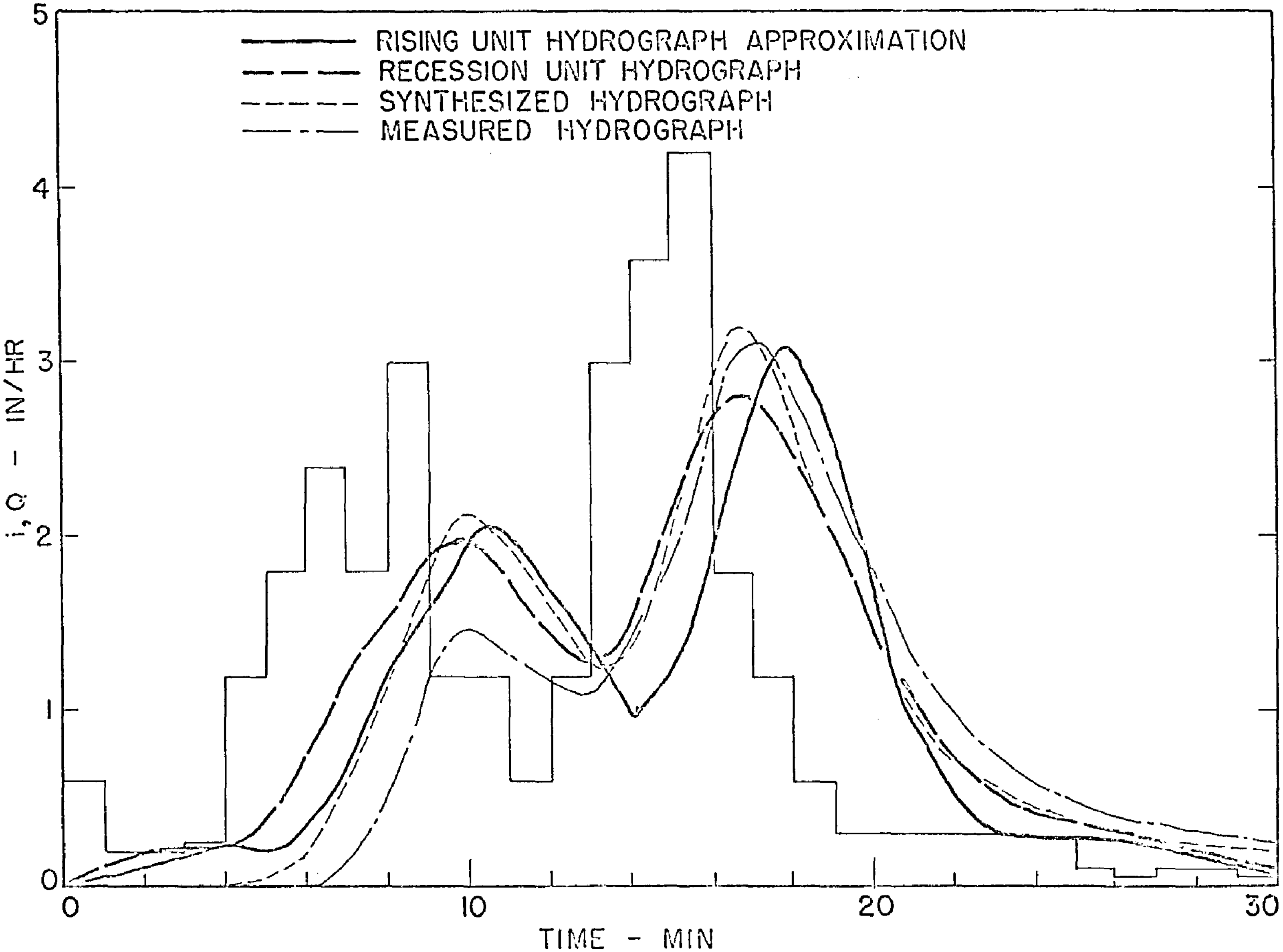

Figure 6-11 Unit Hydrograph Approximation to the Solution of the Equations of Flow - Storm 3SPL1

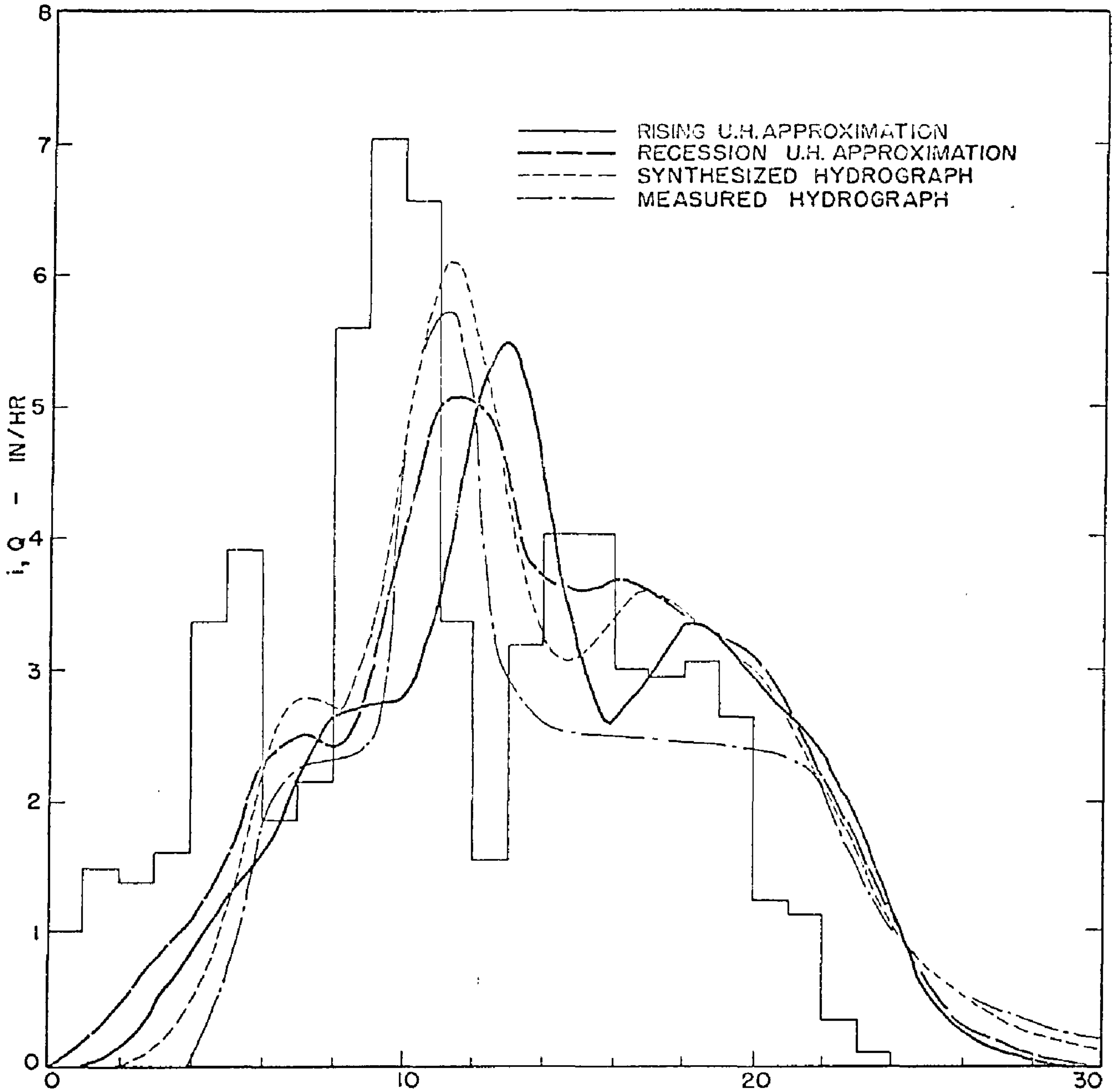

Figure 6-12 Unit Hydrograph Approximation to the Solution of the Equations of Flow - Storm 13SPL1

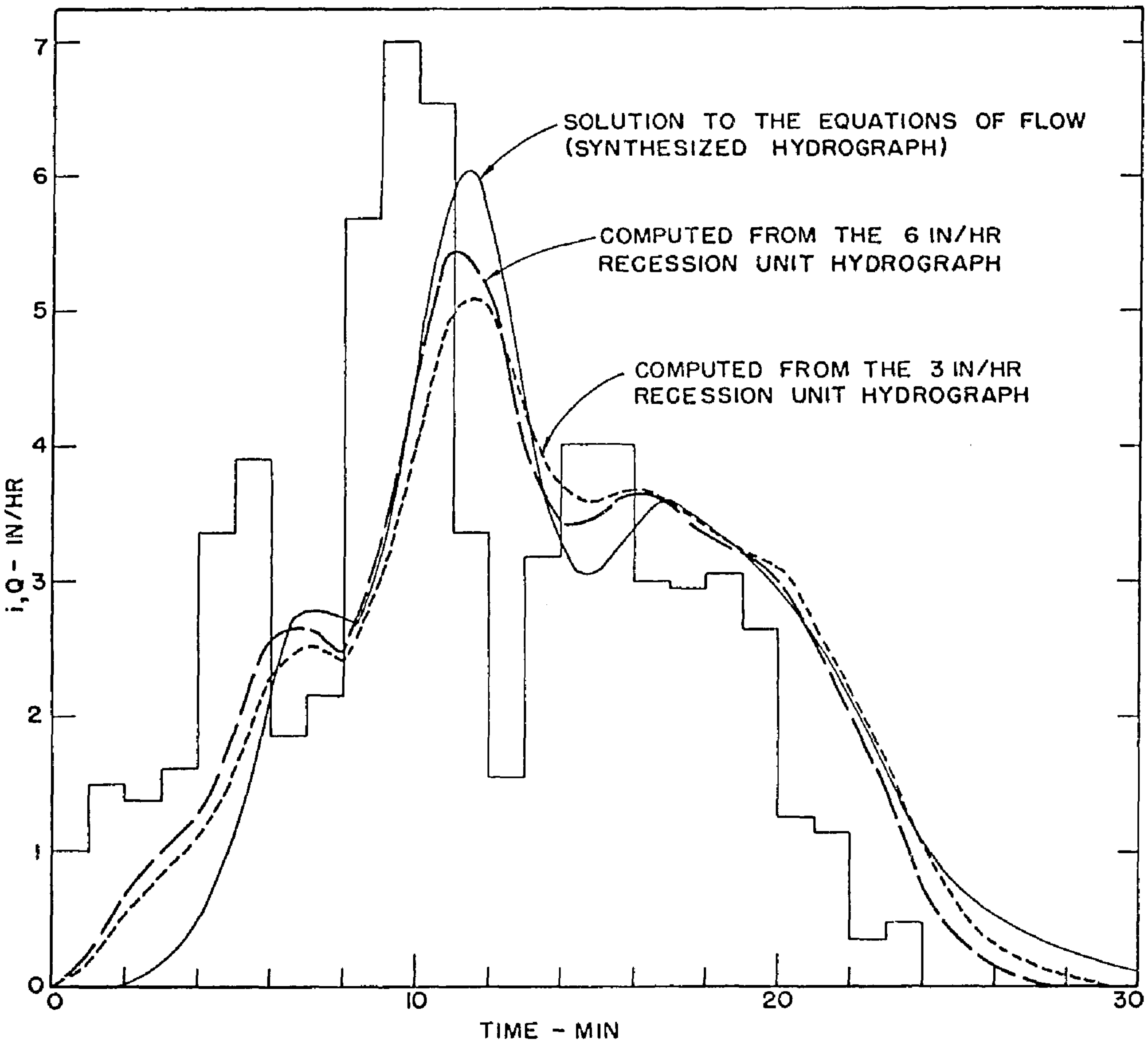

Figure 6-13 Sensitivity of the Computed Runoff Hydrograph to Changes in the Unit Hydrograph - Storm 13SPL1

It later will be shown that the rising unit hydrograph gave better estimates of the peak runoff rate than did the recession unit hydrograph. Generally, the peak runoff rate given by the recession unit hydrograph was too low. It is interesting that in Figure 6-13, where hydrographs computed from the 3 in/hr and 6 in/hr recession unit hydrographs are compared, the greatest difference was in the neighborhood of the peak. This would suggest that better estimates of the peak runoff rate could be obtained using the 6 in/hr recession unit hydrograph rather than 3 in/hr, and good agreement with the remainder of the hydrograph also could be maintained.

It is not surprising that the approximation derived from the recession unit hydrograph is not as good at the beginning of rainfall as after the first few minutes of the storm have elapsed. During the first few minutes of every storm, runoff rates computed from the recession unit hydrograph exceeded the measured and synthesized runoff rates. When using the unit hydrograph to study infiltration losses, this phenomena at the beginning of the storm makes it appear that initial infiltration rates are larger than actually have occurred. Thus, infiltration rates determined from records of rainfall and runoff are <u>apparent</u> rates of infiltration and may not represent actual infiltration rates if a unit hydrograph has been used to describe the mechanics of surface runoff.

Two Additional Methods for Estimating the Peak Runoff Rate

For each of the 12 storms studied in the previous chapter, several methods have been used to obtain estimates of the peak runoff rate. Two of these estimates already have been obtained from the unit hydrograph method where both the rising and recession unit hydrographs were used to compute runoff hydrographs. Also the equations of flow were used to synthesize the runoff hydrograph for each storm. In addition, two other methods have been used to estimate peak runoff rates.

The first of these methods was developed by Viessman (1962). For small paved urban areas Viessman developed the following equation for estimating the peak runoff rate:

$$Q = 59.16(n_b/n_s)(D^{0.99}/T^{0.72})(i_2/i_m)^{0.88}A^{0.95}S^{0.17} \qquad (6\text{-}9)$$

where

Q = the peak runoff rate in cfs

n_b = a base value of the Manning coefficient = .013

n_s = value of the Manning coefficient selected for the drainage area

D = the total depth of rainfall in inches during time T

T = the time of the intense part of the storm defined as the time in minutes extending back from the minute of maximum rainfall intensity to the beginning of the first minute when rainfall intensity exceeds one inch per hour

i_2 = the mean intensity in inches per hour for the peak minute plus the minute preceeding it

$i_m = 60D/T$

A = drainage area in acres

S = average gutter slope in ft/ft

Some of the data used to derive this equation were obtained from the three drainage areas used in this study. Three of the twelve storms used in this study were used by Viessman in deriving Equation 6-9.

In addition to the estimates of the peak rate obtained from the 4 methods so far described, the Rational Formula was also used. This was done because this equation is widely used in the design of storm drainage facilities. However, values for the C-Factor and time of concentration were obtained from equations for these parameters developed by Schaake (1964).

It should be understood that the use of the Rational Formula in this study is considerably different from the use of the Rational Formula in design practice. For design, a frequency of recurrence is associated with the computed peak runoff rate and is assumed to be the same as the frequency of recurrence of the rainfall intensity obtained from a rainfall intensity-time-frequency chart. Here, the rainfall intensity is found as a maximum average rainfall intensity during a storm. Further, the computed peak runoff rate is here compared with the peak runoff rate measured for the particular storm and is not compared with the peak runoff rate expected to occur with the same frequency as the rainfall intensity.

The equations used to estimate the C-Factor and to determine the averaging time, t_a, are

$$C = 0.14 + .65\ Imp + .05\ S \tag{6-10}$$

$$t_a = \frac{0.68 \quad L^{0.27}}{S^{0.13} Imp^{0.38}} \tag{6-11}$$

where

Imp = ratio of paved area to the total area

S = average slope of the main drainage channel in ft/ft

L = length of the main drainage channel in feet

These equations were derived from data collected from 20 gaged drainage areas including the three areas now being studied. The equation used here to estimate t_a originally was derived to estimate the average time lag between the centroid of the rainfall hyetograph and the centroid of the runoff hydrograph.

Estimates of the Peak Runoff Rate

The various estimates of the peak runoff rates for the 12 storms are given in Table 6-1. The per cent difference between the measured and computed values are also given in this

Table 6-1. Comparison of Methods for Estimating Peak Runoff Rates

Storm	Measured Peak (cfs/Acre)	Solution of the Equations of Flow (cfs/Ac)	Solution of the Equations of Flow (% Diff.)	Rising Unit Hydrograph (cfs/Ac)	Rising Unit Hydrograph (% Diff.)	Recession Unit Hydrograph (cfs/Ac)	Recession Unit Hydrograph (% Diff.)	Viessman Formula (cfs/Ac)	Viessman Formula (% Diff.)	Rational Formula (cfs/Ac)	Rational Formula (% Diff.)
15N9	3.87	3.90	+ 0.8	3.90	+ 0.8	3.49	- 9.8	3.31	-14.5	3.87	0.0
19N9	7.85	7.20	- 8.3	6.76	-13.9	6.01	-23.5	6.48	-17.5	6.59	-16.1
23N9	3.87	3.88	+ 0.3	3.74	- 3.4	3.55	- 8.3	4.11	+ 6.2	3.52	- 9.1
25N9	3.65	3.36	- 8.0	3.60	- 1.4	3.03	-17.0	3.44	- 5.8	3.55	- 2.7
47N9	4.67	4.68	+ 0.2	4.47	- 4.3	3.84	-17.8	4.50	- 3.6	3.68	-21.2
8N12	5.77	5.67	- 1.7	5.67	- 1.7	5.58	- 3.3	5.12	-11.3	5.02	-13.0
15N12	2.56	2.78	+ 8.6	3.13	+22.2	2.91	+13.7	2.54	- 0.8	2.95	+15.3
19N12	5.41	5.42	+ 0.2	5.23	- 1.9	5.28	- 2.4	4.58	-15.4	5.12	- 5.4
3SPL1	3.10	3.21	+ 3.6	3.09	- 0.3	2.82	- 9.0	2.34	-24.5	2.51	-19.0
6SPL1	2.43	3.13	+28.9	3.13	+28.9	2.83	+16.5	2.19	- 9.9	2.71	+11.5
13SPL1	5.75	6.11	+ 6.3	5.50	- 4.4	5.03	-12.5	4.57	-20.5	4.47	-22.3
18SPL1	2.48	2.38	- 4.0	2.37	- 4.4	2.17	-12.5	1.23	-50.3	2.07	-16.6

table. Statistical distributions of the differences between measured and computed values are shown in Figure 6-14.

The curves in Figure 6-14 show that the best estimates of the peak runoff rate were obtained from the solutions to the equations of flow. In fact only one of the 12 estimates deviated from the measured rate by more than 10 per cent. This was for a small storm on area SPL1 and the peak occurred during the first few minutes of the storm. For more than half of the storms the difference was less than 5 per cent for this method.

The unit hydrograph method gave surprising results. Estimates based on the rising unit hydrograph agreed better with the measured rates than did estimates based on the recession unit hydrograph. Apparently, it would have been better to use a more intense rainfall than 3 in/hr in obtaining the recession unit hydrograph. This would have improved these estimates considerably. On the other hand, the two curves in Figure 6-14 for the unit hydrograph method illustrates that the unit hydrograph may not give results that are better than those by, say, the Rational Formula; but, the unit hydrograph method could give results about as good as obtained from the solution to the equations of flow.

Both the Viessman Formula and the Rational Formula gave results that were about equally good. In each case,

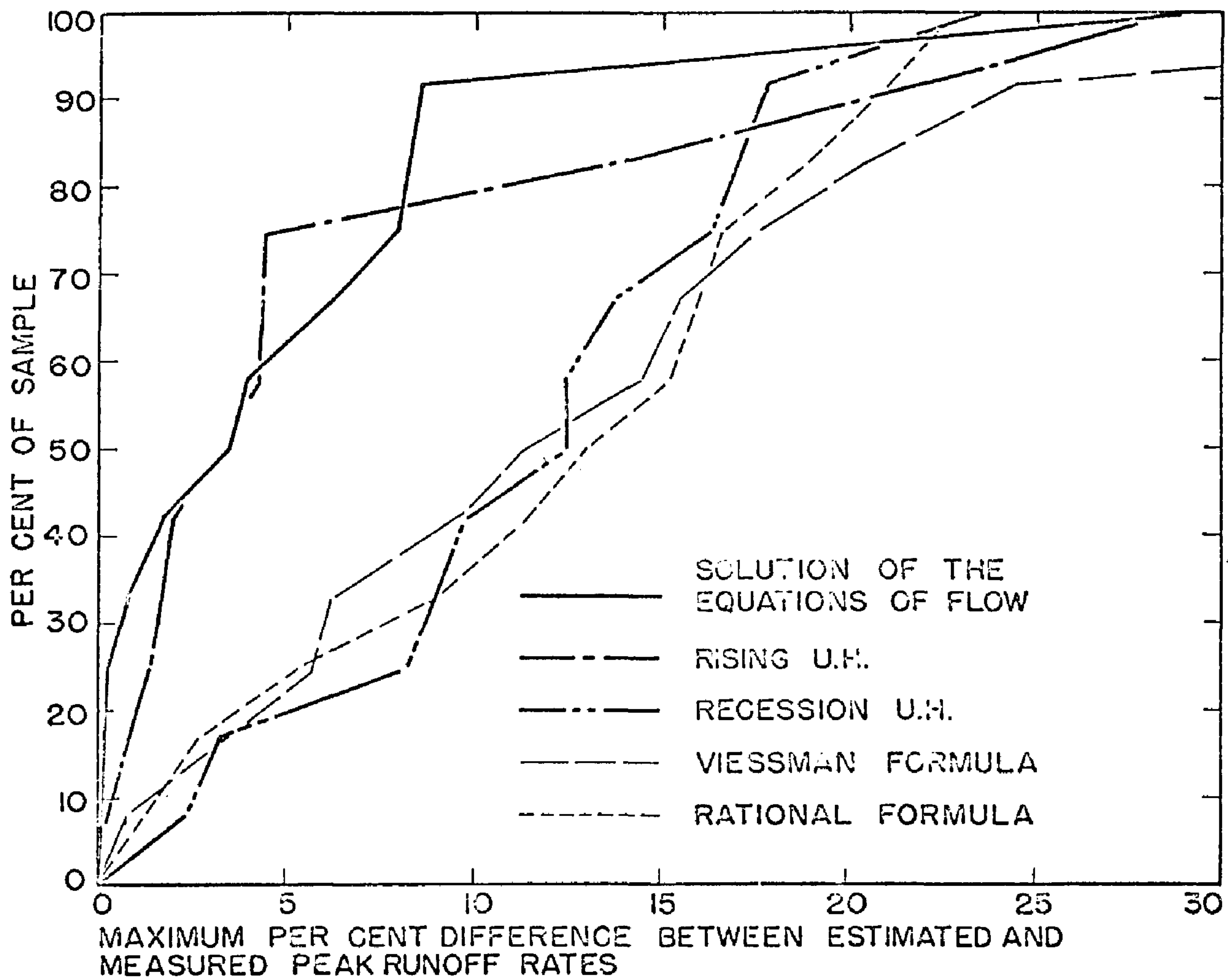

Figure 6-14 Difference between Measured and Computed Peak Runoff Rates

about 80 per cent of these estimates differed from the measured values by less than 20 per cent. It must be emphasized that the Rational Method estimates are based on coefficients obtained from equations derived from observations of rainfall and run-off on 20 urban areas, and the areas studied here were included in these 20 areas. Furthermore, in a recent study (Schaake, Geyer, and Knapp; 1964) C-Factors and concentration times were estimated by engineers using their usual design procedures when applying the Rational Method. The results of that study showed better results were obtained by using Equations 6-10 and 6-11 than were obtained by the engineers' estimates. Therefore, the Rational Method, as presently used in design practice, may not give estimates as good as those obtained here.

Chapter 7

SUMMARY AND CONCLUSIONS

Billions of dollars will be invested during the next few years in storm drainage facilities. If existing design procedures are used to determine the sizes of facilities required, millions of these dollars will be used for facilities larger or smaller than justified by their need. To develop improved design procedures, more accurate estimates of the storm runoff resulting from rainfall are needed.

In this study a method of synthesizing the hydrograph of storm runoff from rainfall has been presented. Only storms on paved drainage areas have thus far been synthesized because improved procedures for estimating infiltration rates on pervious urban areas remain to be developed. Also, friction coefficients for overland flow over turfed areas need to be obtained from hydraulic studies of overland flow.

Using the equations of gradually varied unsteady flow in open channels to describe the mechanics of surface runoff, excellent agreement has been obtained between measured runoff hydrographs from paved drainage areas and computed runoff hydrographs. Data from selected overland flow tests by Izzard in 1942-1943 and data for three paved inlet areas gaged by the Hopkins Storm Drainage Research Project were analyzed.

For each of the gaged drainage areas a synthetic unit hydrograph also was developed from the equations of flow. When synthetic unit hydrographs were used to compute the runoff hydrograph for each of the storms studied, the synthetic unit hydrograph was found to approximate very well the solution to the equations of flow. The runoff hydrographs computed with the synthetic unit hydrographs also agreed well with the measured runoff hydrographs.

Several methods for estimating the peak runoff rate were also studied. All but one estimate given by the equations of flow deviated by less than 10 per cent from the measured peak rate.

APPENDIX A - NOTATION

Symbol		Description
A	=	Area of flow (sq.ft.)
B	=	Top width of flow (ft)
C	=	fR for laminar flow
f	=	Darcy-Weisbach friction coefficient
g	=	Acceleration of gravity (ft/sec^2)
H	=	Depth of flow (ft)
i	=	Rainfall intensity (in/hr)
K	=	$\frac{2g S_o i^3}{\nu L (43200)^2}$
L	=	Length of overland flow (ft)
n	=	Friction coefficient in Manning's equation
q	=	Rate of lateral inflow (S.F./sec)
Q	=	Runoff rate
Q^*	=	Q/K
r	=	Hydraulic radius (ft)
R	=	Reynold's number = VH/ν
S_f	=	Friction slope or slope of the energy gradient for steady flow (ft/ft)
S_o	=	Channel slope (ft/ft)
t	=	Time

Symbol		Description
v	=	Velocity of a fluid element (ft/sec)
V	=	Average velocity (ft/sec)
x	=	Distance along a channel (ft)
λ	=	$V+\sqrt{gA/B}$, $V-\sqrt{gA/B}$ = direction of characteristic curves in the x-t plane (ft/sec)
μ	=	Dynamic viscosity (lb sec/ft^2)
ν	=	Kinematic viscosity = μ/ρ, (ft^2/sec), @ 50°F, $\nu = 1.41 \times 10^{-5}$
ρ	=	Density (slugs/ft^3)
τ	=	Viscous shear stress (lb/ft^2)

BIBLIOGRAPHY

Chow, Ven T., 1959, Open Channel Hydraulics, McGraw-Hill, New York.

Clark, C. O., 1945, Storage and the Unit Hydrograph, Transactions, ASCE, Vol. 110, pp. 1419-1488.

Dooge, James C. I., 1959, A General Theory of the Unit Hydrograph, Journal of Geophysical Research, Vol. 64, No. 2, pp. 241-256.

Dronkers, J. J., 1964, Tidal Computations, John Wiley and Sons, New York.

Fair, Gordon M. and John C. Geyer, 1959, Water Supply and Waste Water Disposal, John Wiley and Sons, New York.

Forsythe, George E. and Wolfgang R. Wasow, 1964, Finite Difference Methods for Partial Differential Equations, John Wiley and Sons, New York.

Horner, W. W. and S. W. Jens, 1942, Surface Runoff Determination from Rainfall without Using Coefficients, Transactions, ASCE, Vol. 107.

Horton, Robert E., 1945, Erosional Development of Streams and Their Drainage Basins; Hydrophysical Approach to Quantitative Morphology, Bulletin of the Geological Society of America, Vol. 56, pp. 275-370.

Izzard, Carl F., 1942-43, Overland Flow Test Data, On file at the Engineering Societies Library, New York.

Izzard, Carl F., 1944, The Surface Profile of Overland Flow, Transactions, AGU, Part VI, pp. 959-968.

Knapp, John W., 1965, An Economic Study of Urban and Highway Drainage Systems, Technical Report No. 2, The Storm Drainage Research Project, The Johns Hopkins University, Baltimore.

Kuichling, Emil, 1889, The Relation between the Rainfall and the Discharges of Sewers in Populous Districts, Transactions, ASCE, Vol. 20, pp. 1-60.

Linsley, Ray K., Max A Kohler, and Joseph T. Paulhaus, 1958, Hydrology for Engineers, McGraw-Hill, New York.

Nash, J. E., 1959, Systematic Determination of Unit Hydrograph Parameters, Journal of Geophysical Research, Vol. 64, No. 1, pp. 111-115.

Rouse, Hunter, 1950, Engineering Hydraulics, John Wiley and Sons, New York.

Schaake, John C., Jr., John C. Geyer, and John W. Knapp, 1964, Technical Report No. 1, The Storm Drainage Research Project, The Johns Hopkins University, Baltimore.

Stoker, J. J., 1957, Water Waves, Interscience Publishers, Inc., New York.

Tholin, A. L. and Clint J. Keifer, 1959, The Hydrology of Urban Runoff, Journal of the Sanitary Engineering Division, Proceedings, ASCE, Vol. 85, No. SA 2.

Woo, Dah-Cheng, and Ernest F. Brater, 1962, Spacially Varied Flow from Controlled Rainfall, Journal of the Hydraulics Division, Proceedings, ASCE, Vol. 88, No. HY 6, pp. 31-56.

Yevdjevich, Vujica M., 1961, Unsteady Free Surface Flow in a Storm Drain, Report to the U. S. Bureau of Public Roads, Colorado State University, Fort Collins.

Yu, Y. S. and John S. McKnown, 1964, Runoff from Impervious Surfaces, Journal of Hydraulic Research, Vol. 2, No. 1, pp. 3-24.

VITA

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